EE 742
Chap 8: Voltage Stability

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Loadability of a simple network

• What are the possible solutions for the network below? What are the limits (if any)?

• Static power-voltage equations:

\[
P_L(V) = VI \cos \varphi = V \frac{IX \cos \varphi}{X} = \frac{EV}{X} \sin \delta
\]  \hspace{1cm} (1)

\[
Q_L(V) = VI \sin \varphi = V \frac{IX \sin \varphi}{X} = \frac{EV}{X} \cos \delta - \frac{V^2}{X}
\]  \hspace{1cm} (2)

\[
\left( \frac{EV}{X} \right)^2 = [P_L(V)]^2 + [Q_L(V) + \frac{V^2}{X}]^2
\]  \hspace{1cm} (3)
Case of ideally stiff load

\[ P_L(V) = P_n \quad \text{and} \quad Q_L(V) = Q_n, \]

- Substitution in (3) yields real power in terms of power factor angle:

\[ p = -v^2 \sin \varphi \cos \varphi + v \cos \varphi \sqrt{1 - v^2 \cos^2 \varphi}, \]

- Where

\[ v = \frac{V}{E}, \quad p = \frac{P_n}{E^2 X}. \]

- The nose curves \( V(P) \) below show the voltage dependency on real power
Case of ideally stiff load

\[ P_L(V) = P_n \quad \text{and} \quad Q_L(V) = Q_n, \]

- Different values of voltage V correspond to different circles in the \((P_n, Q_n)\) plane. The analytical expression of the envelope which encloses all solutions of the network equation (3) is given by:
  - Inside envelope: two solutions
  - On the envelope: one solution
  - Outside the envelope: no solution

\[ Q_n = \frac{E^2}{4X} - \frac{P_n^2}{E^2 / X}, \]
Influence of load characteristics

- Note the infinite solution space for the case where both $P$ and $Q$ vary quadratically with voltage. Herein,

$$P_L(V) = P_n \left( \frac{V}{V_n} \right)^2 = \frac{P_n}{V_n^2} V^2 = G_n V^2,$$

$$Q_L(V) = Q_n \left( \frac{V}{V_n} \right)^2 = \frac{Q_n}{V_n^2} V^2 = B_n V^2,$$

- This is the case of a constant admittance (or impedance) load where there is always a solution (obtained by voltage division).
Consider a 500 kV transmission line with total series impedance = 8+j100Ω and total shunt admittance = 0.0008 S. The line is connected to a stiff source that is fixed at the sending end ($V_s = 1$ pu). Assume the load is stiff.

1) Plot the nose curves (i.e., receiving end voltage as a function of load real power) for a) 0.9 power factor (lag), b) unity power factor, and c) 0.9 power factor (lead).

2) Plot the source reactive power $Q_s$ as a function of receiving end voltage for a) $P = 0$ MW, b) $P = 500$ MW, c) $P = 1000$ MW and d) $P = 1500$ MW, e) $2000$ MW.

3) Determine the critical load power factor for each of the specified load in 2) (b-e) above, and the corresponding receiving end voltage.

4) Repeat 2) and 3) above if a switched shunt capacitor bank is placed at the receiving end with the following values: 300 MVAR (for 1000 MW load), 600 MVAR (for 1500 MW load) and 900 MVAR for a load of 2000 MW.
Stability Criteria

• Voltage stability problem: In case of two solutions with respect to the voltage, which one corresponds to a stable equilibrium point? What are the conditions for stability? Several types of criterion are considered.

• \( d\Delta Q/dV \) Criterion (idea of separating the load and source reactive powers)
  – Reactive power expression in terms of voltage and real power from the network equation (3)

\[
Q_s(V) = \sqrt{\left(\frac{EV}{X}\right)^2 - [P_L(V)]^2 - \frac{V^2}{X}}.
\]
Voltage Stability – $d\Delta Q/dV$ criterion

- The left figure below shows how the source reactive power varies with voltage at different levels or real power.
- Now the load reactive power curve can be drawn on the same figure as shown below. The two reactive powers must be equal to each other at equilibrium.
  - Recall that access reactive power tends to raise the voltage (Chap 3).
  - A small voltage disturbance shows that point $s$ is stable while $u$ is unstable.
  - Condition for stability:
    $$\frac{d(Q_S - Q_L)}{dV} < 0 \quad \text{or} \quad \frac{dQ_S}{dV} < \frac{dQ_L}{dV}.$$
Voltage Stability – $dE/dV$ criterion

- In here, the system equivalent emf $E$ is expressed in terms of $V$:

$$E(V) = \sqrt{\left(V + \frac{Q_L(V)X}{V}\right)^2 + \left(\frac{P_L(V)X}{V}\right)^2},$$

- Stability criterion:

$$\frac{dE}{dV} > 0.$$

- Not convenient when using load flow.
Voltage Stability – $dQ_G/dQ_L$ criterion

- The generated reactive power $Q_G$ includes the network reactive power demand ($I^2X$):

$$Q_L(V) = -\frac{Q_G^2(V)}{E^2/X} + Q_G(V) - \frac{P_L^2(V)}{E^2/X}.$$

- For constant real power, the above equation is a parabola (see fig.)
- Point $s$ is a stable equilibrium point (as generation follows load), while point $u$ is an unstable point.
- Stability criterion:

$$\frac{dQ_G}{dQ_L} > 0.$$

- Convenient with power flow.
Critical load demand and voltage collapse

- The figure below shows a stable state (left), a critical state (center) and a state that does not have an equilibrium point (left).
- An increase in load (both P & Q), lowers the curve $Q_S$ and raises curve $Q_L$. This brings the stable equilibrium point $s$ closer to the critical point $u$. Increasing the load beyond this point results in voltage collapse (where the reactive power demand is larger than the supply).
Avoiding voltage instability

• It is important to stay away from the critical value of the voltage as far as possible. However, such a point is difficult to determine.

• An iterative formula can be used under the following assumptions:

  (a) The power factor is maintained constant

      \[ \frac{P_n(t)}{P_0} = \frac{Q_n(t)}{Q_0} = \xi, \]

  (b) The active and reactive powers vary with the voltage as follows:

      \[ \frac{Q_L}{Q_n} = a_2 \left( \frac{V}{V_n} \right)^2 - a_1 \left( \frac{V}{V_n} \right) + a_0, \quad \frac{P_L}{P_n} = b_1 \left( \frac{V}{V_n} \right), \]

• Then the critical voltage and critical load can be found iteratively by

      \[ \xi_{cr} = \frac{1}{\frac{\alpha_0 X}{V_{cr}^2} - \alpha_2 X}. \]

      \[ V_{cr} = \sqrt{\left( \frac{E}{\beta_1 X} \right)^2 - \xi_{cr}^2 + \frac{\alpha_1}{\beta_1} \xi_{cr}}. \]

• See Example 1 (pp. 313)
Effect of line outage and load characteristic

• Recall that the reactive power curve depends on the system impedance and real power characteristic.
• A network outage may cause voltage stability problems (see Example 2)
• The variation of real power with voltage may improve voltage stability (see Example 3)
• Constant impedance loads are always stable.
Sample of measured P(V) and Q(V) curves on a distribution feeder
Voltage stability assessment through load flow

- Voltage stability of a network with multiple generators and many composite loads is more complicated.
- Recall the power flow equations (Chap. 3)

\[
P_i = V_i^2 Y_{ii} \cos \theta_{ii} + \sum_{j=1; j \neq i}^{N} V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij})
\]

\[
Q_i = -V_i^2 Y_{ii} \sin \theta_{ii} + \sum_{j=1; j \neq i}^{N} V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij}).
\]

- Linearization results in

\[
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} =
\begin{bmatrix}
H & M \\
N & K
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta V
\end{bmatrix}
\]

- Critical demand occurs when
- \( \rightarrow \) monitor the determinant of the Jacobian matrix when running numerous load flows with increased loading.
Static Analysis – Voltage stability indices

- A number of voltage stability indices can be used:
- One is based on the classical dQ/dV criterion (hard to determine)
  \[ k_V = \frac{V_s - V_x}{V_s}, \quad \text{where} \quad \frac{d(Q_s - Q_L)}{dV} \bigg|_{V=V_x} = 0. \]
- Another proximity index is (moderately convenient to use)
  \[ k_{\Delta Q} = \frac{d(Q_s - Q_L)}{dV} \cdot \frac{V_s}{Q_s} \]
- Yet another proximity index is (most convenient to use)
  \[ k_Q = \frac{dQ_G}{dQ_L}. \]

Under light load, \( k_Q \approx 1 \), near the critical load, \( k_Q \to \infty \).
- \( \xi_{cr} \) may also be treated as a measure of voltage stability margin,
Dynamic Analysis

- The static load-voltage we have seen so far can only approximate the real system behavior under slow voltage variations.
- In practice, the above is a process that is influenced by load dynamics, control and protection equipment – more complicated.
Examples of system blackout due to voltage problems
Athens, Greece, 2004

• The load was on the increase (9.39 GW) in mid-day of July 2004.
• A loss of a generator brought the system into an emergency state.
• Load shedding was initiated when another generator was lost.
• See the PV nose curves below - inevitable voltage collapse
  – 1 – loss of first generator
  – 2 – loss of second generator
Examples of power system blackout
USA/Canada, 2003

- Cascaded tripping of transmission lines left 50 million customers without power.
- Each trip caused overload and lower voltages on other lines.
Examples of power system blackout
Scandinavia, 2003

• Generator tripping (due to faulty valve), later followed by a fault that isolated two major transmission lines and a generating plant.

• Although voltage was maintained for by the neighboring system, overloading of other lines caused further tripping and a blackout.
Prevention of voltage collapse

- Action is required at the network planning, operational and monitoring and control stages.
- During planning, reliability criteria must be satisfied for all possible contingencies on at least N-1 type (with maximum voltage drop not exceeded and sufficiently large stability margins).
- During operation and real-time monitoring and control, the desired voltage profile should be continuously maintained. In addition, adequate amounts of real and reactive power reserves at the generators should be maintained.

- Additional defenses include:
  - Emergency back-up reactive power reserve
  - Emergency increase in reactive power generation at the expense of reduction in real power
  - Reduction in real power demand by using OLTC transformers.
  - Under-voltage load shedding.