Overview

- This chapter will cover alternating current.
- A discussion of complex numbers is included prior to introducing phasors.
- Applications of phasors and frequency domain analysis for circuits including resistors, capacitors, and inductors will be covered.
- The concept of impedance and admittance is also introduced.

Alternating Current

- Alternating Current, or AC, is the dominant form of electrical power that is delivered to homes and industry.
- In the late 1800's there was a battle between proponents of DC and AC.
- AC won out due to its efficiency for long distance transmission.
- AC is a sinusoidal current, meaning the current reverses at regular times and has alternating positive and negative values.

Why did AC win?

$P = I^2R$

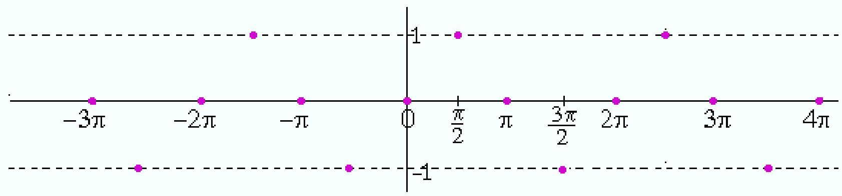
Applying Ohm's Law: $P = VI$

AC Vs DC

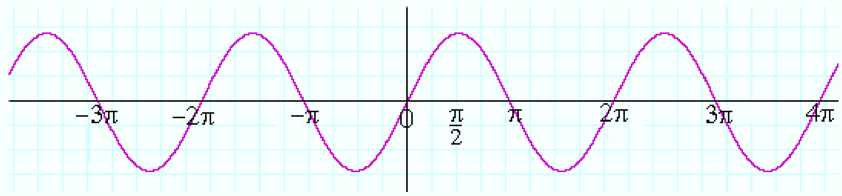
Has AC won? Is it the end of the story? Not so quick.

Sinusoids

- Sinusoids are interesting to us because there are a number of natural phenomenon that are sinusoidal in nature.
- It is also a very easy signal to generate and transmit.
- Also, through Fourier analysis, any practical periodic function can be made by adding sinusoids.
- Lastly, they are very easy to handle mathematically.



Here is the graph of $y = \sin x$:

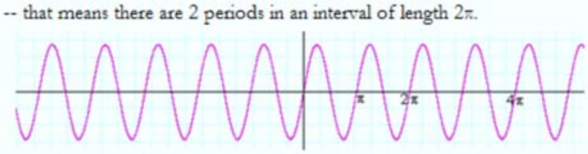


The height of the curve at every point is the line value of the sine.

In the language of functions, $y = \sin x$ is an odd function. It is symmetrical with respect to the origin.
$\sin(-x) = -\sin x.$

$y = \sin 2x$

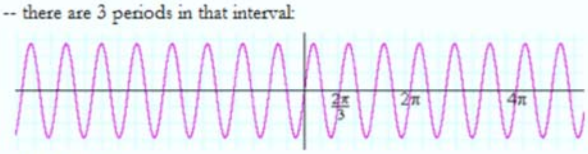
-- that means there are 2 periods in an interval of length 2π.



If $a = 3$ --

$y = \sin 3x$

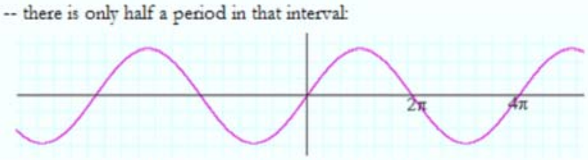
-- there are 3 periods in that interval:



While if $a = ½$ --

$y = \sin ½x$

-- there is only half a period in that interval:



The constant a thus signifies how frequently the function oscillates; so many radians per unit of x.

Sin(x)?

Function grapher

Sinusoids

- A sinusoidal forcing function produces both a transient and a steady state response.
- When the transient has died out, we say the circuit is in sinusoidal steady state.
- A sinusoidal voltage may be represented as:

$$v(t) = V_m \sin \omega t$$

- From the waveforms seen earlier, one characteristic is clear: The function repeats itself every T seconds.
- This is called the period

$$T = \frac{2\pi}{\omega}$$

Sinusoids

- The period is inversely related to another important characteristic, the frequency

$$f = \frac{1}{T}$$

- The units of this is cycles per second, or Hertz (Hz)
- It is often useful to refer to frequency in angular terms:

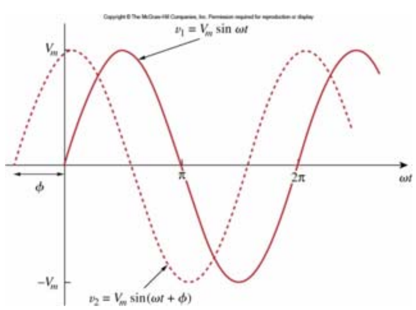
$$\omega = 2\pi f$$

- Here the angular frequency is in radians per second

Sinusoids

- More generally, we need to account for relative timing of one wave versus another.
- This can be done by including a phase shift, ϕ:
- Consider the two sinusoids:

$$v_1(t) = V_m \sin \omega t \quad \text{and} \quad v_2(t) = V_m \sin(\omega t + \phi)$$



Sinusoids

- If two sinusoids are in phase, then this means that they reach their maximum and minimum at the same time.
- Sinusoids may be expressed as sine or cosine.
- The conversion between them is:

$$\sin\left(\omega t \pm 180^\circ\right) = -\sin \omega t$$

$$\cos\left(\omega t \pm 180^\circ\right) = -\cos \omega t$$

$$\sin\left(\omega t \pm 90^\circ\right) = \pm\cos \omega t$$

$$\cos\left(\omega t \pm 90^\circ\right) = \mp\sin \omega t$$

trigonometric identities:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

Revise Trigonometric Formulas

Please use your mathematics book.

Also, several web sites are available.

Sample web site:

http://www.analyzemath.com/trigonometry/trigonometric_formulas.html

Find the amplitude, phase, period, and frequency of the sinusoid

$$v(t) = 12\cos(50t + 10^\circ)$$

Solution:

The amplitude is $V_m = 12$ V.

The phase is $\phi = 10^\circ$.

The angular frequency is $\omega = 50$ rad/s.

The period $T = \frac{2\pi}{\omega} = \frac{2\pi}{50} = 0.1257$ s.

The frequency is $f = \frac{1}{T} = 7.958$ Hz.

Calculate the phase angle between $v_1 = -10\cos(\omega t + 50^\circ)$ and $v_2 = 12\sin(\omega t - 10^\circ)$. State which sinusoid is leading.

$$v_1 = -10\cos(\omega t + 50^\circ) = 10\cos(\omega t + 50^\circ - 180^\circ)$$ Because, - cos(x) = cos (x-pi)

$$v_1 = 10\cos(\omega t - 130^\circ) \quad \text{or} \quad v_1 = 10\cos(\omega t + 230^\circ)$$ Because, cos(x) = cos (x+n(2pi))

and

$$v_2 = 12\sin(\omega t - 10^\circ) = 12\cos(\omega t - 10^\circ - 90^\circ)$$ Because, cos(x-90) = sin (x)

$$v_2 = 12\cos(\omega t - 100^\circ)$$

Therefore, the phase difference is 30°

$$v_2 = 12\cos(\omega t - 130^\circ + 30^\circ) \quad \text{or} \quad v_2 = 12\cos(\omega t + 260^\circ)$$

It is clear that v_2 leads v_1 by 30°

Phasors

- A phasor is a complex number that represents the amplitude and phase of a sinusoid.

Phasors provide a simple means of analyzing linear circuits excited by sinusoidal sources; solutions of such circuits would be intractable otherwise.

Complex Numbers

- A powerful method for representing sinusoids is the phasor.
- But in order to understand how they work, we need to cover some complex numbers first.
- A complex number ***z*** can be represented in rectangular form as:

$$z = x + jy \quad \text{where } j = \sqrt{-1};$$

It can also be written in polar or exponential form as:

$$z = r\angle\phi = re^{j\phi}$$

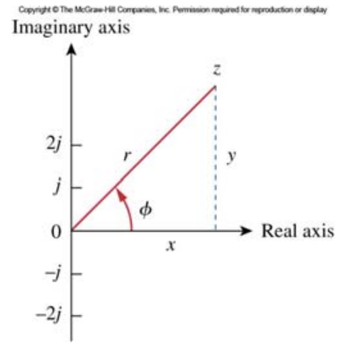
Complex Numbers

- The different forms can be interconverted.
- Starting with rectangular form, one can go to polar:

$$r = \sqrt{x^2 + y^2} \qquad \phi = \tan^{-1}\frac{y}{x}$$

- Likewise, from polar to rectangular form goes as follows:

$$x = r\cos\phi \qquad y = r\sin\phi$$



$$z = x + jy = r\angle\phi = r(\cos\phi + j\sin\phi)$$

Complex Numbers

- The following mathematical operations are important

Addition

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

Subtraction

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

Multiplication

$$z_1 z_2 = r_1 r_2 \angle(\phi_1 + \phi_2)$$

Division

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\angle(\phi_1 - \phi_2)$$

Reciprocal

$$\frac{1}{z} = \frac{1}{r}\angle(-\phi)$$

Square Root

$$\sqrt{z} = \sqrt{r}\angle(\phi/2)$$

Complex Conjugate

$$z^* = x - jy = r\angle -\phi = re^{-j\phi}$$

$$\frac{1}{j} = -j$$

Phasors

- The idea of a phasor representation is based on Euler's identity:

$$e^{\pm j\phi} = \cos\phi \pm j\sin\phi$$

- From this we can represent a sinusoid as the real component of a vector in the complex plane.
- The length of the vector is the amplitude of the sinusoid.
- The vector, V, in polar form, is at an angle ϕ with respect to the positive real axis.

$$\cos\phi = \mathrm{Re}(e^{j\phi})$$
$$\sin\phi = \mathrm{Im}(e^{j\phi})$$

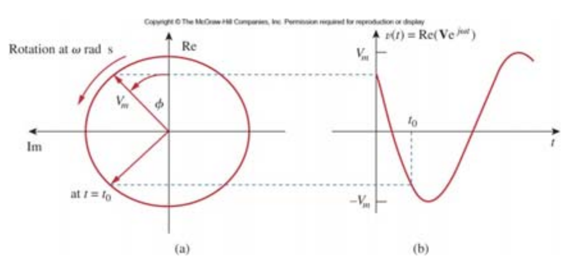
Phasors

- Phasors are typically represented at *t=0*.
- As such, the transformation between time domain to phasor domain is:

$$v(t) = V_m \cos(\omega t + \phi) \quad \Leftrightarrow \quad V = V_m \angle \phi$$

(Time-domain representation) (Phasor-domain representation)

- They can be graphically represented as shown here.



Sinusoid-Phasor Transformation

- Here is a handy table for transforming various time domain sinusoids into phasor domain:

TABLE 9.1

Sinusoid-phasor transformation.

Time domain representation	Phasor domain representation
$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	$V_m \angle \phi - 90^\circ$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle \theta - 90^\circ$

Sinusoid-Phasor Transformation

- Note that the frequency of the phasor is not explicitly shown in the phasor diagram
- For this reason phasor domain is also known as frequency domain.
- Applying a derivative to a phasor yields:

$$\underset{\text{(Time domain)}}{\frac{dv}{dt}} \Leftrightarrow \underset{\text{(Phasor domain)}}{j\omega V}$$

- Applying an integral to a phasor yeilds:

$$\underset{\text{(Time domain)}}{\int v dt} \Leftrightarrow \underset{\text{(Phasor domain)}}{\frac{V}{j\omega}}$$

Example

Using the phasor approach, determine the current $i(t)$ in a circuit described by the integrodifferential equation

$$4i + 8\int i\,dt - 3\frac{di}{dt} = 50\cos(2t + 75^\circ)$$

$\frac{dv}{dt}$ (Time domain)	$\Longleftrightarrow$	$j\omega\mathbf{V}$ (Phasor domain)	$\int v\,dt$ (Time domain)	$\Longleftrightarrow$	$\frac{\mathbf{V}}{j\omega}$ (Phasor domain)

$$4\mathbf{I} + \frac{8\mathbf{I}}{j\omega} - 3j\omega\mathbf{I} = 50\angle 75^\circ$$

But $\omega = 2$, so

$$\mathbf{I}(4 - j4 - j6) = 50\angle 75^\circ$$

$$\mathbf{I} = \frac{50\angle 75^\circ}{4 - j10} = \frac{50\angle 75^\circ}{10.77\angle -68.2^\circ} = 4.642\angle 143.2^\circ \text{ A}$$

Converting this to the time domain,

$$i(t) = 4.642\cos(2t + 143.2^\circ) \text{ A}$$

Homework:

9.2, 9.6, 9.8, 9.14, 9.18, 9.26.

Due: Sept. 08.