Sunlight and its Properties II

EE 446/646

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There are two adjustments that need to be made in order to convert ST to CT:

- **The first is the Longitude adjustment** (the world is divided into 24 1-hour time zones, each spanning 15° of longitude)
  
  - All clocks within a time zone are set to the same time
  - Each time zone is defined by a local time meridian located ideally in the middle of the zone (with the origin of this time system passing through Greenwich, UK at 0° longitude)
  - The longitude correction between local clock time and solar time is the time it takes for the sun to travel between the local time meridian and the observer’s line of longitude (4 minutes/degree)

  - Example 1: San Francisco, CA (longitude 122°) will have solar noon 8 minutes after the sun crosses the 120° local time meridian.
  - Example 2: Las Vegas, NV (longitude 115°) will have solar noon 20 minutes before the sun crosses the 120° local time meridian.
  - Example 3: Fresno, CA (longitude 120°) will have solar noon when the sun crosses the 120° local time meridian.
Time Zones
Day vs. Night

- 12:00 (noon)
- 00:00 (midnight)
- 03:00
- 06:00
- 09:00
- 15:00
- 18:00

International Date Line

- Monday
- Tuesday
- Night/Day
Solar Time (ST) and Civil (clock) Time (CT)

- There are two adjustments that need to be made to ST in order to convert to CT:
  - **The second is a fudge factor** that takes into account the uneven way in which the earth moves around the sun
    - This is a result of the earth’s elliptical orbit which causes the length of a solar day (solar noon to solar noon) to vary throughout the year.
    - The difference between a 24-hour day and a solar day changes according to the following *Equation of Time* (where E is in minutes):

\[
E = 9.87 \sin(2B) - 7.53 \cos B - 1.5 \sin B
\]

where

\[
B = \frac{360}{364} (n - 81)
\]
Equation of Time and Analemma

- **Equation of Time**: Plot of the difference between clock time and solar time at solar noon over a 1-year period.
- **Analemma**: Plot the position of the sun declination angle as a function of equation of time.
Final Relationship between ST and CT

\[ ST = CT + 4 \times (\text{Local Time Meridian} - \text{Local Longitude}) + E \]

• Finally, when daylight savings Time is in effect (34 weeks between early March and early November), one hour must be added to the local CT (except in AZ)

• **Example:** Find Eastern Daylight Time for solar noon (ST = 12:00) in Boston (longitude: 71.1° west) on July 1st. Answer:
  - \( n = 182 \)
  - \( B = 99.89° \)
  - \( E = -3.5 \text{ min} \)
  - The Local Time Meridian: 75°
  - Longitude adjustment: \( 4(75-71.1)=15.6 \text{ min} \)
  - \( CT = 12:00 - 15.6 - (-3.5) = 12:00 - 12.1 = 11:48 \text{ am (EST)} \)
  - Add daylight savings: solar noon will occur at 12:48 pm (EDT)
Countries Practicing Daylight Savings Time
Sunrise and Sunset Revisited

• The sun’s hour angle at sunrise can be determined by setting the altitude angle to zero:

\[ H = \cos^{-1}\left\{ -\tan \delta \tan L \right\} \]

• This hour angle is the converted to solar time of sunrise

\[ ST = 12 - \left( \frac{H}{15} \right) \]

• Finally, ST is converted to CT:

\[ CT = ST - 4x(\text{Local Time Meridian} – \text{Local Longitude}) - \text{E} \]

• Sunrise and sunset times are different due to two factors:
  1) Atmospheric refraction – this bends the sun rays and make them appear to rise 2.4 min earlier (for sunrise) and later (for sunset).
  2) The weather service defines sunrise and sunset when the top of the sun first appears at the horizon (while ours is based on the sun’s center crossing the horizon).
Sunrise and Sunset Revisited

Sunrise / sunset occurs when sun's upper limb appears tangent to horizon.

Sun's apparent position (due to atmospheric refraction):
- 16° angular radius

Sun's actual position:
- 34° atmospheric refraction
- 50’ (0.83°) sun's actual angle below horizon at sunrise / sunset

Sunrise and Sunset Revisited
Sunrise Revisited

- Adjustment factor that should be subtracted for sunrise

\[
Q = \frac{3.467}{\cos L \cos \delta \sin H_{SR}} \quad (\text{min})
\]

- Finally, \( CT = ST - 4x(LTM - \text{Local Longitude}) - E - Q \)
- \( Q \) should be added when calculating sunset.
Sunrise and Sunset Revisited

• Example: Find the time at which sunrise will occur in Boston (longitude: 71.1° west) on July 1st:

• Answer:
  – Declination angle: $\delta = 23.1$ deg
  – Longitude: $L = 71.1$ deg
  – Hour angle at sunrise: $H = 112.86$ deg
  – Solar Time: $ST = 4:28.6$ am
  – Equation of Time: $E = -3.5$ min
  – The Local Time Meridian: 75°
  – Longitude adjustment: $4(75-71.1)=15.6$ min
  – Adjustment factor: $Q = 5.5$ min
  – $CT = 4:28.6 – 15.6\text{min} – (-3.5\text{min}) - 5.5\text{min} = 4:11$ am (EST)
  – Add daylight savings: sunrise will occur at 5:11 am (EDT)
  – Sunset will occur at 8:25pm
Decomposition of solar flux striking a collector

- **Direct-beam radiation** is sunlight that reaches the earth’s surface without scattering.
- **Diffuse radiation**: scattered sunlight (i.e., what makes the sky blue) responsible for the light entering the north-facing windows.
- **Reflected radiation (albedo)**: sunlight that is reflected from the ground and other nearby structures.
- **Global radiation**: the sum of the all three components above.
**Direct beam radiation**

- Day-to-day variation of extra-terrestrial solar insolation (due to change in sun-to-earth distance).

\[
I_0 = SC \cdot \left[ 1 + 0.034 \cos \left( \frac{360n}{365} \right) \right] \quad \text{(W/m}^2) \]

where the solar constant \( SC = 1.377 \text{ kW/m}^2 \).

- The attenuation of incoming radiation is a function of distance travelled through the atmosphere. An exponential model is often used for this

\[
I_B = Ae^{-km}
\]

where
- \( I_B \): beam portion reaching the earth surface
- \( A \): apparent extra-terrestrial flux
- \( k \): optical depth
- \( m \): air mass ratio (defined earlier)
Close fit of optical depth and apparent extra-terrestrial flux

\[ A = 1160 + 75 \sin \left( \frac{360}{365} (n - 275) \right) \quad \text{(W/m}^2) \]

\[ k = 0.174 + 0.035 \sin \left( \frac{360}{365} (n - 100) \right) \]

**Air mass ratio:**
approximate expression: \( m \approx 1/\sin \beta \)

Accurate expression: \( m = \sqrt{(708 \sin \beta)^2 + 1417} - 708 \sin \beta \)

**TABLE 7.6** Optical Depth \( k \), Apparent Extraterrestrial Flux \( A \), and the Sky Diffuse Factor \( C \) for the 21\(^{st} \) Day of Each Month

<table>
<thead>
<tr>
<th>Month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>June</th>
<th>July</th>
<th>Aug</th>
<th>Sept</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A ) (W/m(^2 ))</td>
<td>1230</td>
<td>1215</td>
<td>1186</td>
<td>1136</td>
<td>1104</td>
<td>1088</td>
<td>1085</td>
<td>1107</td>
<td>1151</td>
<td>1192</td>
<td>1221</td>
<td>1233</td>
</tr>
<tr>
<td>( k )</td>
<td>0.142</td>
<td>0.144</td>
<td>0.156</td>
<td>0.180</td>
<td>0.196</td>
<td>0.205</td>
<td>0.207</td>
<td>0.201</td>
<td>0.177</td>
<td>0.160</td>
<td>0.149</td>
<td>0.142</td>
</tr>
<tr>
<td>( C )</td>
<td>0.058</td>
<td>0.060</td>
<td>0.071</td>
<td>0.097</td>
<td>0.121</td>
<td>0.134</td>
<td>0.136</td>
<td>0.122</td>
<td>0.092</td>
<td>0.073</td>
<td>0.063</td>
<td>0.057</td>
</tr>
</tbody>
</table>

Example

- Determine the direct-beam radiation at solar noon in Las Vegas (L = 36 deg.) on January 21.

- Answer:
  - $A = 1,230 \text{ W/m}^2$
  - $K = 0.142$
  - Declination Angle $\delta = -20.1 \text{ deg}$
  - Altitude angle at solar noon $\beta = 33.9 \text{ deg}$
  - air mass ratio $m = 1.793$
  - Clear sky beam radiation: $I_B = 953 \text{ W/m}^2$. 
Beam striking a collector surface

- **Incidence angle** $\theta$: angle between a normal to the collector face and the incoming solar beam.

  $$I_{BC} = I_B \cos \theta$$

- The incidence angle depends on the sun angles (altitude and azimuth) and collector angles (azimuth and tilt)

  $$\cos \theta = \cos \beta \cos(\phi_S - \phi_C) \sin \Sigma + \sin \beta \cos \Sigma$$

- Special case of horizontal surface:

  $$I_{BH} = I_B \cos(90^\circ - \beta) = I_B \sin \beta$$
Example

• Determine the beam radiation at solar noon in Las Vegas (L = 36 deg.) on January 21, on a collector that faces 25 towards the southwest and tilted by 15 degrees.

• Answer (refer to previous example)
  – Altitude angle at solar noon $\beta = 33.9^\circ$
  – Clear sky beam radiation: $I_B = 953 \, \text{W/m}^2$.
  – Sun azimuth = $0^\circ$
  – Collector azimuth = $-25^\circ$
  – Collector tilt = $15^\circ$
  – $\cos\theta = \cos(33.9) \times \cos(0-(-25)) \times \sin(15) + \sin(33.9) \times \cos(15) = .733$
  – Beam radiation on the collector = $953 \times .733 = 684 \, \text{W/m}^2$. 
Diffuse Radiation

- This is difficult to estimate accurately.
- Model that is often used: diffuse radiation on a horizontal surface is proportional to the direct beam radiation all day.

\[ I_{DH} = C \times I_B \]

where \( C \) is the sky diffuse factor that varies daily as follows:

\[ C = 0.095 + 0.04 \sin \left( \frac{360}{365} (n - 100) \right) \]

- The diffuse light received by a collector that is tilted at an angle is approximated by

\[ I_{DC} = I_{DH} \left( \frac{1 + \cos \Sigma}{2} \right) = C \times I_B \left( \frac{1 + \cos \Sigma}{2} \right) \]
Reflected Radiation

• This is also hard to estimate accurately.
• Estimates of ground reflectance $\rho$ range from 0.8 for fresh snow to 0.1 for gravel. Ordinary ground or grass assume a reflectance of 0.2.

• The reflected radiation received by a collector can be estimated by

$$I_{RC} = \rho (I_{BH} + I_{DH}) \left( \frac{1 - \cos \Sigma}{2} \right)$$

or

$$I_{RC} = \rho I_B (\sin \beta + C) \left( \frac{1 - \cos \Sigma}{2} \right)$$
Global radiation is the sum of the three components (direct radiation, diffuse radiation, and reflected radiation):

\[ I_C = I_{BC} + I_{DC} + I_{RC} \]

\[ I_C = A e^{-km} \left[ \cos \beta \cos(\phi_S - \phi_C) \sin \Sigma + \sin \beta \cos \Sigma + C \left( \frac{1 + \cos \Sigma}{2} \right) \right] + \rho (\sin \beta + C) \left( \frac{1 - \cos \Sigma}{2} \right) \]
So far we looked at solar irradiance striking a fixed collector. Now we consider solar irradiance striking tracking collectors.
Two-Axis Tracking:

\[ I_{BC} = I_B \]

\[ I_{DC} = C I_B \left[ \frac{1 + \cos(90^\circ - \beta)}{2} \right] \]

\[ I_{RC} = \rho (I_{BH} + I_{DH}) \left[ \frac{1 - \cos(90^\circ - \beta)}{2} \right] \]
4 Types of Single-Axis Tracking: Rotation about a polar axis (i.e., polar mount: $\Sigma = L$) is most popular.

One-Axis, Polar Mount:

\[
I_{BC} = I_B \cos \delta \\
I_{DC} = CI_B \left[ \frac{1 + \cos(90^\circ - \beta + \delta)}{2} \right] \\
I_{RC} = \rho(I_{BH} + I_{DH}) \left[ \frac{1 - \cos(90^\circ - \beta + \delta)}{2} \right]
\]
Example

• Calculate the clear-sky insolation on both a two-axis tracking mount and single-axis polar mount (located at 40° latitude) at solar noon on the summer solstice. Ignore ground reflectance.

• Answer:
  – Declination angle $\delta = 23.45^\circ$
  – Sun altitude angle at solar noon $\beta = 73.45^\circ$
  – Air Mass ratio $m = 1.043$
  – From Table 7.6, $A = 1088$ (W/m$^2$) and $k = 0.205$

<table>
<thead>
<tr>
<th></th>
<th>Two-Axis Tracker</th>
<th>Single-Axis Pole Mount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct beam rad. (W/m$^2$)</td>
<td>879</td>
<td>806</td>
</tr>
<tr>
<td>Diffuse rad. (W/m$^2$)</td>
<td>115</td>
<td>104</td>
</tr>
<tr>
<td>Total radiation (W/m$^2$)</td>
<td>994</td>
<td>910</td>
</tr>
</tbody>
</table>