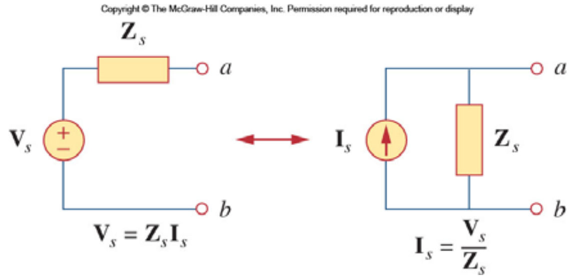
HW Chapter 10:
14, 20, 26, 44, 52, 64, 74, 92.

Source Transformation

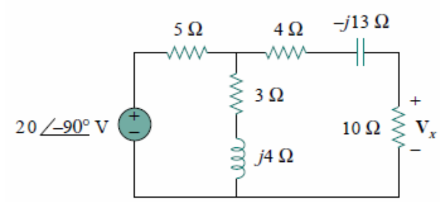
- Source transformation in frequency domain involves transforming a voltage source in series with an impedance to a current source in parallel with an impedance.
- Or vice versa:

$$V_s = Z_s I_s \quad \Leftrightarrow \quad I_s = \frac{V_s}{Z_s}$$



Example:

Calculate $\mathbf{V}_x$ using the source transformation method.



We transform the voltage source to a current source and obtain the circuit.

$$\mathbf{I}_s = \frac{20\angle -90^\circ}{5} = 4\angle -90^\circ = -j4 \text{ A}$$

The parallel combination of 5-Ω resistance and $(3 + j4)$ impedance gives

$$\mathbf{Z}_1 = \frac{5(3 + j4)}{8 + j4} = 2.5 + j1.25\ \Omega$$

$$x = r\cos\phi \qquad y = r\sin\phi$$

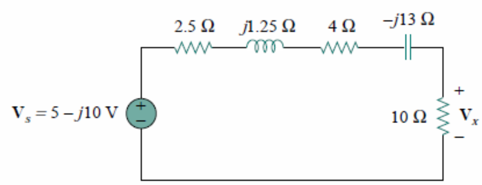
$$r = \sqrt{x^2 + y^2} \qquad \phi = \tan^{-1}\frac{y}{x}$$

Convert the current source to a voltage source

$$\mathbf{V}_s = \mathbf{I}_s\mathbf{Z}_1 = -j4(2.5 + j1.25) = 5 - j10 \text{ V}$$

By voltage division,

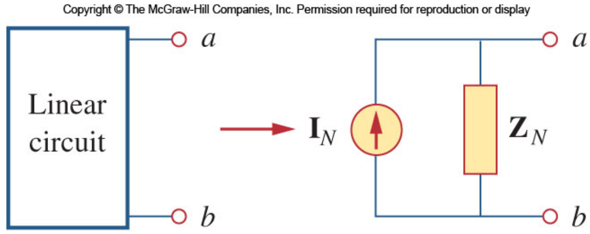
$$\mathbf{V}_x = \frac{10}{10 + 2.5 + j1.25 + 4 - j13}(5 - j10) = 5.519\angle -28^\circ \text{ V}$$



Thevenin and Norton Equivalency

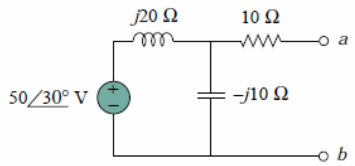
- Both Thevenin and Norton's theorems are applied to AC circuits the same way as DC.
- The only difference is the fact that the calculated values will be complex.

$$V_{Th} = Z_N I_N \qquad Z_{Th} = Z_N$$

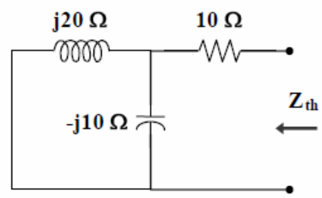


Example

- Find the Thevenin and Norton equivalent circuits at terminals *a*-*b* for each of the circuit.



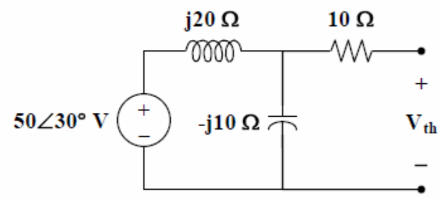
To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (a).



(a)

$$\mathbf{Z}_N = \mathbf{Z}_{th} = 10 + j20 \,||\, (-j10) = 10 + \frac{(j20)(-j10)}{j20 - j10}$$
$$= 10 - j20 = \mathbf{22.36\angle\text{-}63.43°\ \Omega}$$

To find $\mathbf{V}_{th}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{V}_{th} = \frac{-j10}{j20 - j10}(50\angle 30°) = \mathbf{\text{-}50\angle 30°\ V}$$

$$\mathbf{I}_N = \frac{\mathbf{V}_{th}}{\mathbf{Z}_{th}} = \frac{-50\angle 30°}{22.36\angle -63.43°} = \mathbf{2.236\angle 273.4°\ A}$$

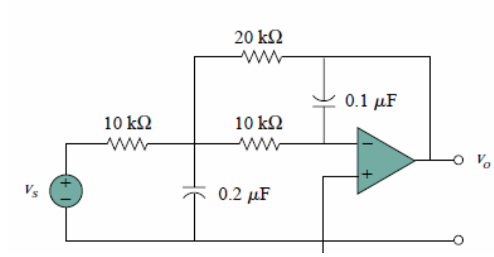
Op Amp AC Circuits

- As long as the op amp is working in the linear range, frequency domain analysis can proceed just as it does for other circuits.
- It is important to keep in mind the two qualities of an ideal op amp:
  - No current enters either input terminals.
  - The voltage across its input terminals is zero with negative feedback.

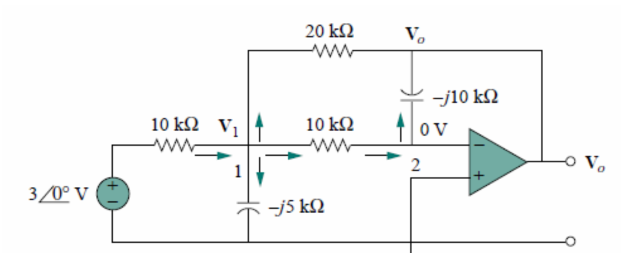
Example

Determine $v_o(t)$ for the op amp circuit in Fig.

if v_s=3 cos 1000t V.

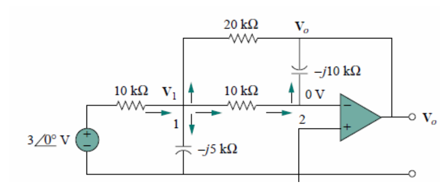


frequency-domain equivalent



Example contd…

Applying KCL at node 1,



we obtain

$$\frac{3\angle 0^\circ - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j5} + \frac{\mathbf{V}_1 - 0}{10} + \frac{\mathbf{V}_1 - \mathbf{V}_o}{20}$$

or

$$6 = (5 + j4)\mathbf{V}_1 - \mathbf{V}_o$$

At node 2, KCL gives

$$\frac{\mathbf{V}_1 - 0}{10} = \frac{0 - \mathbf{V}_o}{-j10}$$

which leads to

$$\mathbf{V}_1 = -j\mathbf{V}_o$$

$$6 = -j(5 + j4)\mathbf{V}_o - \mathbf{V}_o = (3 - j5)\mathbf{V}_o$$

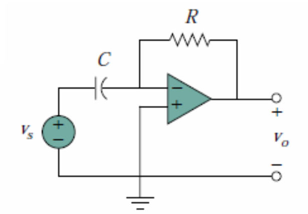
$$\mathbf{V}_o = \frac{6}{3 - j5} = 1.029\angle 59.04^\circ$$

Hence,

$$v_o(t) = 1.029 \cos(1000t + 59.04^\circ) \text{ V}$$

Example

For the differentiator shown in Fig. obtain $\mathbf{V}_o/\mathbf{V}_s$. Find $v_o(t)$ when $v_s(t) = V_m \sin \omega t$ and $\omega = 1/RC$.



This is an inverting op amp so that

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-\mathbf{Z}_f}{\mathbf{Z}_i} = \frac{-R}{1/j\omega C} = -\mathbf{j\omega RC}$$

When $\mathbf{V}_s = V_m$ and $\omega = 1/RC$,

$$\mathbf{V}_o = -j \cdot \frac{1}{RC} \cdot RC \cdot V_m = -jV_m = V_m \angle -90°$$

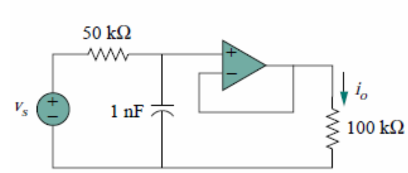
Therefore,

$$v_o(t) = V_m \sin(\omega t - 90°) = -\mathbf{V_m \cos(\omega t)}$$

Example

Compute $i_o(t)$ in the op amp circuit in Fig.

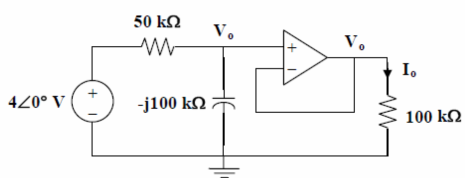
$v_s = 4\cos 10^4 t$ V.



$$4\cos(10^4 t) \longrightarrow 4\angle 0°, \quad \omega = 10^4$$

$$1\text{ nF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^4)(10^{-9})} = -j100\text{ k}\Omega$$

Consider the circuit as shown below.



At the noninverting node,

$$\frac{4 - \mathbf{V}_o}{50} = \frac{\mathbf{V}_o}{-j100} \longrightarrow \mathbf{V}_o = \frac{4}{1 + j0.5}$$

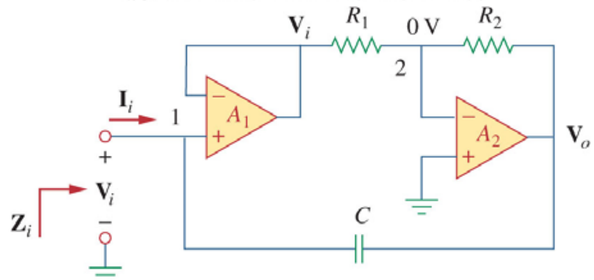
$$\mathbf{I}_o = \frac{\mathbf{V}_o}{100k} = \frac{4}{(100)(1 + j0.5)}\text{mA} = 35.78\angle -26.56°\ \mu\text{A}$$

Therefore,

$$i_o(t) = \mathbf{35.78\cos(10^4 t - 26.56°)\ \mu A}$$

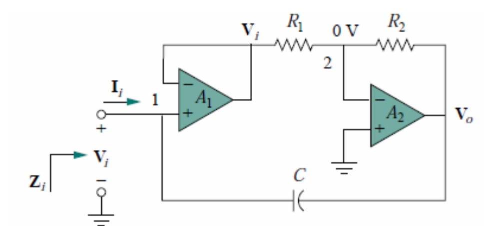
Application

- The op amp circuit shown here is known as a capacitance multiplier.
- It is used in integrated circuit technology to create large capacitances out of smaller ones.



Capacitor Multiplier

- The first op amp is acting as a voltage follower, while the second one is an inverting amplifier.



- At node 1:

$$I_i = \frac{V_i - V_o}{1/j\omega C} = j\omega C\left(V_i - V_o\right)$$

- Applying KCL at node 2 gives

$$V_o = -\frac{R_2}{R_1}V_i$$

Capacitor Multiplier II

- Combining these two gives:

$$\frac{I_i}{V_i} = j\omega\left(1+\frac{R_2}{R_1}\right)C$$

- Which can be expressed as an input impedance.

$$Z_i = \frac{V_i}{I_i} = \frac{1}{j\omega C_{eq}}$$

Capacitor Multiplier III

- Where:

$$C_{eq} = \left(1+\frac{R_2}{R_1}\right)C$$

- By selecting the resistors, the circuit can produce an effective capacitance between its terminal and ground.

Capacitor Multiplier IV

- It is important to understand a limitation of this circuit.
- The larger the multiplication factor, the easier it is for the inverting stage to go out of the linear range.
- Thus the larger the multiplier, the smaller the allowable input voltage.
- A similar op amp circuit can simulate inductance, eliminating the need to have physical inductors in an IC.

Oscillators

- It is straightforward to imagine a DC voltage source.
- One typically thinks of a battery.
- But how to make an AC source?
- Mains power comes to mind, but that is a single frequency.
- This is where an oscillator comes into play.
- They are designed to generate an oscillating voltage at a frequency that is often easily changed.

Barkhausen Criteria

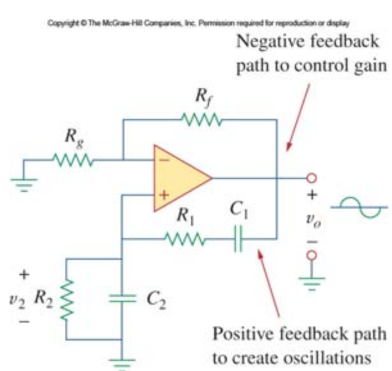
- In order for a sine wave oscillator to sustain oscillations, it must meet the Barkhausen criteria:

1. The overall gain of the oscillator must be unity or greater. Thus losses must be compensated for by an amplifying device.
2. The overall phase shift (from the output and back to the input) must be zero

- Three common types of sine wave oscillators are phase-shift, twin T, and Wein-bridge oscillators.

Wein-bridge

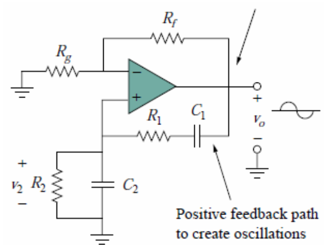
- Here we will only consider the Wein-bridge oscillator
- It is widely used for generating sinusoids in the frequency range below 1 MHz.
- It consists of a RC op amp with a few easily tunable components.



Wein-bridge II

- The circuit is an amplifier in a non-inverting configuration.
- There are two feedback paths:
  - The positive feedback path to the non-inverting input creates oscillations
  - The negative feedback path to the inverting input controls the gain.
- We can define the RC series and parallel combinations as Z_s and $Z_{p.}$

$$Z_s = R_1 - \frac{j}{\omega C_1} \quad Z_p = \frac{R_2}{1 + j\omega R_2 C_2}$$



Wein-bridge III

- The resulting feedback ratio is:

$$\frac{V_2}{V_o} = \frac{Z_p}{Z_s + Z_p}$$

- Expanded out:

$$\frac{V_2}{V_o} = \frac{\omega R_2 C_1}{\omega\left(R_2 C_1 + R_1 C_1 + R_2 C_2\right) + j\left(\omega^2 R_1 C_1 R_2 C_2 - 1\right)}$$

- To satisfy the second Barkhausen criterion, V_2 must be in phase with V_o.
- This means the ratio of V_2/V_o must be real

Wein-bridge IV

- This requires that the imaginary part be set to zero:

$$\omega_o^2 R_1 C_1 R_2 C_2 - 1 = 0$$

- Or

$$\omega_o = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

- In most practical cases, the resistors and capacitors are set to the same values.

Wein-bridge V

- In most cases the capacitors and resistors are made equal.
- The resulting frequency is thus:

$$\omega_o = \frac{1}{RC}$$

$$f_o = \frac{1}{2\pi RC}$$

- Under this condition, the ratio of V_2/V_o is:

$$\frac{V_2}{V_o} = \frac{1}{3}$$

Wein-bridge VI

- Thus in order to satisfy the first Barkhausen criteria, the amplifier must provide a gain of 3 or greater.
- Thus the feedback resistors must be:

$$R_f = 2R_g$$