# EE 221:Circuits II

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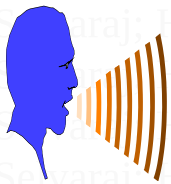
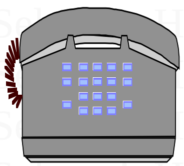
# Our Communication



Room TBE B336



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# Lecture Hours

Mondays and Wednesdays 1:00 pm. – 2:15 pm.

TBE B176

# Office Hours

Mondays and Wednesdays 4:00 pm. – 5:00 pm.

OR

By appointment

# **Textbook**:

Fundamentals of Electric Circuits

Charles K. Alexander and Matthew N.O. Sadiku

5th Edition

- **Prerequisite:**

  EE 220, and CSC 117 or CSC 135

**Semester Grades will be computed as follows:**

| | | |
|---|---|---|
| Exam 1: | 20 points | September 29. |
| Exam 2: | 25 points | October 27. |
| final exam: | 35 points | |
| home work: | 20 points | |
| Total | 100 points | |

Guaranteed Grades: A- ( > 90%); B- ( > 80%); C- ( > 70%);

# TOPICS



- Sinusoidal Steady-State Analysis: sinusoids, phasor concept, impedance and admittance, phasor diagram.
- AC Steady-State Analysis: nodal, mesh, superposition, Thevenin and Norton equivalents, Op Amp circuits.
- AC Power Analysis: instantaneous and average power, complex power, power factor, power superposition principle, conservation of power.
- Three-Phase Circuits: Y and Delta circuits, balanced and unbalanced circuits, power in three-phase circuits, two-wattmeter power measurement.
- Magnetically Coupled Circuits. Mutual inductance, energy in coupled circuits, linear transformer, ideal transformer.
- Frequency Response: transfer functions, gain and phase shift, resonant circuits, Bode plots, passive and active filters.
- Two-Port Networks: impedance, admittance, hybrid, and transmission line parameters, interconnection of networks and applications.

**Course Objectives are to learn to**

- solve circuits problems under sinusoidal steady-state conditions using phasors and impedances,
- calculate various powers and how to correct the power factor in sinusoidal steady-state circuits,
- analyze circuits with coupled inductors and ideal transformers,
- analyze both balanced and unbalanced three-phase circuits,
- measure and calculate real power in three-phase circuits,
- derive the frequency response of electric circuits using Bode plots,
- apply two-port networks to circuit problems.

# COURSE OUTCOMES

**Students should be able to**

- analyze steady-state sinusoidal circuits using phasors and impedance,
- calculate real, reactive, apparent and complex powers, and correct the power factor in a given circuit,
- analyze three-phase circuits and calculate real power,
- derive and plot the frequency response of a given circuit,
- analyze circuits using two-port networks.

# ABET COURSE OUTCOMES

- The appropriate technical knowledge and skills:
- an ability to apply mathematics through differential and integral calculus,
- an ability to apply advanced mathematics such as differential equations, linear algebra, complex variables, and discrete mathematics,
- an ability to apply knowledge of basic sciences,
- an ability to apply knowledge of engineering,
- an ability to identify, formulate, and solve engineering problems,
- an ability to analyze and design complex electrical and electronic devices,
- an ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.

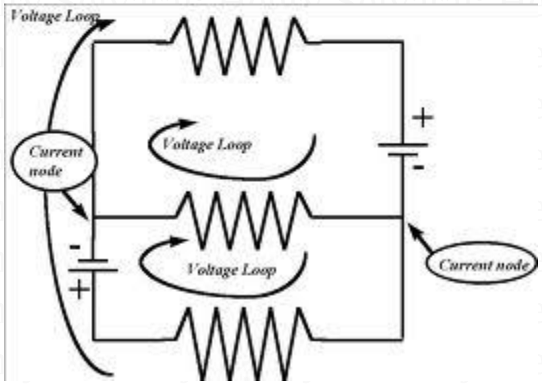
# What did we learn in EE220

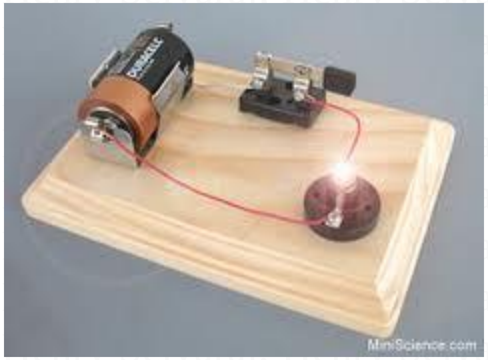
- Introduction, basic definitions, basic electric elements
- Ohm's Law and Kirchhoff's Laws
- Network Equivalence, Series Resistors and Voltage Division, Parallel Resistors and Current Division, Analysis of Series-Parallel Circuits
- Analysis techniques (Nodal, Mesh and Loop Analysis)
- Linear Property, superposition, source transformation theorems
- Thevenin and Norton Equivalent circuits, maximum power transfer
- Op-amps and Op-amp-based circuits
- The energy storage elements
- First order RL and RC circuits
- Second-Order circuits

# What is circuit

- Energy is communicated or transferred from one point to another. We require an interconnection of electric devices to do this. Such interconnection is referred to as *electric circuit* and each component of the circuit is known as an *element*.
- An **electric circuit** is an interconnection of elements.

# Electric circuits




# Systems and Units

International System of Units (SI), adopted by the General Conference on Weights and measurements in 1960.

## Basic Units

| Quantity | Basic unit | Symbol |
|---|---|---|
| Length | meter | m |
| Mass | kilogram | Kg |
| Time | second | s |
| Electric current | ampere | A |
| Thermodynamic temperature | kelvin | K |
| Luminous intensity | candela | cd |

## Derived units

| Quantity | Unit | Symbol |
|---|---|---|
| electric charge | coulomb | C |
| electric potential | volt | V |
| resistance | ohm | Ω |
| conductance | siemens | S |
| inductance | henry | H |
| capacitance | farad | F |
| frequency | hertz | Hz |
| force | newton | N |
| energy, work | joule | J |
| power | watt | W |
| magnetic flux | weber | Wb |
| magnetic flux density | tesla | T |

## Decimal multiples and submultiples of SI units

| Factor | Prefix | Symbol |
|---|---|---|
| $10^9$ | giga | G |
| $10^6$ | mega | M |
| $10^3$ | kilo | k |
| $10^{-2}$ | centi | c |
| $10^{-3}$ | milli | m |
| $10^{-6}$ | micro | $\mu$ |
| $10^{-9}$ | nano | n |
| $10^{-12}$ | pico | p |

# Electric charge

- Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C).
- The charge e on one electron is negative and equal in magnitude to $1.602 \times 10^{-19}$ C which is called as electronic charge. The charges that occur in nature are integral multiples of the electronic charge.

# Current

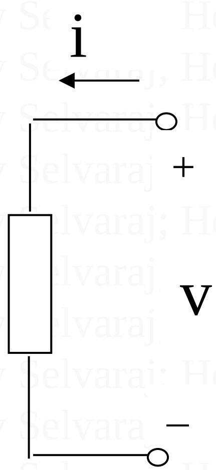
- Electric current $i = dq/dt$. The unit of ampere can be derived as $1\ A = 1 C/s$.
- A direct current (dc) is a current that remains constant with time.
- An alternating current (ac) is a current that varies sinusoidally with time. (reverse direction)

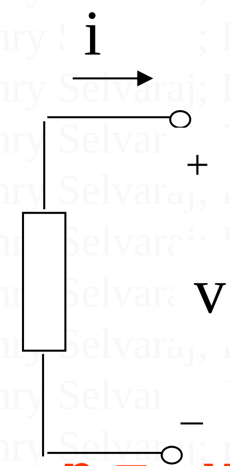
# Voltage

- Voltage (or potential difference) is the energy required to move a unit charge through an element, measured in volts (V).
- Mathematically, $v_{ab} = dw/dq$ (volt)
  - $w$ is energy in joules (J) and $q$ is charge in coulomb (C).
- Electric voltage, $v_{ab}$, is always across the circuit element or between two points in a circuit.
  - $v_{ab} > 0$ means the potential of a is higher than potential of b.
  - $v_{ab} < 0$ means the potential of a is lower than potential of b.

# Power and Energy

- Power is the time rate of expending or absorbing energy, measured in watts (W).
- Mathematical expression: $$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = vi$$




**Passive sign convention**

**P = +vi**
**absorbing power**

**p = –vi**
**supplying power**

# Power and Energy

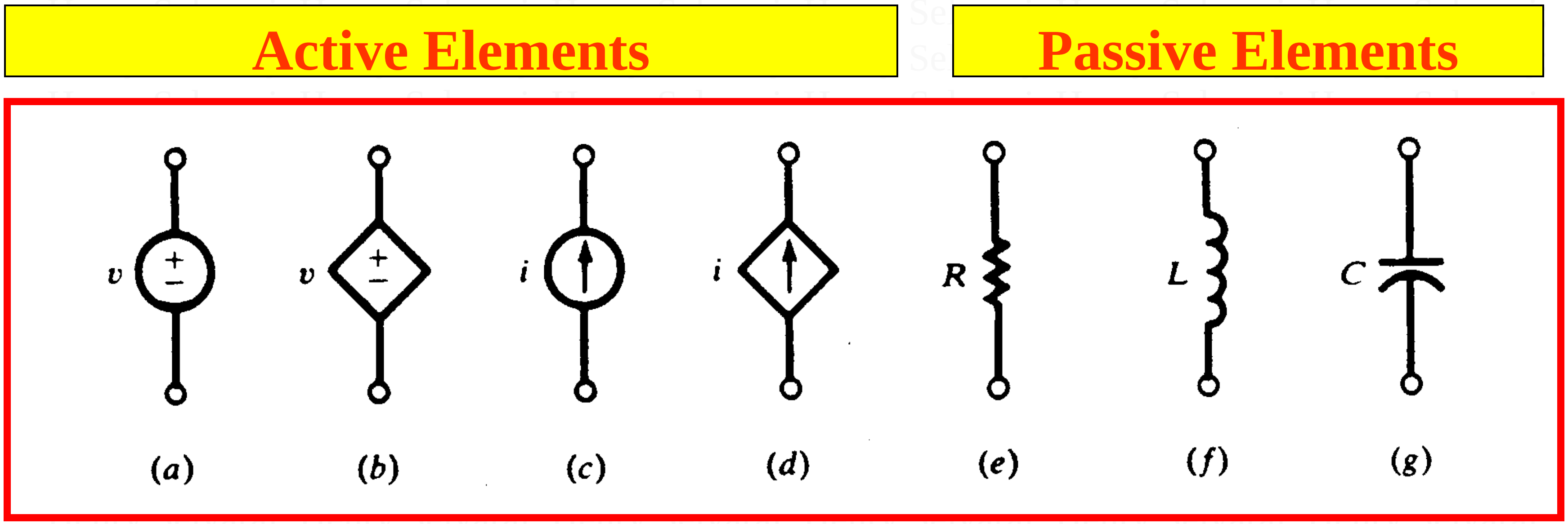


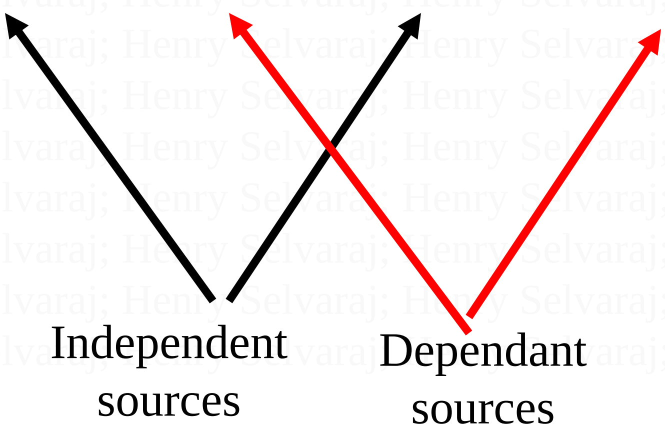
- The law of conservation of energy

$$\sum p = 0$$

- Energy is the capacity to do work, measured in joules (J).
- Mathematical expression $w = \int_{t_0}^{t} p\,dt = \int_{t_0}^{t} vi\,dt$

# Circuit Elements




- **A dependent source is an active element in which the source quantity is controlled by another voltage or current.**
- **They have four different types: VCVS, CCVS, VCCS, CCCS. Keep in mind the signs of dependent sources.**

- The Coulomb is a large unit of charges. In 1 C of charge, there are $1/(1.602 \times 10^{-19}) = 6.24 \times 10^{18}$ electrons. Thus realistic or laboratory values of charges are on the order of pC, nC, or μC.
- Only charges that occur in nature are integral multiples of the electronic charge $e = -1.602 \times 10^{-19}$
- **Law of conservation of charge: charge can neither be created nor destroyed, only transferred.**

# Revision of definitions

Electric current is the time rate of change of charge, measured in *Amperes (A)*. $i \triangleq \frac{dq}{dt}$

Voltage (or potential difference) is the energy required to move a unit charge through an element, measured in volts (V).

$$v_{ab} = dw / dq$$

Power is the time rate of expending or absorbing energy, measured in watts (W). $p \triangleq \frac{dw}{dt}$

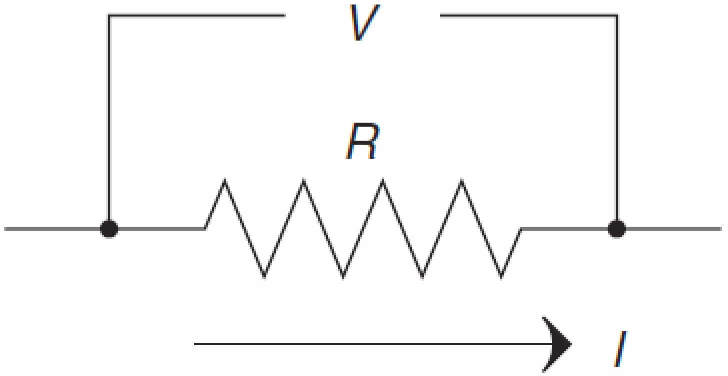
Energy is the capacity to do work, measured in joules (J). $w = \int_{t_0}^{t} p\,dt = \int_{t_0}^{t} vi\,dt$

# Ohm's Law



The voltage *v* across a linear element is directly proportional to the current *i* flowing through the element under constant temperature and pressure.

$$v = i\,\mathrm{R}$$




$$V = I \cdot R$$

$$I = \frac{V}{R}$$

$$R = \frac{V}{I}$$

The power dissipated by a resistor can be expressed in terms of *R*.

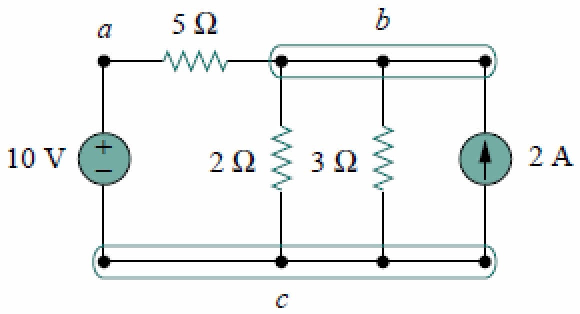
$$p = vi = i^2 R = \frac{v^2}{R}$$

The power dissipated by a resistor may also be expressed in terms of *G* as:

$$p = vi = v^2 G = \frac{i^2}{G}$$

Since *R* and *G* are positive quantities, the power dissipated in a resistor is always positive. Thus, a resistor always absorbs power from the circuit. This confirms the idea that a resistor is a passive element, incapable of generating energy.

# Nodes, branches and loops




A branch represents a single element such as a voltage source or a resistor.

A node is the point of connection between two or more branches.

The circuit has five branches and three nodes.

# A loop is any closed path in a circuit.

A loop is a closed path formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once. A loop is said to be *independent* if it contains a branch which is not in any other loop. Independent loops or paths result in independent sets of equations.

# KCL

- Kirchoff's current law is based on conservation of charge
- It states that the algebraic sum of currents entering a node (or a closed boundary) is zero.
- It can be expressed as:

$$\sum_{n=1}^{N} i_n = 0$$

# KVL

- Kirchoff’s voltage law is based on conservation of energy
- It states that the algebraic sum of voltages around a closed path (or loop) is zero.
- It can be expressed as:

$$\sum_{m=1}^{M} v_m = 0$$

# Nodal Analysis

- If instead of focusing on the voltages of the circuit elements, one looks at the voltages at the nodes of the circuit, the number of simultaneous equations to solve for can be reduced.
- Given a circuit with n nodes, without voltage sources, the nodal analysis is accomplished via three steps:
  1. Select a node as the reference node. Assign voltages $v_1, v_2, \ldots v_n$ to the remaining *n-1* nodes, voltages are relative to the reference node.
  2. Apply KCL to each of the *n-1* non-reference nodes. Use Ohm's law to express the branch currents in terms of node voltages
  3. Solve the resulting *n-1* simultaneous equations to obtain the unknown node voltages.
- The reference, or datum, node is commonly referred to as the ground since its voltage is by default zero.

# Supernode



- A supernode is formed by enclosing a voltage source (dependant or independent) connected between two non-reference nodes and any elements connected in parallel with it.
- Why?
  - Nodal analysis requires applying KCL
  - The current through the voltage source cannot be known in advance (Ohm's law does not apply)
  - By lumping the nodes together, the current balance can still be described
- In the example circuit node 2 and 3 form a supernode
- The current balance would be:
- Or this can be expressed as:

$$i_1 + i_4 = i_2 + i_3$$

$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2 - 0}{8} + \frac{v_3 - 0}{6}$$

# Mesh Analysis

- Another general procedure for analyzing circuits is to use the mesh currents as the circuit variables.
- Recall:
  - A loop is a closed path with no node passed more than once
  - A mesh is a loop that does not contain any other loop within it
- Mesh analysis uses KVL to find unknown currents
- Mesh analysis is limited in one aspect: It can only apply to circuits that can be rendered planar.
- A planar circuit can be drawn such that there are no crossing branches.

# Supermesh

- Similar to the case of nodal analysis where a voltage source shared two non-reference nodes, current sources (dependent or independent) that are shared by more than one mesh need special treatment
- The two meshes must be joined together, resulting in a supermesh.
- The supermesh is constructed by merging the two meshes and excluding the shared source and any elements in series with it
- A supermesh is required because mesh analysis uses KVL
- But the voltage across a current source cannot be known in advance.
- Intersecting supermeshes in a circuit must be combined to for a larger supermesh.

# Mesh analysis when…

- If the network contains:
  - Many series connected elements
  - Voltage sources
  - Supermeshes
  - A circuit with fewer meshes than nodes
- If branch or mesh currents are what is being solved for.
- Mesh analysis is the only suitable analysis for transistor circuits
- It is not appropriate for operational amplifiers because there is no direct way to obtain the voltage across an op-amp.

# Nodal analysis if…

- If the network contains:
  - Many parallel connected elements
  - Current sources
  - Supernodes
  - Circuits with fewer nodes than meshes
- If node voltages are what are being solved for
- Non-planar circuits can only be solved using nodal analysis
- This format is easier to solve by computer

# KCL and KVL for a BJT

- The currents from each terminal can be related to each other as follows:

$$I_E = I_B + I_C$$

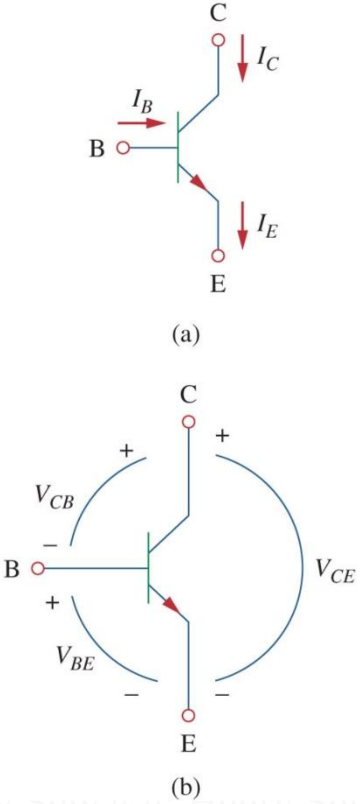
- The base and collector current can be related to each other by the parameter β, which can range from 50-1000

$$I_C = \beta I_B$$

- Applying KVL to the BJT gives:

$$V_{CE} + V_{EB} + V_{BC} = 0$$






(a)

(b)

# Superposition

- If there are two or more independent sources there are two ways to solve for the circuit parameters:
  - Nodal or mesh analysis
  - Use superposition
- The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

# Applying Superposition

- Using superposition means applying one independent source at a time
- Dependent sources are left alone
- The steps are:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using the techniques covered in Chapters 2 and 3.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

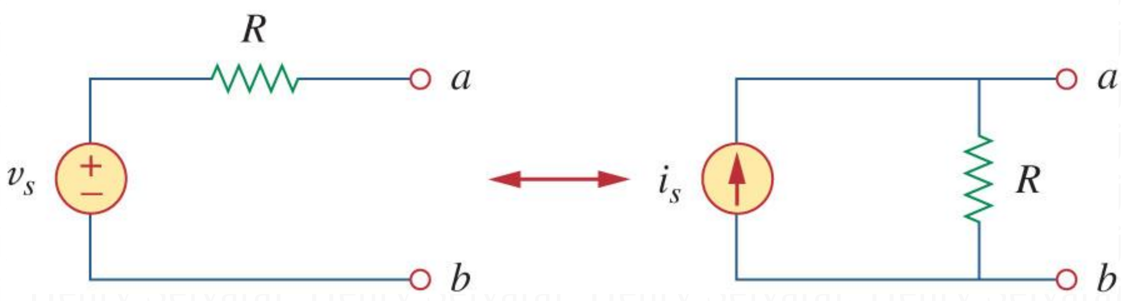
# Source Transformation

- Much like the delta-wye transformation, it is possible to transform a source from one form to another
- This can be useful for simplifying circuits
- The principle behind all of these transformations is equivalence

# Source Transformation II

- A source transformation is the process of replacing a voltage source $v_s$ in series with a resistor $R$ by a current source $i_s$ in parallel with a resistor $R$, or vice versa.

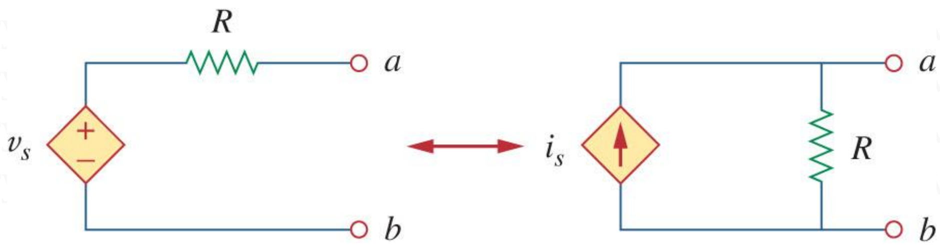




# Dependent Sources

- Source transformation also applies to dependent sources
- But, the dependent variable must be handled carefully
- The same relationship between the voltage and current holds here:





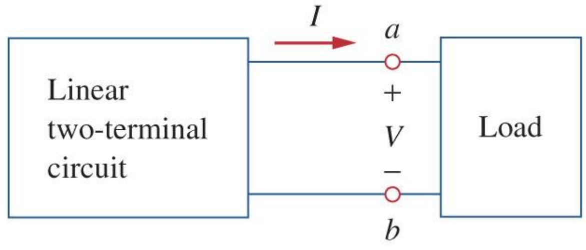
# Thevenin's Theorem

- In many circuits, one element will be variable
- An example of this is mains power; many different appliances may be plugged into the outlet, each presenting a different resistance
- This variable element is called the load
- Ordinarily one would have to reanalyze the circuit for each change in the load

# Thevenin's Theorem II

- Thevenin's theorem states that a linear two terminal circuit may be replaced with a voltage source and resistor
- The voltage source's value is equal to the open circuit voltage at the terminals
- The resistance is equal to the resistance measured at the terminals when the independent sources are turned off.



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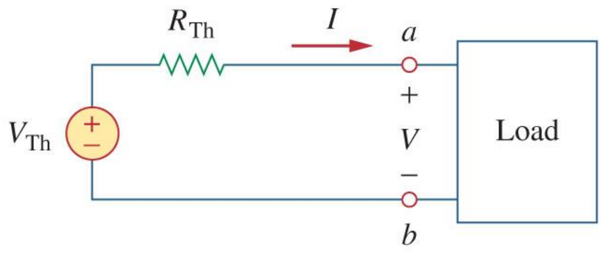
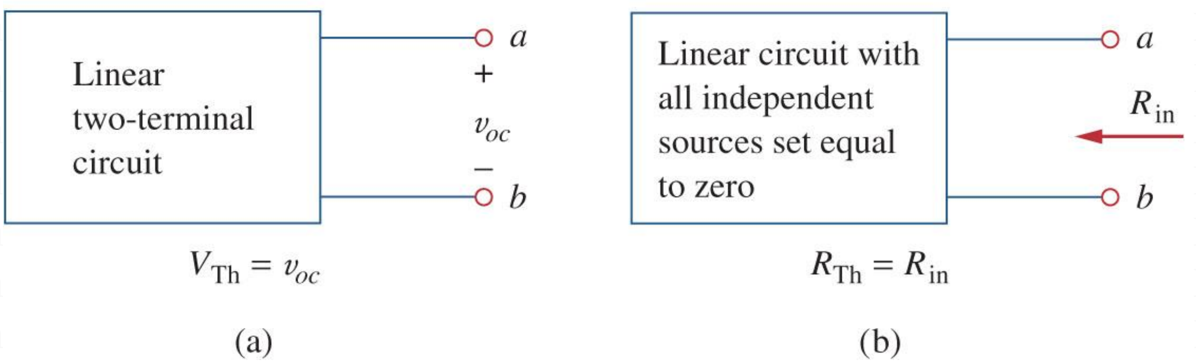



(b)

# Thevenin's Theorem III

- There are two cases to consider when finding the equivalent resistance
- Case 1: If there are no dependent sources, then the resistance may be found by simply turning off all the sources

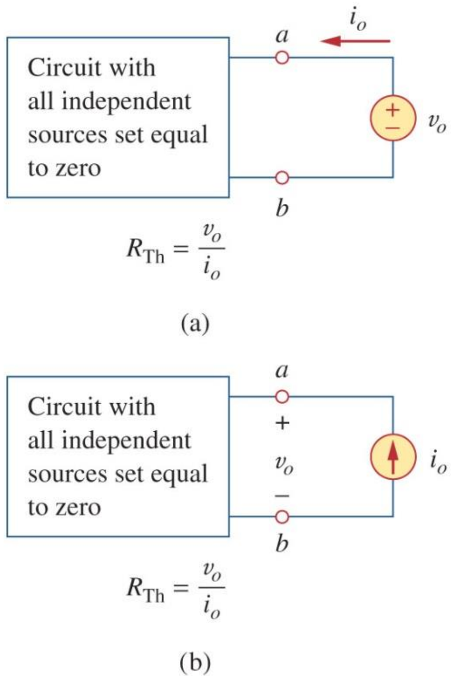




# Thevenin's Theorem IV



- Case 2: If there are dependent sources, we still turn off all the independent sources.
- Now apply a voltage $v_0$ (or current $i_0$)to the terminals and determine the current $i_0$ (voltage $v_0$).



# Thevenin's Theorem V

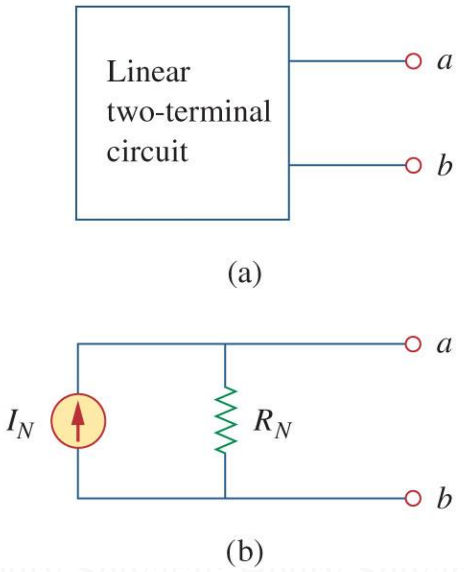


- Thevenin's theorem is very powerful in circuit analysis.
- It allows one to simplify a circuit
- A large circuit may be replaced by a single independent voltage source and a single resistor.
- The equivalent circuit behaves externally exactly the same as the original circuit.

# Norton's Theorem



- Similar to Thevenin's theorem, Norton's theorem states that a linear two terminal circuit may be replaced with an equivalent circuit containing a resistor and a current source
- The Norton resistance will be exactly the same as the Thevenin



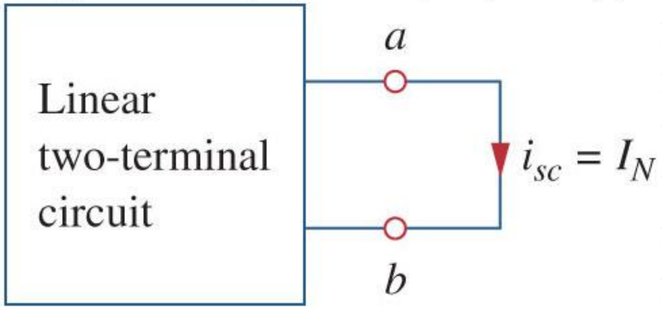
# Norton's Theorem II



- The Norton current $I_N$ is found by short circuiting the circuit's terminals and measuring the resulting current

$$I_N = i_{sc}$$





# Norton vs. Thevenin



- These two equivalent circuits can be related to each other
- One need only look at source transformation to understand this
- The Norton current and Thevein voltage are related to each other as follows:

$$I_N = \frac{V_{Th}}{R_{Th}}$$

# Norton vs. Thevenin II

- With $V_{TH}$, $I_N$, and ($R_{TH}$=$R_N$) related, finding the Thevenin or Norton equivalent circuit requires that we find:
- The open-circuit voltage across terminals *a and b*.
- The short-circuit current at terminals *a and b*.
- The equivalent or input resistance at terminals *a and b when* all independent sources are turned off.

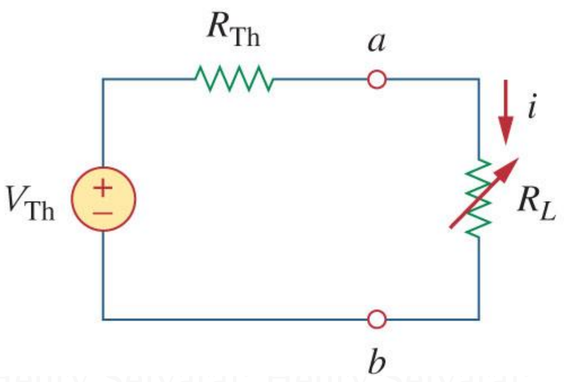
# Maximum Power Transfer

- In many applications, a circuit is designed to power a load
- Among those applications there are many cases where we wish to maximize the power transferred to the load
- Unlike an ideal source, internal resistance will restrict the conditions where maximum power is transferred.

# Maximum Power Transfer II

- We can use the Thevenin equivalent circuit for finding the maximum power in a linear circuit
- We will assume that the load resistance can be varied
- Looking at the equivalent circuit with load included, the power transferred is:



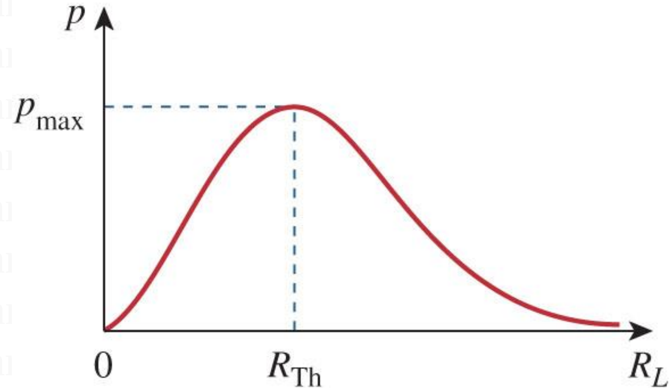



$$p=\left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L$$

# Maximum Power Transfer III

- For a given circuit, $V_{TH}$ and $R_{TH}$ are fixed. By varying the load resistance $R_L$, the power delivered to the load varies as shown
- You can see that as $R_L$ approaches 0 and ∞ the power transferred goes to zero.
- In fact the maximum power transferred is when $R_L=R_{TH}$

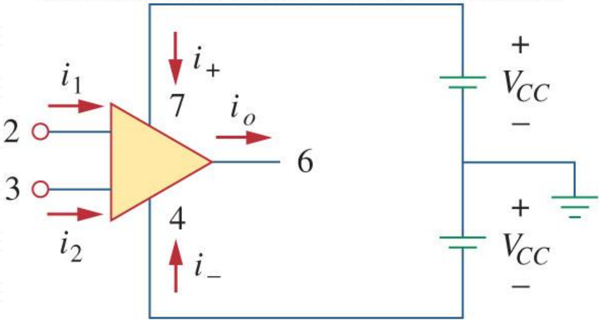




# Powering an Op-amp

- As an active element, the op-amp requires a power source
- Often in circuit diagrams the power supply terminals are obscured
- It is taken for granted that they must be connected
- Most op-amps use two voltage sources, with a ground reference between them
- This gives a positive and negative supply voltage.

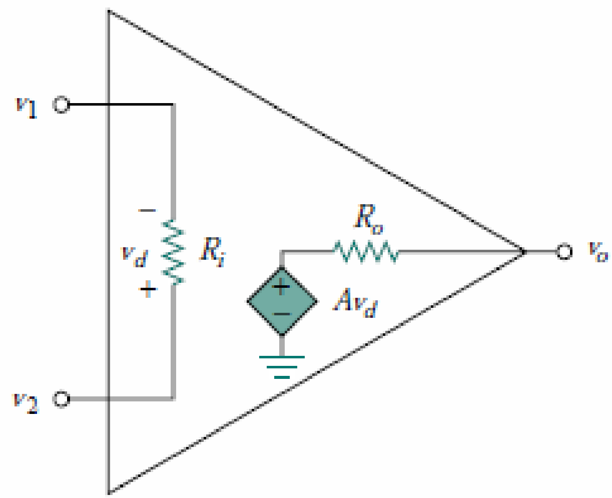





$$i_o = i_1 + i_2 + i_+ + i_-$$

# Equivalent circuit of nonideal op amp

$v_d = v_2 - v_1$




$$v_o = Av_d = A(v_2 - v_1)$$

A: open loop voltage gain.

$$A\,dB = 20 \log_{10} A$$