EE 740 – Transmission Lines
High Voltage Power Lines (overhead)

- Common voltages in north America: 138, 230, 345, 500, 765 kV
- Bundled conductors are used in extra-high voltage lines
- Stranded instead of solid conductors are used.
HVDC Transmission

- Because of the large fixed cost necessary to convert ac to dc and then back to ac, dc transmission is only practical in specialized applications
  - long distance overhead power transfer (> 400 miles)
  - long underwater cable power transfer
  - providing an asynchronous means of joining different power systems.
Electrical Characteristics

- Transmission lines are characterized by a **series resistance**, **inductance**, and **shunt capacitance** per unit length.
- These values determine the power-carrying capacity of the transmission line and the voltage drop across it at full load.

![Diagram of transmission line with R, L, and C symbols]

- The DC resistance of a conductor is expressed in terms of resistively, length and cross sectional area as follows:

\[
R_{DC} = \frac{\rho l}{A}
\]
Cable resistance

• The resistivity increases linearly with temperature over normal range of temperatures.
• If the resistivity at one temperature and material temperature constant are known, the resistivity at another temperature can be found by

\[ \rho_{T_2} = \frac{M + T_2}{M + T_1} \rho_{T_1} \]

<table>
<thead>
<tr>
<th>Material</th>
<th>Resistivity at 20(^{\circ})C ([\Omega\cdot m])</th>
<th>Temperature constant ([^{\circ})C])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annealed copper</td>
<td>1.72(\cdot)10(^{-8})</td>
<td>234.5</td>
</tr>
<tr>
<td>Hard-drawn copper</td>
<td>1.77(\cdot)10(^{-8})</td>
<td>241.5</td>
</tr>
<tr>
<td>Aluminum</td>
<td>2.83(\cdot)10(^{-8})</td>
<td>228.1</td>
</tr>
<tr>
<td>Iron</td>
<td>10.00(\cdot)10(^{-8})</td>
<td>180.0</td>
</tr>
<tr>
<td>Silver</td>
<td>1.59(\cdot)10(^{-8})</td>
<td>243.0</td>
</tr>
</tbody>
</table>
Cable Resistance

• AC resistance of a conductor is always higher than its DC resistance due to the skin effect forcing more current flow near the outer surface of the conductor. The higher the frequency of current, the more noticeable skin effect would be.

• Wire manufacturers usually supply tables of resistance per unit length at common frequencies (50 or 60 Hz) and different temperatures. Therefore, the resistance can be determined from such tables.

### Aluminum Conductor Steel Reinforced

#### Electrical Properties

<table>
<thead>
<tr>
<th>CODE WORD</th>
<th>AWG or kcmil</th>
<th>Aluminum/Steel</th>
<th>DC (Ohms/1000 Ft.) @20°C</th>
<th>AC-50-Hz (Ohms/1000 Ft.) @25°C</th>
<th>Capacitive (Megaohms-1000 Ft.) @25°C</th>
<th>Inductive (Ohms/1000 Ft.) @25°C</th>
<th>GMR (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WAXWING</td>
<td>26/7</td>
<td>18/1</td>
<td>0.0894</td>
<td>0.0873</td>
<td>0.0857</td>
<td>0.0873</td>
<td>0.0894</td>
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<tr>
<td>PARTRIDGE</td>
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<td>16/1</td>
<td>0.0887</td>
<td>0.0873</td>
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<td>0.0873</td>
<td>0.0894</td>
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<td>15/1</td>
<td>0.0510</td>
<td>0.0532</td>
<td>0.0574</td>
<td>0.0547</td>
<td>0.0560</td>
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<tr>
<td>LINNET</td>
<td>336.4</td>
<td>26/1</td>
<td>0.0508</td>
<td>0.0534</td>
<td>0.0574</td>
<td>0.0547</td>
<td>0.0560</td>
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<td>ORIOLE</td>
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<td>PELICAN</td>
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<td>0.0389</td>
<td>0.0405</td>
<td>0.0441</td>
<td>0.0528</td>
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<tr>
<td>FLICKER</td>
<td>477.0</td>
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<td>0.0367</td>
<td>0.0403</td>
<td>0.0430</td>
<td>0.0524</td>
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<tr>
<td>HAWK</td>
<td>477.0</td>
<td>26/1</td>
<td>0.0357</td>
<td>0.0366</td>
<td>0.0402</td>
<td>0.0438</td>
<td>0.0522</td>
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<tr>
<td>HEN</td>
<td>477.0</td>
<td>30/1</td>
<td>0.0354</td>
<td>0.0362</td>
<td>0.0389</td>
<td>0.0434</td>
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<td>OSPREY</td>
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<td>0.0348</td>
<td>0.0379</td>
<td>0.0518</td>
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<tr>
<td>PARAKEET</td>
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<td>0.0314</td>
<td>0.0347</td>
<td>0.0377</td>
<td>0.0514</td>
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</tbody>
</table>
Line inductance

- The series inductance of a transmission line consists of two components: internal and external inductances, which are due to the magnetic flux inside and outside the conductor respectively.

- The inductance of a transmission line is defined as the number of flux linkages [Wb-turns] produced per ampere of current flowing through the line:

\[ v = \frac{d\psi}{dt} = L \frac{di}{dt} \quad \rightarrow \quad L = \frac{d\psi}{di} \approx \frac{\psi}{i} \]

- The inductance of a single-phase transmission line is given by (see derivation in the book): (r : conductor radius - assumed solid, D: distance between cables, \( \mu = 4\pi \times 10^{-7} \) H/m, \( r' = r e^{-1/4} = .7788 r \))

\[ l = \frac{\mu}{\pi} \left( \frac{1}{4} + \ln \frac{D}{r} \right) \quad [H/m] \]

\[ L = 4x10^{-7} \ln \left( \frac{D}{r'} \right) \] H/m
Inductance of 3-phase transposed transmission line

\[ L = 2 \times 10^{-7} \ln \left( \frac{GMD}{GMR} \right) \text{ H/m} \]

where the Geometric Mean Distance (GMD) is defined by

\[ GMD = \sqrt[3]{D_1 D_2 D_3} \]

where \( D_1, D_2, \) and \( D_3 \) are the distances between the 3 conductors. The Geometric Mean Radius (GMR) is supplied by the manufacturer (takes into account the cable strands). For a solid conductor, \( GMR = 0.7788 \text{ r} \).

For a 60 Hz system, the reactance of the line is

\[ X_L = 0.754 \times 10^{-4} \ln \left( \frac{GMD}{GMR} \right) \text{ Ohms/m} \]
\[ X_L = 0.1213 \ln \left( \frac{GMD}{GMR} \right) \text{ Ohms/mi} \]

**Note:** in your book, the authors call GMR – self GMD and GMD – mutual GMD.
Remarks on line inductance

- **The greater the spacing between the phases of a transmission line, the greater the inductance of the line.**
  - Since the phases of a high-voltage overhead transmission line must be spaced further apart to ensure proper insulation, a high-voltage line will have a higher inductance than a low-voltage line.
  - Since the spacing between lines in buried cables is very small, series inductance of cables is much smaller than the inductance of overhead lines.

- **The greater the radius of the conductors in a transmission line, the lower the inductance of the line.** In practical transmission lines, instead of using heavy and inflexible conductors of large radii, two and more conductors are bundled together to approximate a large diameter conductor, and reduce corona loss.

\[
GMR_2 = \sqrt{GMR \cdot d}
\]
\[
GMR_3 = \sqrt[3]{GMR \cdot d^2}
\]
\[
GMR_4 = 1.09 \sqrt[4]{GMR \cdot d^3}
\]
Shunt capacitance

• Since a voltage $V$ is applied to a pair of conductors separated by a dielectric (air), charges $q$ of equal magnitude but opposite sign will accumulate on the conductors. Capacitance $C$ between the two conductors is defined by

$$C = \frac{q}{V}$$

• The capacitance of a single-phase transmission line is given by (see derivation in the book): ($\varepsilon = 8.85 \times 10^{-12}$ F/m)

$$C = \frac{2\pi\varepsilon}{\ln\left(\frac{D}{r}\right)} \text{ F/m}$$

D: distance between conductors  
$\varepsilon$: permittivity of free space  
r: radius of conductor
Capacitance of 3-phase transposed transmission line

- The capacitance per phase is computed by
  \[
  C = \frac{2\pi \epsilon}{\ln\left(\frac{GMD}{r}\right)} \text{ F/m}
  \]
- The shunt admittance per phase at 60 Hz is given by
  \[
  y = 2\pi f C = \frac{0.35 \times 10^{-8}}{\ln\left(\frac{GMD}{r}\right)} \text{ S.m}
  \]
- The shunt capacitive reactance per phase at 60 Hz is given by
  \[
  X_c = 47.7 \times 10^6 \ln \left(\frac{GMD}{r}\right) \text{ Ohm.m}
  \]
  \[
  X_c = 0.02965 \ln \frac{GMD}{r} (M\Omega.m)
  \]

**Note:** For bundled conductors, replace GMR with r in expressions in slide 10
Remarks on line capacitance

1. The greater the spacing between the phases of a transmission line, the lower the capacitance of the line.
   - Since the phases of a high-voltage overhead transmission line must be spaced further apart to ensure proper insulation, a high-voltage line will have a lower capacitance than a low-voltage line.
   - Since the spacing between lines in buried cables is very small, shunt capacitance of cables is much larger than the capacitance of overhead lines.

2. The greater the radius of the conductors in a transmission line, the higher the capacitance of the line. Therefore, bundling increases the capacitance.

Note: The presence of the earth increases the line capacitance (see section 3.7)
Use of Tables

• Inductive reactance (in $\Omega$/mi):

$$X_L = 0.1213 \ln \frac{GMD}{GMR} = 0.1213 \ln \frac{1}{GMR} + 0.1213 \ln GMD$$

  – The first term is defined as $X_a$: the inductive reactance at 1-foot spacing
  – The second term is defined as $X_d$: the inductive reactance spacing factor
  – The first component is often given in the table of cables.

• Capacitive reactance (in $M\Omega$.mi):

$$X_C = 0.02965 \ln \frac{GMD}{r} = 0.02965 \ln \frac{1}{r} + 0.02965 \ln GMD$$

  – The first term is defined as $X'_a$: the capacitive reactance at 1-foot spacing
  – The second term is defined as $X'_d$: the capacitive reactance spacing factor
  – The first component is often given in the table of cables.
# ACSR Conductor Table Data

**TABLE A8.1. BARE ALUMINUM CONDUCTORS, STEEL REINFORCED (ACSR) ELECTRICAL PROPERTIES OF MULTILAYER SIZES (Cont’d)**

<table>
<thead>
<tr>
<th>Code Word</th>
<th>Size (kcmil)</th>
<th>Stranding Al./St.</th>
<th>Number of Aluminum Layers</th>
<th>dc 20°C (Ohms/Mile)</th>
<th>ac-60 Hz 25°C (Ohms/Mile)</th>
<th>ac-60 Hz 50°C (Ohms/Mile)</th>
<th>ac-60 Hz 75°C (Ohms/Mile)</th>
<th>GMR (ft)</th>
<th>Inductive Ohms/Mile $X_a$</th>
<th>Capacitive Megohm-Miles $X'_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flicker</td>
<td>477</td>
<td>24/7</td>
<td>2</td>
<td>0.1889</td>
<td>0.194</td>
<td>0.213</td>
<td>0.232</td>
<td>0.0283</td>
<td>0.432</td>
<td>0.0992</td>
</tr>
<tr>
<td>Hawk</td>
<td>477</td>
<td>26/7</td>
<td>2</td>
<td>0.1883</td>
<td>0.193</td>
<td>0.212</td>
<td>0.231</td>
<td>0.0290</td>
<td>0.430</td>
<td>0.0988</td>
</tr>
<tr>
<td>Hen</td>
<td>477</td>
<td>30/7</td>
<td>2</td>
<td>0.1869</td>
<td>0.191</td>
<td>0.210</td>
<td>0.229</td>
<td>0.0304</td>
<td>0.424</td>
<td>0.0980</td>
</tr>
<tr>
<td>Osprey</td>
<td>556.5</td>
<td>18/1</td>
<td>2</td>
<td>0.1629</td>
<td>0.168</td>
<td>0.184</td>
<td>0.200</td>
<td>0.0284</td>
<td>0.432</td>
<td>0.0981</td>
</tr>
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<td>24/7</td>
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<td>0.1620</td>
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<td>0.183</td>
<td>0.199</td>
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<tr>
<td>Dove</td>
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<td>26/7</td>
<td>2</td>
<td>0.1613</td>
<td>0.166</td>
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<td>30/7</td>
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<td>0.1602</td>
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<td>0.0328</td>
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</tr>
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<td>Peacock</td>
<td>605</td>
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<td>0.1490</td>
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<td>0.168</td>
<td>0.183</td>
<td>0.0319</td>
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<td>0.182</td>
<td>0.0327</td>
<td>0.415</td>
<td>0.0953</td>
</tr>
</tbody>
</table>

**Geometric Mean Radius**

**Inductive and Capacitive Reactance for 1-foot Spacing**
Short line model

- Overhead transmission lines shorter than 50 miles can be modeled as a series resistance and inductance, since the shunt capacitance can be neglected over short distances.

- The total series resistance and series reactance can be calculated as

\[
R = rd \\
X = xd
\]

- where \( r \), \( x \) are resistance and reactance per unit length and \( d \) is the length of the transmission line.
Short line model

- Two-port network model:

\[ \begin{align*}
    I_S &= I_R \\
    V_S &= AV_R + BI_R \\
    I_S &= CV_R + DI_R \\
    V_R &= V_S - RI - jX_L I
\end{align*} \]

- The equation is similar to that of a synchronous generator and transformer (w/o shunt impedance)
Voltage Regulation:

\[ VR = \left( \frac{V_{nl} - V_{fl}}{V_{fl}} \right) \times 100\% \]

1. If lagging (inductive) loads are added at the end of a line, the voltage at the end of the transmission line decreases significantly – large positive VR.

2. If unity-PF (resistive) loads are added at the end of a line, the voltage at the end of the transmission line decreases slightly – small positive VR.

3. If leading (capacitive) loads are added at the end of a line, the voltage at the end of the transmission line increases – negative VR.
If the resistance of the line is ignored, then

\[ I \cos \theta = \frac{V_s \sin \delta}{X_L} \]

Therefore, the power flow through a transmission line depends on the angle between the input and output voltages.

Maximum power flow occurs when \( \delta = 90^\circ \).

Notes:

- The maximum power handling capability of a transmission line is a function of the square of its voltage.
- The maximum power handling capability of a transmission line is inversely proportional to its series reactance (some very long lines include series capacitors to reduce the total series reactance).
- The angle \( \delta \) controls the power flow through the line. Hence, it is possible to control power flow by placing a phase-shifting transformer.
Line Characteristics

• To prevent excessive voltage variations in a power system, the ratio of the magnitude of the receiving end voltage to the magnitude of the ending end voltage is generally within

\[ 0.95 \leq \frac{V_S}{V_R} \leq 1.05 \]

• The angle \( \delta \) in a transmission line should typically be \( \leq 30^\circ \) to ensure that the power flow in the transmission line is well below the static stability limit.

• Any of these limits can be more or less important in different circumstances.
  – In short lines, where series reactance \( X \) is relatively small, the \textbf{resistive heating} usually limits the power that the line can supply.
  – In longer lines operating at lagging power factors, the \textbf{voltage drop} across the line is usually the limiting factor.
  – In longer lines operating at leading power factors, the \textbf{maximum angle} \( \delta \) can be the limiting factor.
Example

• A line with reactance $X$ and negligible resistance supplies a pure resistive load from a fixed source $V_S$. Determine the maximum power transfer, and the load voltage $V_R$ at which this occurs. *(Hint: recall the maximum power transfer theorem from your basic circuits course)*

• Ans: $P_{\text{max}} = \frac{V_S^2}{2X}$, $V_R = \frac{V_S}{\sqrt{2}}$
Medium Line (50-150 mi)

- the shunt admittance must be included in calculations. However, the total admittance is usually modeled (π model) as two capacitors of equal values (each corresponding to a half of total admittance) placed at the sending and receiving ends.

- The total series resistance and series reactance are calculated as before. Similarly, the total shunt admittance is given by

\[ Y = yd \]

- where \( y \) is the shunt admittance per unit length and \( d \) is the length of the transmission line.
Two-port network:

\[ V_S = AV_R + BI_R \]
\[ I_S = CV_R + DI_R \]

\[ A = \frac{ZY}{2} + 1 \]
\[ B = Z \]
\[ C = Y \left( \frac{ZY}{4} + 1 \right) \]
\[ D = \frac{ZY}{2} + 1 \]
Long Lines ( > 150 mi)

• For long lines, both the shunt capacitance and the series impedance must be treated as distributed quantities. The voltages and currents on the line are found by solving differential equations of the line.

• However, it is possible to model a long transmission line as a π model with a *modified* series impedance $Z'$ and a *modified* shunt admittance $Y'$ and to perform calculations on that model using ABCD constants. These modified values are

$$Z' = Z \frac{\sinh \gamma d}{\gamma d}$$

$$Y' = Y \frac{\tanh (\gamma d/2)}{\gamma d/2}$$

where the propagation constant is defined by

$$\gamma = \sqrt{yz}$$
Surge Impedance Loading

- The surge impedance of a line is defined as
  \[ Z_C = \sqrt{z/y} \approx \sqrt{L/C} \]
- Surge Impedance Loading (SIL) is the power delivered by a line to a pure resistive load that is equal to its surge impedance:
  \[ SIL = 3 \frac{V_\phi^2}{\sqrt{L/C}} = \frac{V_L^2}{\sqrt{L/C}} \text{ MW} \]
- Under such loading, the line consumes as much reactive power as it generates and the terminal voltages are equal to each other.
- Power system engineers sometime find it convenient to express the power transmitted by a line in terms of per-unit of SIL.
Note that a transmission line both absorbs and generates reactive power:

- Under light load, the line generates more reactive power than it consumes.
- Under “surge impedance loading”, the line generates and consumes the same amount of reactive power.
- Under heavy load, the line absorbs more reactive power than it generates.
Input/Output Power and efficiency

- **Input powers**

\[
P_{in} = 3V_S I_S \cos \theta_S = \sqrt{3}V_{LL,S} I_S \cos \theta_S \\
Q_{in} = 3V_S I_S \sin \theta_S = \sqrt{3}V_{LL,S} I_S \sin \theta_S \\
S_{in} = 3V_S I_S = \sqrt{3}V_{LL,S} I_S
\]

- **Output powers**

\[
P_{out} = 3V_R I_R \cos \theta_R = \sqrt{3}V_{LL,R} I_R \cos \theta_R \\
Q_{out} = 3V_R I_R \sin \theta_R = \sqrt{3}V_{LL,R} I_R \sin \theta_R \\
S_{out} = 3V_R I_R = \sqrt{3}V_{LL,R} I_R
\]

- **Efficiency**

\[
\eta = \frac{P_{out}}{P_{in}} \cdot 100\%
\]
Power Flow Through a Transmission Line

• Let

\[ A = |A| \angle \alpha, \quad B = |B| \angle \beta, \quad V_S = |V_S| \angle \delta, \quad V_R = |V_R| \angle 0^\circ \]

• Then the complex power at the receiving end is given by

\[ P_R + jQ_R = V_R I_R^* = \frac{V_S V_R}{|B|} \angle (\beta - \delta) - \frac{A|V_R|^2}{|B|} \angle (\beta - \alpha) \]
Power Diagram (by shifting origin of coordinate axes)

• For fixed values of both voltage and as the load changes, point k moves on a circle of center n.
  – Any change in $P_R$ will require a change in $Q_R$.
  – The limit of the power that can be transmitted occurs when $\beta = \delta$.
  – The maximum power transfer is
    $$P_{R,\text{max}} = \frac{|V_S||V_R|}{|B|} - \frac{|A||V_R|^2}{|B|} \cos(\beta - \alpha)$$
  – This above requires a large leading current.
  – Normally,
    • $\delta \leq 35^\circ$
    • $0.95 \leq \frac{|V_S|}{|V_R|} \leq 1.05$
Long line series and shunt compensation

- **Shunt reactors** are used to compensate the line shunt capacitance under light load or no load to regulate voltage.
- **Series capacitors** are often used to compensate the line inductive reactance in order to transfer more power.
• Find both the inductive and capacitive reactances of the double circuit 3-phase transposed lines shown in Fig. P-2.11.

• Solve problem 5.11. Now assume the load consists of an impedance whose magnitude is variable and phase angle fixed at 30°.
  – Plot how the receiving end voltage vary with the load.
  – Repeat the above with a series compensation factor of 50%.