EE 741
Power Transformers
Overview

• Two-winding transformer
• Autotransformers
• Three-phase transformer
  – Delta-Y-grounded
  – Ungrounded-Y-Delta
  – Y-grounded-Y-Grounded
  – Delta-Delta
  – Open-Y-Open-Delta
Two-winding transformer

- Exact circuit

- Approximate ckt.

\[ Z_t = n_t^2 \cdot Z_1 + Z_2 \]

\[ n_t = \frac{N_2}{N_1} \]

\[ V_S = a \cdot V_L + b \cdot I_2 \]

\[ I_S = c \cdot V_L + d \cdot I_2 \]

\[ a = \frac{1}{n_t} \]

\[ b = \frac{Z_t}{n_t} \]

\[ c = \frac{Y_m}{n_t} \]

\[ d = \frac{Y_m \cdot Z_t}{n_t} + n_t \]
Autotransformer

Step-up

Series winding

Step-down

Shunt winding

(+): sign for step-up, and (-): sign for step-down

\[ V_S = a \cdot V_L + b \cdot I_L \]

\[ I_S = c \cdot V_L + d \cdot I_2 \]

\[ a = \frac{1}{1 \pm n_t} \]

\[ b = \frac{Z_t}{1 \pm n_t} \]

\[ c = \frac{Y_m}{1 \pm n_t} \]

\[ d = \frac{Y_m \cdot Z_t}{1 \pm n_t} \]
3-phase transformer

- Generalized matrices:
  \[
  [V_{LN_{ABC}}] = [a_t] \cdot [V_{LN_{abc}}] + [b_t] \cdot [I_{abc}]
  \]
  \[
  [I_{ABC}] = [c_t] \cdot [V_{LN_{abc}}] + [d_t] \cdot [I_{abc}]
  \]

- Phase shift in Y-Delta connection (American Standard),

  **Step-Up Connection**

  \( V_{ab} \) leads \( V_{AB} \) by 30 degrees
  \( I_a \) leads \( I_A \) by 30 degrees
Delta – Y-grounded step-down connection
Delta – Y-grounded step-down connection

- Matrices
  (ignore shunt admittance)

\[
[a_t] = \frac{-n_t}{3} \cdot \begin{bmatrix}
0 & 2 & 1 \\
1 & 0 & 2 \\
2 & 1 & 0
\end{bmatrix}
\]

\[
b_t = \frac{-n_t}{3} \cdot \begin{bmatrix}
0 & 2 \cdot Z_{t_b} & Z_{t_c} \\
Z_{t_a} & 0 & 2 \cdot Z_{t_c} \\
2 \cdot Z_{t_a} & Z_{t_b} & 0
\end{bmatrix}
\]

\[
d_t = \frac{1}{n_t} \cdot \begin{bmatrix}
1 & -1 & 0 \\
0 & 1 & -1 \\
-1 & 0 & 1
\end{bmatrix}
\]

\[
c_t = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]
Ungrounded-Y – Delta step-down connection
Ungrounded-Y – Delta step-down connection

- Matrices

\[
[a_t] = n_t \cdot \begin{bmatrix}
1 & -1 & 0 \\
0 & 1 & -1 \\
-1 & 0 & 1
\end{bmatrix}
\]

\[
[b_t] = \frac{n_t}{3} \cdot \begin{bmatrix}
Zt_{ab} & -Zt_{ab} & 0 \\
Zt_{bc} & 2 \cdot Zt_{bc} & 0 \\
-2 \cdot Zt_{ca} & -Zt_{ca} & 0
\end{bmatrix}
\]

\[
c_t = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
d_t = \frac{1}{3 \cdot n_T} \cdot \begin{bmatrix}
1 & -1 & 0 \\
1 & 2 & 0 \\
-2 & -1 & 0
\end{bmatrix}
\]
Example 1

• Consider the following:
  – The load be unbalanced with Sab = 100 kVA @ .9 PF lag, and Sbc = Sca = 50 kVA @ 0.8 PF lag,
  – The voltage at the load is balanced at 240 V (line-to-line)
  – Transformer across a-b is rated at 100 kVA with Ztab = .01 +j.4 pu
  – Transformers across b-c and c-a are rated at 50 kVA with Ztbc = Ztca = .015 +j.435 pu

• Compute a) the secondary line currents, (b) the primary line currents, c) the primary phase and line voltages, the kVA loading on each transformer.
Grounded-Y – grounded-Y step-down connection
Grounded-Y – grounded-Y step-down connection

• Matrices

\[
[a_t] = \begin{bmatrix}
    n_t & 0 & 0 \\
    0 & n_t & 0 \\
    0 & 0 & n_t \\
\end{bmatrix}
\]

\[
[b_t] = \begin{bmatrix}
    n_t \cdot Zt_a & 0 & 0 \\
    0 & n_t \cdot Zt_b & 0 \\
    0 & 0 & n_t \cdot Zt_c \\
\end{bmatrix}
\]

\[
[d_t] = \begin{bmatrix}
    \frac{1}{n_t} & 0 & 0 \\
    0 & \frac{1}{n_t} & 0 \\
    0 & 0 & \frac{1}{n_t} \\
\end{bmatrix}
\]

\[
[c_t] = \begin{bmatrix}
    0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
Delta-Delta Connection

No phase shift
Delta-Delta Connection

- Matrices

\[
[b_t] = \frac{1}{Z_{t_{ab}} + Z_{t_{bc}} + Z_{t_{ca}}} \cdot \begin{bmatrix}
Z_{t_{ca}} & -Z_{t_{bc}} & 0 \\
Z_{t_{ca}} & Z_{t_{ab}} + Z_{t_{ca}} & 0 \\
-Z_{t_{ab}} - Z_{t_{bc}} & -Z_{t_{bc}} & 0
\end{bmatrix} \begin{bmatrix}
n_t \cdot Z_{t_{a}} & 0 & 0 \\
0 & n_t \cdot Z_{t_{b}} & 0 \\
0 & 0 & n_t \cdot Z_{t_{c}}
\end{bmatrix}
\]

\[
[d_t] = \begin{bmatrix}
\frac{1}{n_t} & 0 & 0 \\
0 & \frac{1}{n_t} & 0 \\
0 & 0 & \frac{1}{n_t}
\end{bmatrix}
\]

\[
[c_t] = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]
Open-Y – open Delta connection
Open-Y – open Delta connection

• Matrices

\[
[a_t] = \begin{bmatrix}
  n_t & -n_t & 0 \\
  0 & n_t & -n_t \\
  0 & 0 & 0
\end{bmatrix}
\]

\[
[b_t] = \begin{bmatrix}
  n_t \cdot Zt_{ab} & 0 & 0 \\
  0 & 0 & -n_t \cdot Zt_{bc} \\
  0 & 0 & 0
\end{bmatrix}
\]

\[
[d_t] = \begin{bmatrix}
  \frac{1}{n_t} & 0 & 0 \\
  0 & 0 & -\frac{1}{n_t} \\
  0 & 0 & 0
\end{bmatrix}
\]

\[
[c_t] = \begin{bmatrix}
  0 & 0 & 0 \\
  0 & 0 & 0 \\
  0 & 0 & 0
\end{bmatrix}
\]
Example # 2

• Repeat Example 1 by using only phases A and B (i.e., removing the transformer whose secondary is connected between phases a-c). As in Example 1, assume the voltage is balanced at the load