Power Flow Analysis

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A power flow study (load-flow study) is a steady-state analysis whose target is to determine the voltages, currents, and real and reactive power flows in a system under a given load conditions.

The purpose of power flow studies is to plan ahead and account for various hypothetical situations. For example, if a transmission line is be taken off line for maintenance, can the remaining lines in the system handle the required loads without exceeding their rated values.
Power-flow analysis equations

The basic equation for power-flow analysis is derived from the nodal analysis equations for the power system: For example, for a 4-bus system,

\[
\begin{bmatrix}
Y_{11} & Y_{12} & Y_{13} & Y_{14} \\
Y_{21} & Y_{22} & Y_{23} & Y_{24} \\
Y_{31} & Y_{32} & Y_{33} & Y_{34} \\
Y_{41} & Y_{42} & Y_{43} & Y_{44}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4
\end{bmatrix}
= 
\begin{bmatrix}
I_1 \\
I_2 \\
I_3 \\
I_4
\end{bmatrix}
\]

where \( Y_{ij} \) are the elements of the bus admittance matrix, \( V_i \) are the bus voltages, and \( I_j \) are the currents injected at each node. The node equation at bus \( i \) can be written as

\[
I_i = \sum_{j=1}^{n} Y_{ij} V_j
\]
Power-flow analysis equations

Relationship between per-unit real and reactive power supplied to the system at bus $i$ and the per-unit current injected into the system at that bus:

$$S_i = V_i I_i^* = P_i + jQ_i$$

where $V_i$ is the per-unit voltage at the bus; $I_i^*$ - complex conjugate of the per-unit current injected at the bus; $P_i$ and $Q_i$ are per-unit real and reactive powers. Therefore,

$$I_i^* = (P_i + jQ_i) / V_i \quad \Rightarrow I_i = (P_i - jQ_i) / V_i^*$$

$$\Rightarrow P_i - jQ_i = V_i^* \sum_{j=1}^{n} Y_{ij} V_j = \sum_{j=1}^{n} Y_{ij} V_j V_i^*$$
Power flow equations

Let

\[ Y_{ij} = |Y_{ij}| \angle \theta_{ij} \quad \text{and} \quad V_i = |V_i| \angle \delta_i \]

Then

\[ P_i - jQ_i = \sum_{j=1}^{n} |Y_{ij}| \| V_j \| \| V_i \| \angle(\theta_{ij} + \delta_j - \delta_i) \]

Hence,

\[ P_i = \sum_{j=1}^{n} |Y_{ij}| \| V_j \| \| V_i \| \cos(\theta_{ij} + \delta_j - \delta_i) \]

and

\[ Q_i = -\sum_{j=1}^{n} |Y_{ij}| \| V_j \| \| V_i \| \sin(\theta_{ij} + \delta_j - \delta_i) \]
Formulation of power-flow study

- There are 4 variables that are associated with each bus:
  - P,
  - Q,
  - V,
  - δ.

- Meanwhile, there are two power flow equations associated with each bus.

- In a power flow study, two of the four variables are defined and the other two are unknown. That way, we have the same number of equations as the number of unknown.

- The known and unknown variables depend on the type of bus.
Each bus in a power system can be classified as one of three types:

1. **Load bus (P-Q bus)** – a bus at which the real and reactive power are specified, and for which the bus voltage will be calculated. All busses having no generators are load busses. In here, $V$ and $\delta$ are unknown.

2. **Generator bus (P-V bus)** – a bus at which the magnitude of the voltage is defined and is kept constant by adjusting the field current of a synchronous generator. We also assign real power generation for each generator according to the economic dispatch. In here, $Q$ and $\delta$ are unknown.

3. **Slack bus (swing bus)** – a special generator bus serving as the reference bus. Its voltage is assumed to be fixed in both magnitude and phase (for instance, $1 \angle 0^\circ$ pu). In here, $P$ and $Q$ are unknown.
Formulation of power-flow study

- Note that the power flow equations are non-linear, thus cannot be solved analytically. A numerical iterative algorithm is required to solve such equations. A standard procedure follows:
  1. Create a bus admittance matrix $Y_{bus}$ for the power system;
  2. Make an initial estimate for the voltages (both magnitude and phase angle) at each bus in the system;
  3. Substitute in the power flow equations and determine the deviations from the solution.
  4. Update the estimated voltages based on some commonly known numerical algorithms (e.g., Newton-Raphson or Gauss-Seidel).
  5. Repeat the above process until the deviations from the solution are minimal.
Consider a 4-bus power system below. Assume that

- bus 1 is the slack bus and that it has a voltage $V1 = 1.0\angle0^\circ$ pu.
- The generator at bus 3 is supplying a real power $P3 = 0.3$ pu to the system with a voltage magnitude 1 pu.
- The per-unit real and reactive power loads at busses 2 and 4 are $P2 = 0.3$ pu, $Q2 = 0.2$ pu, $P4 = 0.2$ pu, $Q4 = 0.15$ pu.

Line impedances (in pu):
- Line 1-2: $Z = 0.1+j0.4$
- Line 2-3: $Z = 0.1+j0.5$
- Line 2-4: $Z = 0.1+j-0.4$  
- Line 3-4: $Z = 0.05+j0.2$
- Line 4-1: $Z = 0.05+j0.2$
Example (cont.)

• Y-bus matrix (refer to example in book)

\[ Y_{bus} = \begin{bmatrix}
1.7647 - j7.0588 & -0.5882 + j2.3529 & 0 & -1.1765 + j4.7059 \\
-0.5882 + j2.3529 & 1.5611 - j6.6290 & -0.3846 + j1.9231 & -0.5882 + j2.3529 \\
0 & -0.3846 + j1.9231 & 1.5611 - j6.6290 & -1.1765 + j4.7059 \\
-1.1765 + j4.7059 & -0.5882 + j2.3529 & -1.1765 + j4.7059 & 2.9412 - j11.7647
\end{bmatrix} \]

\[ V_1 = 1.0\angle0^\circ \ \text{pu} \]
\[ V_2 = 0.964\angle-0.97^\circ \ \text{pu} \]
\[ V_3 = 1.0\angle1.84^\circ \ \text{pu} \]
\[ V_4 = 0.98\angle-0.27^\circ \ \text{pu} \]

• Power flow solution:

• By knowing the node voltages, the power flow (both active and reactive) in each branch of the circuit can easily be calculated.