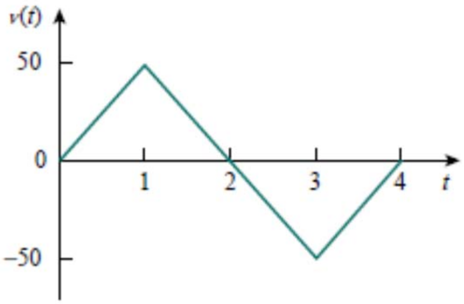
Determine the current through a 200 μF capacitor whose voltage is as in the figure.



Solution:

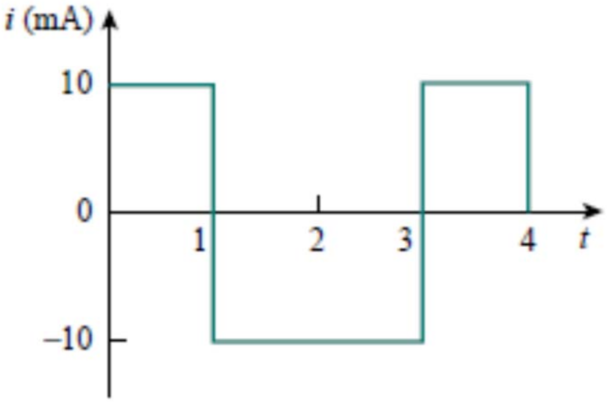
The voltage waveform can be described mathematically as

$$v(t) = \begin{cases} 50t \text{ V} & 0 < t < 1 \\ 100 - 50t \text{ V} & 1 < t < 3 \\ -200 + 50t \text{ V} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

Since $i = C\, dv/dt$ and $C = 200\ \mu\text{F}$, we take the derivative of v to obtain

$$i(t) = 200 \times 10^{-6} \times \begin{cases} 50 & 0 < t < 1 \\ -50 & 1 < t < 3 \\ 50 & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} 10 \text{ mA} & 0 < t < 1 \\ -10 \text{ mA} & 1 < t < 3 \\ 10 \text{ mA} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$



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Parallel Capacitors

- We learned with resistors that applying the equivalent series and parallel combinations can simply many circuits.
- Starting with N parallel capacitors, one can note that the voltages on all the caps are the same
- Applying KCL:

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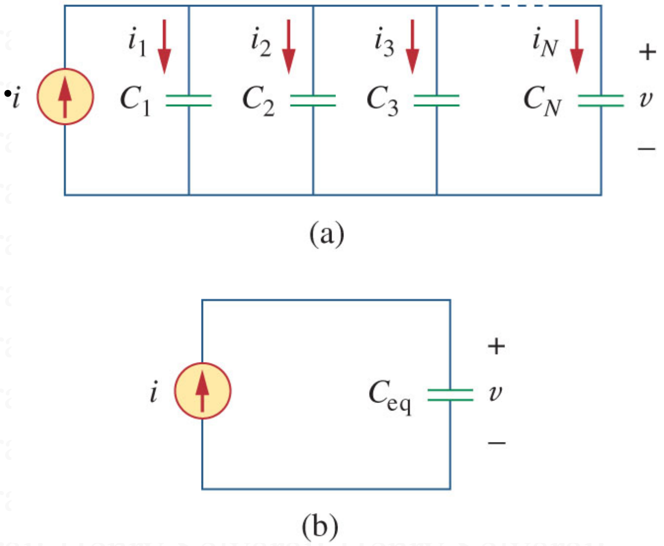

$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

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- Taking into consideration the current voltage relationship of each capacitor:

$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} + \cdots + C_N \frac{dv}{dt}$$

$$= \left(\sum_{k=1}^{N} C_k \right) \frac{dv}{dt} = C_{eq} \frac{dv}{dt}$$

- Where

$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N$$

- From this we find that parallel capacitors combine as the sum of all capacitance

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Series Capacitors

- Turning our attention to a series arrangement of capacitors:
- Here each capacitor shares the same current
- Applying KVL to the loop:

$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

- Now apply the voltage current relationship

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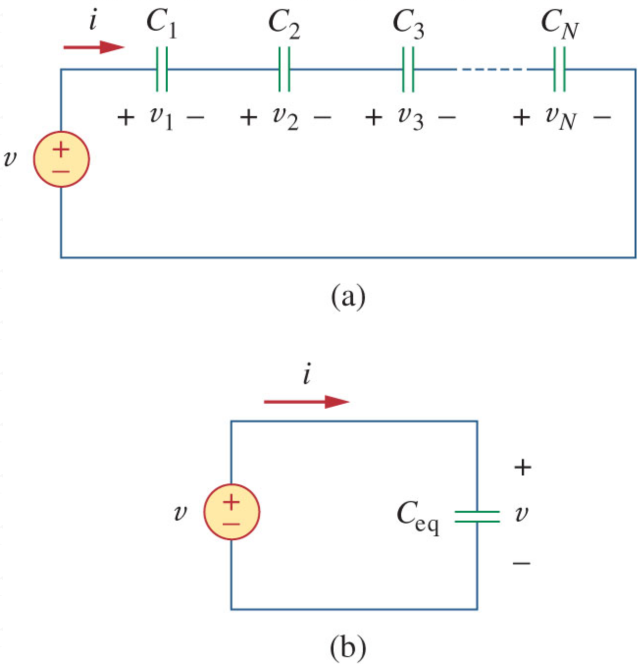

(a)

(b)

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$$v = \frac{1}{C_1}\int_{t_0}^{t} i(\tau)\,d\tau + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i(\tau)\,d\tau + v_2(t_0) + \frac{1}{C_3}\int_{t_0}^{t} i(\tau)\,d\tau + v_3(t_0) + \cdots + \frac{1}{C_N}\int_{t_0}^{t} i(\tau)\,d\tau + v_N(t_0)$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}\right)\int_{t_0}^{t} i(\tau)\,d\tau + v_1(t_0) + v_2(t_0) + v_3(t_0) + \cdots + v_N(t_0)$$

$$= \frac{1}{C_{eq}}\int_{t_0}^{t} i(\tau)\,d\tau + v(t_0)$$

- Where

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}$$

- From this we see that the series combination of capacitors resembles the parallel combination of resistors.

Series and Parallel Caps

Another way to think about the combinations of capacitors is this:

- Combining capacitors in parallel is equivalent to increasing the surface area of the capacitors:
- This would lead to an increased overall capacitance (as is observed)
- A series combination can be seen as increasing the total plate separation
- This would result in a decrease in capacitance (as is observed)

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- If a current is passed through an inductor, the voltage across it is directly proportional to the time rate of change in current

$$v = L\frac{di}{dt}$$

- Where, *L*, is the unit of inductance, measured in Henries, H.
- One Henry is 1 volt-second per ampere.
- The voltage developed tends to oppose a changing flow of current.

Current in an Inductor

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$$di = \frac{1}{L} v \, dt$$

- The current voltage relationship for an inductor is:

$$I = \frac{1}{L} \int_{t_0}^{t} v(\tau) d\tau + i(t_0)$$

- The power delivered to the inductor is: $p = vi = \left(L \frac{di}{dt} \right) i$
- The energy stored is: $w = \frac{1}{2} L i^2$

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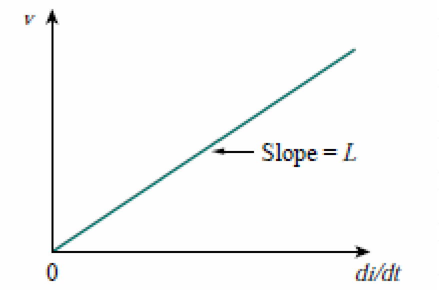
Voltage-Current relationship:



Properties of Inductors

- If the current through an inductor is constant, the voltage across it is zero
- **Thus an inductor acts like a short for DC**
- **The current through an inductor cannot change instantaneously**
- If this did happen, the voltage across the inductor would be infinity!
- This is an important consideration if an inductor is to be turned off abruptly; it will produce a high voltage

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- Like the ideal capacitor, the ideal inductor does not dissipate energy stored in it.
- Energy stored will be returned to the circuit later
- In reality, inductors do have internal resistance due to the wiring used to make them.
- A real inductor thus has a winding resistance in series with it.
- There is also a small winding capacitance due to the closeness of the windings
- These two characteristics are typically small, though at high frequencies, the capacitance may matter.

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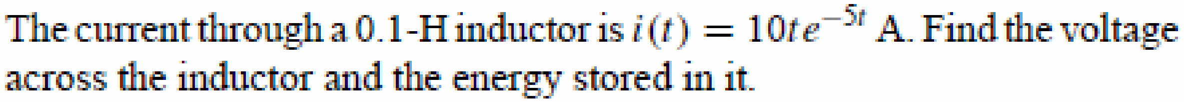
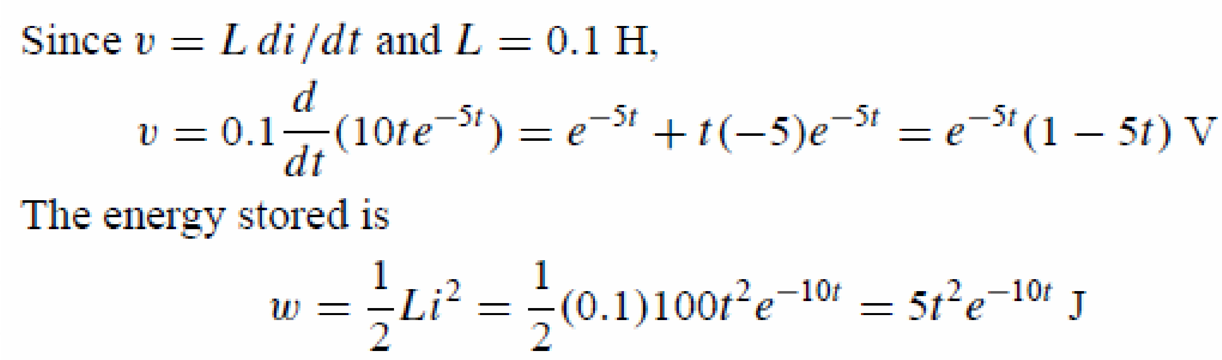


The current through a 0.1-H inductor is $i(t) = 10te^{-5t}$ A. Find the voltage across the inductor and the energy stored in it.



Since $v = L\,di/dt$ and $L = 0.1$ H,

$$v = 0.1\frac{d}{dt}(10te^{-5t}) = e^{-5t} + t(-5)e^{-5t} = e^{-5t}(1 - 5t) \text{ V}$$

The energy stored is

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.1)100t^2e^{-10t} = 5t^2e^{-10t} \text{ J}$$

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Under dc conditions, we replace the capacitor with an open circuit and the inductor with a short circuit

Series Inductors

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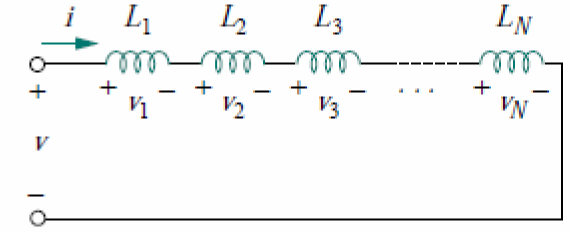
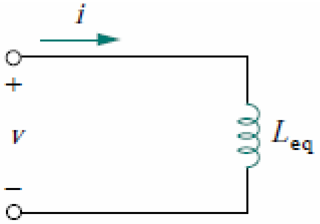
- We now need to extend the series parallel combinations to inductors
- First, let’s consider a series combination of inductors
- Applying KVL to the loop:





$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

The equivalent inductance of series-connected inductors is the sum of the individual inductances.

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Series Inductors II

Factoring in the voltage current relationship

$$v = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt} + \cdots + L_N \frac{di}{dt}$$

$$= \left(\sum_{k=1}^{N} L_k \right) \frac{di}{dt} = L_{eq} \frac{di}{dt}$$

Where $L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N$

Here we can see that the inductors have the same behavior as resistors

Parallel Inductors

- Now consider a parallel combination of inductors:
- Applying KCL to the circuit:

$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

- When the current voltage relationship is considered, we have:

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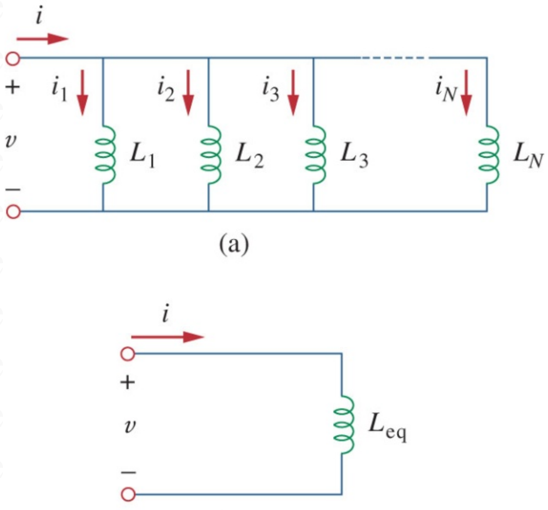

(a)

(b)

$$i = \left(\sum_{k=1}^{N}\frac{1}{L_k}\right)\int_{t_0}^{t} v\,dt + \sum_{k=1}^{N} i_k\left(t_0\right) = \frac{1}{L_{eq}}\int_{t_0}^{t} v\,dt + i\left(t_0\right)$$

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Parallel Inductors II

- The equivalent inductance is thus:

$$\frac{1}{L_{eq}}=\frac{1}{L_1}+\frac{1}{L_2}+\frac{1}{L_3}+\cdots+\frac{1}{L_N}$$

- Once again, the parallel combination resembles that of resistors
- On a related note, the Delta-Wye transformation can also be applied to inductors and capacitors in a similar manner, as long as all elements are the same type.

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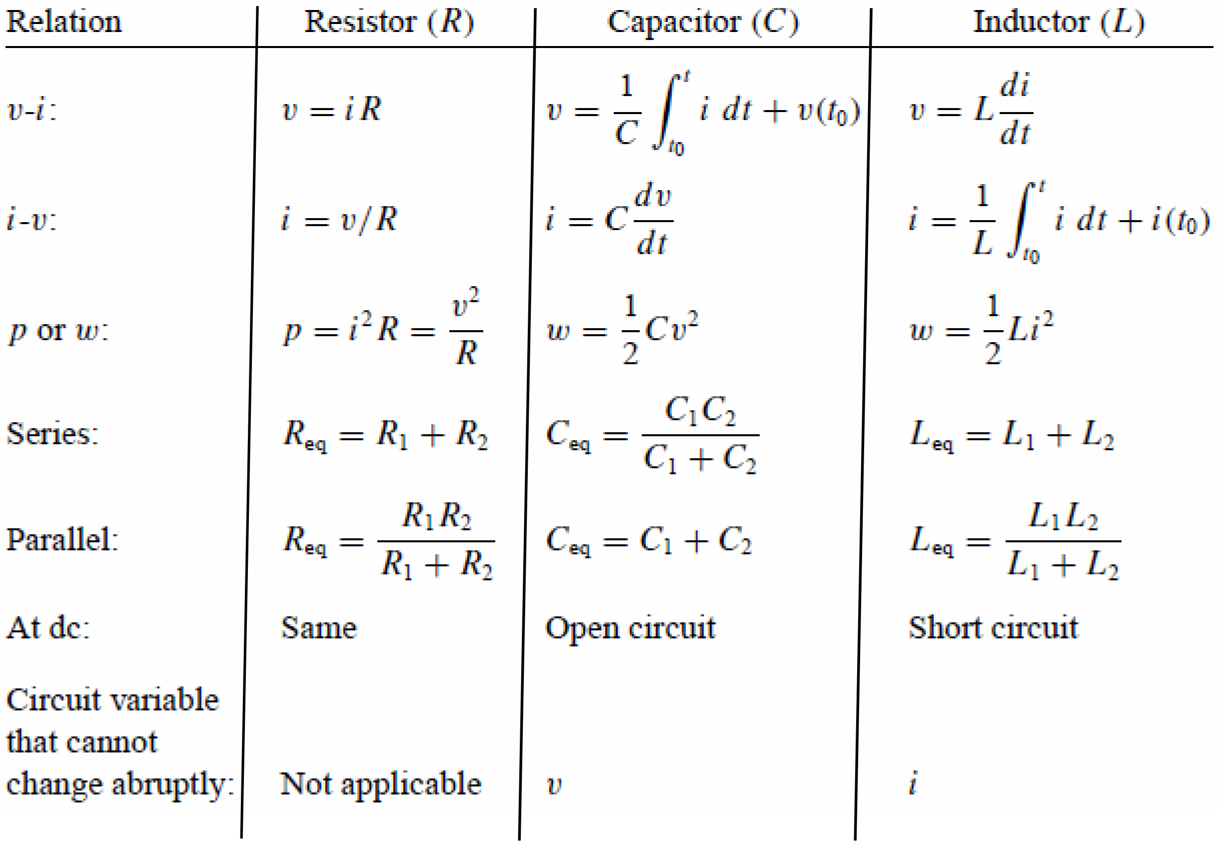
Summary of Capacitors and Inductors



Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} i\,dt + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i

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First Order Circuits

- A first order circuit is characterized by a first order differential equation.
- There are two types of first order circuits:
- Resistive capacitive, called RC
- Resistive inductive, called RL
- There are also two ways to excite the circuits:
- Initial conditions
- Independent sources

Source Free RC Circuit

- A source free RC circuit occurs when its dc source is suddenly disconnected.
- The energy stored in the capacitor is released to the resistors.
- Consider a series combination of a resistor and a initially charged capacitor as shown:

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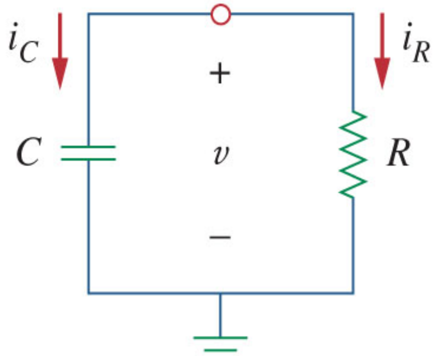

Source Free RC Circuit

- Since the capacitor was initially charged, we can assume at t=0 the initial voltages is: $v(0) = V_0$
- Applying KCL at the top node:

$$i_C + i_R = 0$$

Or

$$\frac{dv}{dt} + \frac{v}{RC} = 0$$

- This is a first order differential equation.

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Source Free RC Circuit

- Rearranging the equation and solving both sides yields:

$$\ln v = -\frac{t}{RC} + \ln A$$

- Where A is the integration constant
- Taking powers of e produces

$$v(t) = Ae^{-t/RC}$$

- With the initial conditions:

$$v(t) = V_0 e^{-t/RC}$$

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Natural Response

- The result shows that the voltage response of the RC circuit is an exponential decay of the initial voltage.
- Since this is the response of the circuit without any external applied voltage or current, the response is called the natural response.

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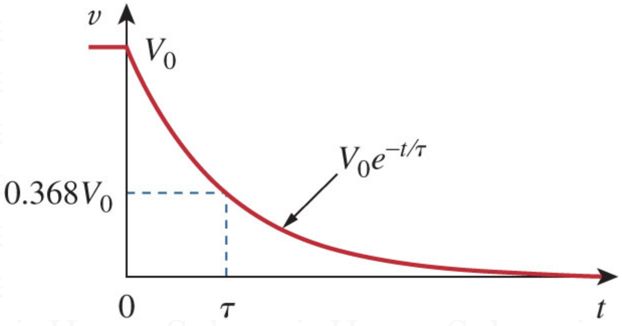

Time Constant

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- The speed at which the voltage decays can be characterized by how long it takes the voltage to drop to *1/e* of the initial voltage.
- This is called the time constant and is represented by τ.
- By selecting *1/e* as the reference voltage:

$$\tau = RC$$

- The voltage can thus be expressed as: $v(t) = V_0 e^{-t/\tau}$

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Time Constant II

- After five time constants the voltage on the capacitor is less than one percent.
- After five time constants a capacitor is considered to be either fully discharged or charged
- A circuit with a small time constant has a fast response and vice versa.

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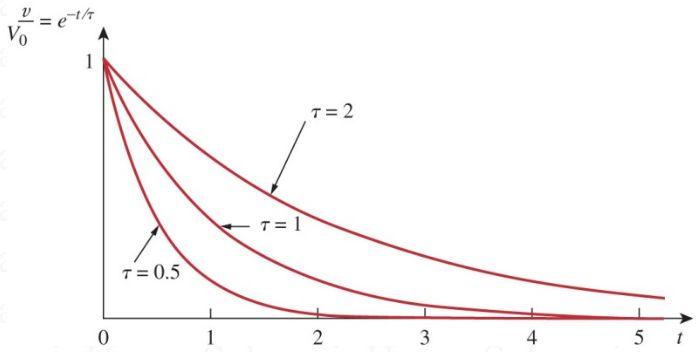

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RC Discharge

- With the voltage known, we can find the current:

$$i_R(t) = \frac{V_0}{R} e^{-t/\tau}$$

- The power dissipated in the resistor is:

$$p(t) = \frac{V_0^2}{R} e^{-2t/\tau}$$

- The energy absorbed by the resistor is:

$$w_R(t) = \frac{1}{2} C V_0^2 \left(1 - e^{-2t/\tau}\right)$$

Source Free RC Circuit Summary

- The key to working with this type of situation is:
- Start with the initial voltage across the capacitor and the time constant.
- With these two items, the voltage as a function of time can be known.
- From the voltage, the current can be known by using the resistance and Ohm’s law.
- The resistance of the circuit is often the Thevenin equivalent resistance.

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Source Free RL Circuit

- Now lets consider the series connection of a resistor and inductor.
- In this case, the value of interest is the current through the inductor.
- Since the current cannot change instantaneously, we can determine its value as a function of time.
- Once again, we will start with an initial current passing through the inductor.

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Source Free RL Circuit

- We will take the initial current to be:

$$i(0) = I_0$$

- Applying KVL around the loop:

$$v_L + v_R = 0$$

Or

$$L\frac{di}{dt} + Ri = 0$$

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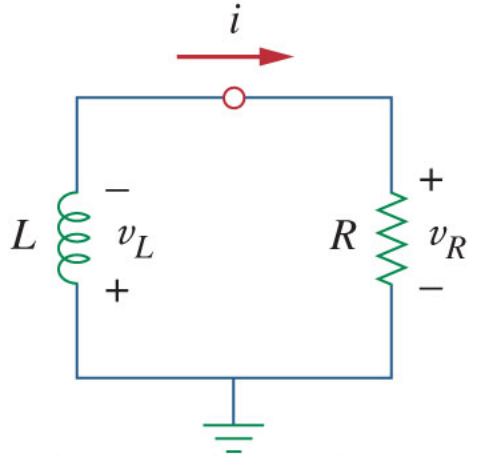

Source Free RL Circuit

- After integration:

$$i(t) = I_0 e^{-Rt/L}$$

- Once again, the natural response is an exponentially decaying current.
- The time constant in this case is:

$$\tau = \frac{L}{R}$$

- The same principles as the RC circuit apply here.

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