EE 740
Economic Dispatch
Operating Costs

- Factors influencing the minimum cost of power generation
  - operating efficiency of prime mover and generator
  - fuel costs
  - transmission losses

- The most efficient generator in the system does not guarantee minimum costs
  - may be located in an area with high fuel costs
  - may be located far from the load centers and transmission losses are high

- The problem is to determine generation at different plants to minimize the total operating costs
Operating Costs

- Generator heat rate curves lead to the fuel cost curves

*The fuel cost is commonly express as a quadratic function*

\[ C_i = \alpha_i + \beta_i P_i + \gamma_i P_i^2 \]

*The derivative is known as the incremental fuel cost*

\[ \frac{dC_i}{dP_i} = \beta_i + 2\gamma_i P_i \]

Power Systems I
Economic Dispatch

- The simplest problem is when system losses and generator limits are neglected
  - minimize the objective or cost function over all plants
  - a quadratic cost function is used for each plant

\[
C_{total} = \sum_{i=1}^{n_{gen}} C_i = \sum_{i=1}^{n_{gen}} \alpha_i + \beta_i P_i + \gamma_i P_i^2
\]

- the total demand is equal to the sum of the generators’ output; the equality constraint

\[
\sum_{i=1}^{n_{gen}} P_i = P_{Demand}
\]
Economic Dispatch

- A typical approach using the Lagrange multipliers

\[ L = C_{\text{total}} + \lambda \left( P_{\text{Demand}} - \sum_{i=1}^{n_{\text{gen}}} P_i \right) \]

\[ \frac{\partial L}{\partial P_i} = \frac{\partial C_{\text{total}}}{\partial P_i} + \lambda (0 - 1) = 0 \quad \rightarrow \quad \frac{\partial C_{\text{total}}}{\partial P_i} = \lambda \]

\[ C_{\text{total}} = \sum_{i=1}^{n_{\text{gen}}} C_i \quad \rightarrow \quad \frac{\partial C_{\text{total}}}{\partial P_i} = \frac{dC_i}{dP_i} = \lambda \quad \forall i = 1, \ldots, n_g \]

\[ \lambda = \frac{dC_i}{dP_i} = \beta_i + 2\gamma_i P_i \]
Economic Dispatch

- the second condition for optimal dispatch

\[
\frac{dL}{d\lambda} = \left( P_{\text{Demand}} - \sum_{i=1}^{n_{\text{gen}}} P_i \right) = 0 \quad \Rightarrow \quad \sum_{i=1}^{n_{\text{gen}}} P_i = P_{\text{Demand}}
\]

- rearranging and combining the equations to solve for \( \lambda \)

\[
P_i = \frac{\lambda - \beta_i}{2\gamma_i}
\]

\[
\sum_{i=1}^{n_{\text{gen}}} \frac{\lambda - \beta_i}{2\gamma_i} = P_{\text{Demand}} \quad \lambda = \frac{P_{\text{Demand}} + \sum_{i=1}^{n_{\text{gen}}} \frac{\beta_i}{2\gamma_i}}{\sum_{i=1}^{n_{\text{gen}}} \frac{1}{2\gamma_i}}
\]
Example

- Neglecting system losses and generator limits, find the optimal dispatch and the total cost in $/hr for the three generators and the given load demand

\[ C_1 = 500 + 5.3P_1 + 0.004P_1^2 \text{ [$/MWhr]} \]
\[ C_2 = 400 + 5.5P_2 + 0.006P_2^2 \]
\[ C_3 = 200 + 5.8P_3 + 0.009P_3^2 \]
\[ P_{Demand} = 800 \text{MW} \]
Example

\[
\lambda = \frac{P_{\text{Demand}} + \sum_{i=1}^{n_{\text{gen}}} \frac{\beta_i}{2\gamma_i}}{\sum_{i=1}^{n_{\text{gen}}} \frac{1}{2\gamma_i}} = \frac{800 + \frac{5.3}{0.008} + \frac{5.5}{0.012} + \frac{5.8}{0.018}}{\frac{1}{0.008} + \frac{1}{0.012} + \frac{1}{0.018}} = \$8.5/\text{MWhr}
\]

\[
P_i = \frac{\lambda - \beta_i}{2\gamma_i} \implies P_1 = \frac{8.5 - 5.3}{2(0.004)} = 400 \text{ MW}
\]

\[
P_2 = \frac{8.5 - 5.5}{2(0.006)} = 250 \text{ MW}
\]

\[
P_3 = \frac{8.5 - 5.8}{2(0.009)} = 150 \text{ MW}
\]

\[
P_{\text{Demand}} = 800 \text{ MW} = 400 + 250 + 150 \text{ MW}
\]
Example

Incremental cost curves

$\text{$/MWh}$

$P, \text{ MW}$
Economic Dispatch with Generator Limits

- The power output of any generator should not exceed its rating nor be below the value for stable boiler operation
  - Generators have a minimum and maximum real power output limits

- The problem is to find the real power generation for each plant such that cost are minimized, subject to:
  - Meeting load demand - equality constraints
  - Constrained by the generator limits - inequality constraints

- The Kuhn-Tucker conditions
  \[
  \frac{dC_i}{dP_i} = \lambda \quad \leftarrow \quad P_i(\text{min}) < P_i < P_i(\text{max}) \\
  \frac{dC_i}{dP_i} \leq \lambda \quad \leftarrow \quad P_i = P_i(\text{max}) \\
  \frac{dC_i}{dP_i} \geq \lambda \quad \leftarrow \quad P_i = P_i(\text{min})
  \]
Example

- Neglecting system losses, find the optimal dispatch and the total cost in $/hr for the three generators and the given load demand and generation limits

\[ C_1 = 500 + 5.3P_1 + 0.004P_1^2 \quad [\$/\text{MWhr}] \]
\[ C_2 = 400 + 5.5P_2 + 0.006P_2^2 \]
\[ C_3 = 200 + 5.8P_3 + 0.009P_3^2 \]
\[ 200 \leq P_1 \leq 450 \]
\[ 150 \leq P_2 \leq 350 \]
\[ 100 \leq P_3 \leq 225 \]
\[ P_{\text{Demand}} = 975 \text{ MW} \]
Example

\[
\lambda = \frac{P_{\text{Demand}} + \sum_{i=1}^{n_{\text{gen}}} \frac{\beta_i}{2\gamma_i}}{\sum_{i=1}^{n_{\text{gen}}} \frac{1}{2\gamma_i}} = \frac{975 + \frac{5.3}{0.008} + \frac{5.5}{0.012} + \frac{5.8}{0.018}}{\frac{1}{0.008} + \frac{1}{0.012} + \frac{1}{0.018}} = \$9.163/\text{MWh}
\]

Upper limit violated:
\[\rightarrow P_1 = 450 \text{ MW}\]
\[\rightarrow \text{solve the dispatch problem with two generators:}\]
\[P_2 + P_3 = 525 \text{ MW}\]
\[\rightarrow \lambda = \$9.4/\text{MWh}\]
\[\rightarrow P_2 = 315 \text{ MW}\]
\[\rightarrow P_3 = 210 \text{ MW}\]

\[P_1 = \frac{9.16 - 5.3}{2(0.004)} = 483 \text{ MW}\]
\[P_2 = \frac{9.16 - 5.5}{2(0.006)} = 305 \text{ MW}\]
\[P_3 = \frac{9.16 - 5.8}{2(0.009)} = 187 \text{ MW}\]

\[P_{\text{Demand}} = 975 = 450 + 315 + 210\]
Example

Graph showing incremental cost ($/MWh) vs. Power (MW) for three different generators labeled G1, G2, and G3.
Economic Dispatch including Losses

- For large interconnected system where power is transmitted over long distances with low load density areas
  - transmission line losses are a major factor
  - losses affect the optimum dispatch of generation

- One common practice for including the effect of transmission losses is to express the total transmission loss as a quadratic function of the generator power outputs
  - simplest form: \( P_L = \sum_{i=1}^{n_{\text{gen}}} \sum_{j=1}^{n_{\text{gen}}} P_i B_{ij} P_j \)
  - Kron’s loss formula: \( P_L = \sum_{i=1}^{n_{\text{gen}}} \sum_{j=1}^{n_{\text{gen}}} P_i B_{ij} P_j + \sum_{j=1}^{n_{\text{gen}}} B_{0j} P_j + B_{00} \)
Economic Dispatch including Losses

- $B_{ij}$ are called the loss coefficients
  - they are assumed to be constant
  - reasonable accuracy is expected when actual operating conditions are close to the base case conditions used to compute the coefficients
- The economic dispatch problem is to minimize the overall generation cost, $C$, which is a function of plant output
- Constraints:
  - the generation equals the total load demand plus transmission losses
  - each plant output is within the upper and lower generation limits - inequality constraints
Economic Dispatch including Losses

\[ f : \quad C_{\text{total}} = \sum_{i=1}^{n_{\text{gen}}} C_i = \sum_{i=1}^{n_{\text{gen}}} \alpha_i + \beta_i P_i + \gamma_i P_i^2 \]

\[ g : \quad \sum_{i=1}^{n_{\text{gen}}} P_i = P_{\text{demand}} + P_{\text{losses}} \]

\[ u : \quad P_{i(\text{min})} \leq P_i \leq P_{i(\text{max})} \quad i = 1, \ldots, n_{\text{gen}} \]

The resulting optimization equation

\[ L = C_{\text{total}} + \lambda \left( P_{\text{demand}} + P_{\text{losses}} - \sum_{i=1}^{n_{\text{gen}}} P_i \right) + \sum_{i=1}^{n_{\text{gen}}} \mu_{i(\text{max})} (P_{i(\text{max})} - P_i) \]

\[ + \sum_{i=1}^{n_{\text{gen}}} \mu_{i(\text{min})} (P_i - P_{i(\text{min})}) \]

\[ P_i < P_{i(\text{max})} : \quad \mu_{i(\text{max})} = 0 \quad \quad \quad \quad P_i > P_{i(\text{min})} : \quad \mu_{i(\text{min})} = 0 \]
Economic Dispatch including Losses

- When generator limits are not violated:

\[
\frac{\partial L}{\partial P_i} = 0 = \frac{\partial C_{total}}{\partial P_i} + \lambda \left( 0 + \frac{\partial P_L}{\partial P_i} - 1 \right)
\]

\[
\frac{\partial C_{total}}{\partial P_i} = \frac{\partial}{\partial P_i} \left( C_1 + C_2 + \cdots + C_{n_{gen}} \right) = \frac{dC_i}{dP_i}
\]

\[
\therefore \quad \lambda = \frac{dC_i}{dP_i} + \lambda \frac{\partial P_L}{\partial P_i} = \left( \frac{1}{1 - \partial P_L/\partial P_i} \right) \frac{dC_i}{dP_i} = L_i \frac{dC_i}{dP_i}
\]

- The effect of transmission losses introduces a penalty factor that depends on the location of the plant
- The minimum cost is obtained when the incremental cost of each plant multiplied by its penalty factor is the same for all plants
Example

- Find the optimal dispatch and the total cost in $/hr
  - fuel costs and plant output limits
    \[ C_1 = 200 + 7.0P_1 + 0.008P_1^2 \text{ [$/hr]} \quad 10 \leq P_1 \leq 85 \text{ MW} \]
    \[ C_2 = 180 + 6.3P_2 + 0.009P_2^2 \quad 10 \leq P_2 \leq 80 \]
    \[ C_3 = 140 + 6.8P_3 + 0.007P_3^2 \quad 10 \leq P_3 \leq 70 \]
  - real power loss and total load demand
    \[ P_{loss} = 0.000218P_1^2 + 0.000228P_2^2 + 0.000179P_3^2 \]
    \[ P_{Demand} = 150 \text{ MW} \]
Example

\[
\frac{1}{1 - 0.000436P_1}(7 + 0.016P_1) = \frac{1}{1 - 0.000456P_2}(6.3 + 0.018P_2) = \frac{1}{1 - 0.000358P_3}(6.8 + 0.014P_3),
\]

\[P_1 + P_2 + P_3 - 150 = P_{loss}\]

Results (obtained numerically):

- \(P_1 = 35.1\) MW
- \(P_2 = 64.1\) MW
- \(P_3 = 52.5\) MW
- \(P_{loss} = 1.7\) MW
- \(P_{demand} = 150\) MW
Optima Power Flow
Unit Commitment