HW #4 Orthogonality, Eigenvalue and Eigenvector of Matrix

1. Express $y = \begin{bmatrix} 6 \\ 2 \\ 7 \end{bmatrix}$ as a linear combination of the orthogonal basis $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.

2. Express $y = \begin{bmatrix} 3 \\ 5 \\ -4 \end{bmatrix}$ as a linear combination of the orthogonal basis $\left\{ \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.

3. Let $V = R^3$, U is the orthogonal complement to $[1, 2, 1]^T$
   a) Find a basis of U
   b) Find an orthogonal basis of U
   c) Find the distance between $v = [1,2,3]$ and U

4. Let $A = \begin{bmatrix} 1 & -2 \\ 4 & 7 \end{bmatrix}$. Find its eigenvalue λ and eigenvector $\vec{x}$

5. Find the least squares solution to the system of linear equations:
$$\begin{bmatrix} 2 & 0 \\ -1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

6. Suppose W is a flat plane spanned by $\{x_1, x_2\}$, where $x_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and $x_2 = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$. Find an orthogonal basis $\{v_1, v_2\}$ for W.

7. Find an orthogonal basis for the space $W = span([1,3,0], [2,1,4])$ of $\mathbb{R}^3$

8. Find the eigenvalue λ and eigenvector $\vec{x}$ for matrix
$$A = \begin{pmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{pmatrix}$$

9. Use Matlab to draw vectors: $V_1 = (\sqrt{2}, \sqrt{2}), V_2 = -2V_1, V_3 = V_1\angle 75°$ (rotate V_1 clockwise by 60°), $V_4 = -\frac{3}{2}V_1\angle -90°$ (rotate V_1 counter clockwise by 90°).

10. Find the value for coefficient "a", such that following equations have no real solutions for x and y.
$$ax + 4y = 12$$
$$3x - y = 1$$