Overview

- This chapter applies the circuit analysis introduced in the DC circuit analysis for AC circuit analysis.
- Nodal and mesh analysis are discussed.
- Superposition and source transformation for AC circuits are also covered.
- Applications in op-amps and oscillators are reviewed.

Steps to Analyze an AC Circuit

- There are three steps to analyzing an AC circuit.
- They make use of the fact that frequency domain analysis is simpler because it can make use of the nodal and mesh techniques developed for DC

1. Transform the circuit to the phasor or frequency domain
2. Solve the problem using circuit techniques
3. Transform back to time domain.

Nodal Analysis

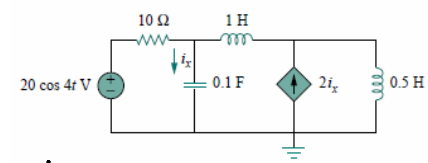
- It is possible to use KCL to analyze a circuit in frequency domain.
- The first step is to convert a time domain circuit to frequency domain by calculating the impedances of the circuit elements at the operating frequency.
- Note that AC sources appear as DC sources with their values expressed as their amplitude.

Nodal Analysis II

- Impedances will be expressed as complex numbers.
- Sources will have amplitude and phase noted.
- At this point, KCL analysis can proceed as normal.
- It is important to bear in mind that complex values will be calculated, but all other treatments are the same.

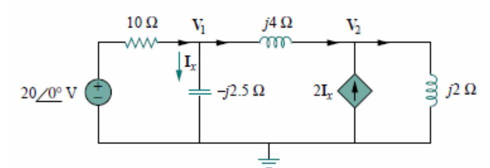
Nodal Analysis Example

Find i_x.



Transform to frequency domain.

$$
\begin{aligned}
20\cos 4t &\implies 20\angle 0^\circ, \quad \omega = 4 \text{ rad/s}\\
1\text{ H} &\implies j\omega L = j4\\
0.5\text{ H} &\implies j\omega L = j2\\
0.1\text{ F} &\implies \frac{1}{j\omega C} = -j2.5
\end{aligned}
$$



Applying KCL at node 1,

$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

or

$$(1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20$$

At node 2,

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

But $\mathbf{I}_x = \mathbf{V}_1/-j2.5$. Substituting this gives

$$\frac{2\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

By simplifying, we get

$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0$$

$$\begin{bmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

We obtain the determinants as

$$\Delta = \begin{vmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = 15 - j5$$

$$\Delta_1 = \begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix} = 300, \qquad \Delta_2 = \begin{vmatrix} 1 + j1.5 & 20 \\ 11 & 0 \end{vmatrix} = -220$$

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = \frac{300}{15 - j5} = 18.97\angle 18.43^\circ \text{ V}$$

$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = \frac{-220}{15 - j5} = 13.91\angle 198.3^\circ \text{ V}$$

The current $\mathbf{I}_x$ is given by

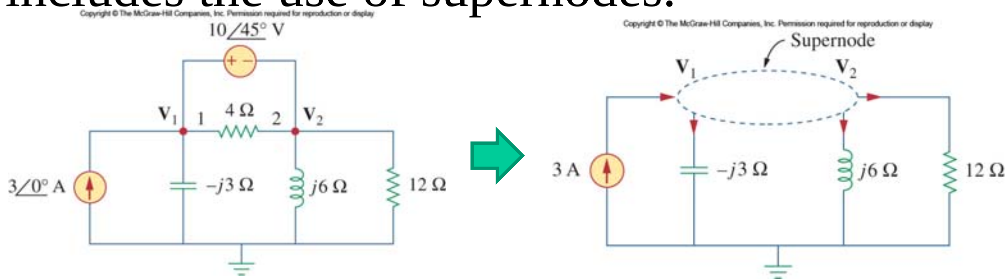
$$\mathbf{I}_x = \frac{\mathbf{V}_1}{-j2.5} = \frac{18.97\angle 18.43^\circ}{2.5\angle -90^\circ} = 7.59\angle 108.4^\circ \text{ A}$$

Transforming this to the time domain,

$$i_x = 7.59\cos(4t + 108.4^\circ) \text{ A}$$

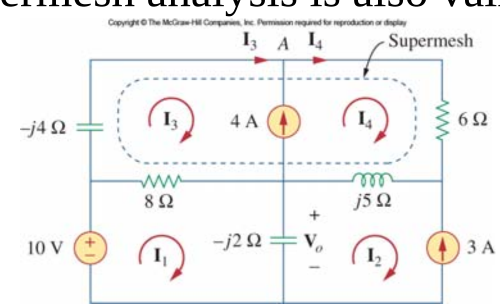
Nodal Analysis III

- The final voltages and current calculated are the real component of the derived values.
- The equivalency of the frequency domain treatment compared to the DC circuit analysis includes the use of supernodes.



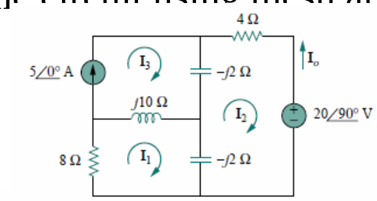
Mesh Analysis

- Just as in KCL, the KVL analysis also applies to phasor and frequency domain circuits.
- The same rules apply: Convert to frequency domain first, then apply KVL as usual.
- In KVL, supermesh analysis is also valid.



Example Mesh Analysis:

- Determine current $\mathbf{I}_o$ in the circuit using mesh analysis.



Applying KVL to mesh 1, we obtain:

$$(8 + j10 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - j10\mathbf{I}_3 = 0$$

For mesh 2,

$$(4 - j2 - j2)\mathbf{I}_2 - (-j2)\mathbf{I}_1 - (-j2)\mathbf{I}_3 + 20\angle 90^\circ = 0$$

For mesh 3, $\mathbf{I}_3 = 5$.

$$(8 + j8)\mathbf{I}_1 + j2\mathbf{I}_2 = j50$$
$$j2\mathbf{I}_1 + (4 - j4)\mathbf{I}_2 = -j20 - j10$$

$$\begin{bmatrix} 8+j8 & j2 \\ j2 & 4-j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

from which we obtain the determinants

$$\Delta = \begin{vmatrix} 8+j8 & j2 \\ j2 & 4-j4 \end{vmatrix} = 32(1+j)(1-j) + 4 = 68$$

$$\Delta_2 = \begin{vmatrix} 8+j8 & j50 \\ j2 & -j30 \end{vmatrix} = 340 - j240 = 416.17\angle -35.22^\circ$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{416.17\angle -35.22^\circ}{68} = 6.12\angle -35.22^\circ \text{ A}$$

The desired current is

$$\mathbf{I}_o = -\mathbf{I}_2 = 6.12\angle 144.78^\circ \text{ A}$$

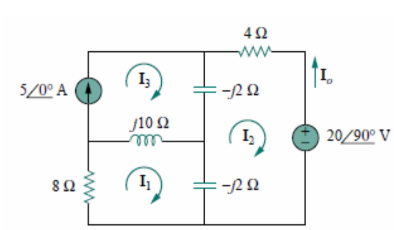
Superposition

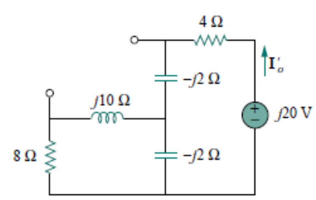
- Since AC circuits are linear, it is also possible to apply the principle of superposition.
- This becomes particularly important if the circuit has sources operating at different frequencies.
- The complication is that each source must have its own frequency domain equivalent circuit.

Superposition II

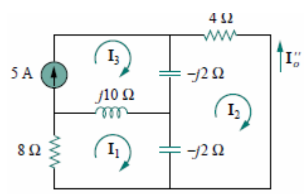
- The reason for this is that each element has a different impedance at different frequencies.
- Also, the resulting voltages and current must be converted back to time domain before being added.
- This is because there is an exponential factor $e^{j\omega t}$ implicit in sinusoidal analysis.

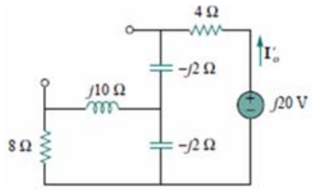
Superposition Example





$$I_o = I'_o + I''_o$$





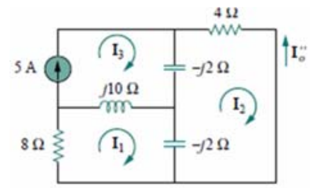
$$\mathbf{Z} = \frac{-j2(8 + j10)}{-2j + 8 + j10} = 0.25 - j2.25$$

and current $\mathbf{I}_o'$ is

$$\mathbf{I}_o' = \frac{j20}{4 - j2 + \mathbf{Z}} = \frac{j20}{4.25 - j4.25}$$

or

$$\mathbf{I}_o' = -2.353 + j2.353$$



For mesh 1, $(8 + j8)\mathbf{I}_1 - j10\mathbf{I}_3 + j2\mathbf{I}_2 = 0$

For mesh 2,

$$(4 - j4)\mathbf{I}_2 + j2\mathbf{I}_1 + j2\mathbf{I}_3 = 0$$

For mesh 3,

$$\mathbf{I}_3 = 5$$

$$(4 - j4)\mathbf{I}_2 + j2\mathbf{I}_1 + j10 = 0$$

Expressing $\mathbf{I}_1$ in terms of $\mathbf{I}_2$ gives

$$\mathbf{I}_1 = (2 + j2)\mathbf{I}_2 - 5$$

$$(8 + j8)[(2 + j2)\mathbf{I}_2 - 5] - j50 + j2\mathbf{I}_2 = 0$$

or

$$\mathbf{I}_2 = \frac{90 - j40}{34} = 2.647 - j1.176$$

Current $\mathbf{I}_o''$ is obtained as

$$\mathbf{I}_o'' = -\mathbf{I}_2 = -2.647 + j1.176$$

$$\mathbf{I}_o = \mathbf{I}_o' + \mathbf{I}_o'' = -5 + j3.529 = 6.12\angle 144.78^\circ \text{ A}$$