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Routing and wavelength assignment for core-based tree in WDM networks ☆

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Abstract

In this paper, we address the routing and wavelength assignment problem for the core-based tree (CBT) service in a wavelength-division-multiplexing (WDM) network, where k sources need to send data to a common core node. We formally model the problem as a problem of finding k shortest lightpaths from sources to the core with the constraint of wavelength collision free. To address different objectives, we define and study several subproblems. For the feasibility and the minimum total cost problems of k shortest lightpaths, we show how the classical network flow algorithms can be modified and applied efficiently on the network flow model constructed on the transformed wavelength graph. For the minimum max-cost and the constrained feasibility problems, we prove their NP-completeness and propose two efficient heuristic algorithms. Simulation results show that the proposed heuristic algorithms achieve performance very close to the calculated lower bounds.

Keywords: WDM; Core-based tree; Routing and wavelength assignment; Network flow; NP-completeness

1. Introduction

Wavelength-division-multiplexing (WDM) networks are taking shape as the consequence of the rapid growth of the Internet together with emerging network applications which demand high bandwidth and low communication delay. In a WDM network, a link (fiber) between two nodes has multiple wavelengths, each of which can transmit signals independently. Usually, each wavelength is assumed to have different cost and availability. Therefore, in addition to finding a path which specifies a sequence of links between the two nodes, an available wavelength on each link also needs to be reserved for the connection request from a sender to a receiver. The resultant problem is called the routing and wavelength assignment problem. In the WDM network, cost occurs not only on the links, but also on a node when different wavelengths are used on the incoming link and outgoing link on that node. This cost is called wavelength conversion cost. Finding the shortest path (the path with the least cost) in a WDM network was originally studied by Chlamtac et al. [4], in which they defined a lightpath as a sequence of connected links with the same assigned wavelength, and a semilightpath as a sequence of connected lightpaths with wavelength conversion between two lightpaths. For simplicity, in this paper, we refer to a semilightpath as a lightpath [4].

Multicast is a means of one-to-many communication, which is an important communication scheme in the Internet. The concepts and applications of multicast can be found in [14]. The multicast routing problem for legacy

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packet switching network has been well studied and in use in the past decade. In the WDM network, multicast can be implemented at higher layers and the WDM layer is used as a transmission medium. A more efficient way is to make the WDM layer to support multicast, which is referred to as WDM multicast [5,15,18]. The problem of routing and wavelength assignment for WDM multicast is to find a multicast tree rooted at the source node with wavelengths assigned to minimize various criteria, such as those studied in [3,8,10,12,16,17,19].

An extension of the one-to-many communication is the many-to-many communication scheme. Core-based tree (CBT) is such an extension in which there are multiple senders (sources) that need to send data to a common group of receivers. In a CBT, there is a core node which receives messages from all sources and then sends the messages to receivers. One of the advantages of the CBT is its scalability. Constructing a source-based multicast tree for each source may suffer the scalability problem of routing when the number of multicast sessions increases. By sending messages to one core node, multiple sources can share the same multicast tree rooted at the core node. The other advantage is the efficient bandwidth utilization. The core node can multiplex and send data from multiple sources by using only one wavelength on each link in the multicast tree.

Core-based tree has been extensively studied in conventional IP networks, such as those works done in [2,7,11]. However, we find that virtually no work has been done to efficiently implement CBT in WDM networks. In this paper, we try to fill this gap. The routing problem for a CBT essentially contains two subproblems: routing for the sources to send data to the core, which is to find a lightpath from each source to the core, and routing for the core, which is to find a multicast tree rooted at the core. In this paper, we only address the first problem since the second problem is purely a multicast routing problem. When the CBT is used in IP networks, the first problem can be easily solved by establishing a shortest path from each source to the core. However, in the WDM networks, the problem becomes complex due to the fact that one wavelength on a link can only be used by one lightpath.

In this paper, we study the routing and wavelength assignment problem for sources by introducing the so-called k shortest lightpaths problem, where k is the number of sources in the CBT. To address different objectives, we further define several subproblems, including the feasibility problem, the minimum total cost problem, the minimum max-cost problem, and the constrained feasibility problem. For the first two subproblems, we show how the classical network flow algorithms can be modified and efficiently applied on the network flow model constructed on the transformed wavelength graph [4]. The proposed algorithms generalize the algorithms proposed in [4]. For the last two subproblems, we prove their NP-completeness and propose two heuristic algorithms. Simulations for heuristic algorithms have been conducted and simulation results show that the proposed heuristic algorithms perform very close to the calculated lower bounds.

The rest of the paper is organized as follows. In Section 2, we describe the network model and the problem statement and propose several objectives for setting up the k shortest lightpaths. In Section 3, we present the solutions to the k shortest lightpaths problems. In Section 4, we present and discuss simulation results. In Section 5, we summarize the paper.

2. Network model and problem statement

A WDM network can be modeled as a directed graph $G = (V, E)$, where $V = (v_1, v_2, \ldots, v_n)$ stands for the set of network nodes and E stands for the set of directed links between nodes. The undirected version of the network can be modeled by replacing an undirected link with two oppositely directed links.

Suppose that a set $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_w\}$ of wavelengths are provided by the network. Let $\Lambda(e) \subset \Lambda$ be the set of wavelengths available on link $e \in E$. For each link e and wavelength $\lambda_i \in \Lambda(e)$, a non-negative weight $f_w(e, \lambda_i)$ is associated, representing the "cost" of using wavelength λ_i on link e. $f_w(e, \lambda_i)$ can be formulated to reflect the cost of wavelength (bandwidth) and/or the propagation delay on e.

The "cost" of wavelength conversion at node v is modeled via cost factors of the form $f_c(v, \lambda_{v\_\text{in}}, \lambda_{v\_\text{out}})$, which is the cost of wavelength conversion from an available input wavelength λ_{v_in} to an available output wavelength λ_{v_out} when such a conversion is available at node v. In particular, $f_c(v, \lambda_{v\_\text{in}}, \lambda_{v\_\text{out}}) = 0$ if λ_{v_in} and λ_{v_out} are both available and $\lambda_{v\_\text{in}} = \lambda_{v\_\text{out}}$. Suppose link e is in lightpath L and wavelength $\lambda(e)$ is selected. Then the cost of a lightpath L is the summation of the total wavelength cost of the links in the lightpath and the total wavelength conversion cost occurred at the nodes in the lightpath, i.e.,

$$F(L) = \sum_{e \,\text{in}\, L} f_w(e, \lambda) + \sum_{v \,\text{in}\, L} f_c(v, \lambda_{v\_\text{in}}, \lambda_{v\_\text{out}}).$$

Assume there are k sources, $v_{s_1}, v_{s_2}, \ldots, v_{s_k}$, that need to send data to the core v_c simultaneously. Thus, for each source v_{s_i}, $i = 1, \ldots, k$, we need to construct a lightpath L_{s_i} that connects v_{s_i} to v_c. To avoid wavelength collision, if two lightpaths share a common physical link, they must be assigned different wavelengths. At the same time, certain cost measurements (to be defined) regarding to the k lightpaths must be minimized. We call the problem of constructing such k lightpaths in WDM networks subject to the constraint of wavelength collision free as the k shortest lightpaths problem.

Based on different requirements of the CBT applications, there are different objectives in the k shortest lightpaths problems. In this paper, we focus our study on the following four problems.

(1) *The feasibility problem.* The feasibility problem is to ask whether there exist k lightpaths that satisfy the constraint of wavelength collision free. This is the basic problem to ask before we do any optimization, especially when the network resources (including both wavelength and conversion resources) are rare and the infeasibility may happen frequently.
(2) *The minimum total cost problem.* In this problem, we consider constructing k lightpaths of which the total cost is minimized. The motivation for this objective is to use network resource efficiently. If the network resource is limited, we should minimize the overall network resource consumption, i.e., the total cost. Considering the feasibility issue, this problem is formally defined as finding $k' \leqslant k$ lightpaths for which the total cost is minimized subject to that k' is maximized.
(3) *The minimum max-cost problem.* This problem is concerned with the fairness of the network. The objective of minimizing total cost may result in that the cost for some sources is very small, while the cost for some sources is very large. To achieve good fairness, we try to minimize the maximum cost among these k lightpaths. Considering the feasibility issue, this problem is formally defined as finding $k' \leqslant k$ lightpaths for which the maximum cost among all lightpaths is minimized subject to that k' is maximized.
(4) *The constrained feasibility problem.* This objective is motivated by the concept of delay-sensitive differentiated service. Each source may belong to a different service class, which has its own cost tolerance, especially with respect to propagation and wavelength conversion delay. Assume that $F^*_{s_i}$ is the minimum possible cost of a lightpath L_{s_i} and Δ_{s_i} is the cost tolerance for node v_{s_i}. The problem is to find k lightpaths such that the cost $F(L_{s_i})$ of the lightpath L_{s_i} is no more than $F^*_{s_i} + \Delta_{s_i}$ for all $i = 1, 2, \ldots, k$. Note that this is also a feasibility problem and $F^*_{s_i}$ can be readily obtained from a shortest lightpath [4]. A similar criterion has been studied in QoS WDM multicast [8].

In the following, we first convert a k shortest lightpaths problem on a WDM network into a k shortest paths problem in a traditional network using an auxiliary graph, called the *wavelength graph* (WG), which was first introduced in [4]. Given a graph $G = (V, E)$ with wavelength set Λ, where $V = (v_1, v_2, \ldots, v_n)$ and $\Lambda = (\lambda_1, \lambda_2, \ldots, \lambda_w)$, n is the number of nodes in G and w is the number of wavelengths in Λ, the major steps for constructing its WG are listed as follows.

Step 1. Take wn nodes, namely v'_{ij}, for $i = 1, 2, \ldots, w$ and $j = 1, 2, \ldots, n$, and arrange the nodes into a matrix-like structure with w rows and n columns, where row i represents wavelength λ_i and column j represents node v_j in G.

Step 2. For row i, $i = 1, 2, \ldots, w$, add a horizontal directed link $l(v'_{ij}, v'_{ih})$ from column j to column h if there exists a link e in G from node v_j to node v_h and the wavelength λ_i is available on this link. Assign $f_w(e, \lambda_i)$ to $l(v'_{ij}, v'_{ih})$ as its cost.

Step 3. For column j, $j = 1, 2, \ldots, n$, add a vertical directed link $l(v'_{ij}, v'_{hj})$ from row i to row h if wavelength conversion from λ_i to λ_h is available at node j. Assign $f_c(v_j, \lambda_i, \lambda_h)$ to $l(v'_{ij}, v'_{hj})$ as its cost.

Step 4. For $j = s_1, s_2, \ldots, s_k$, add a node v'_{0j}. For $i = 1, 2, \ldots, w$, add a directed link $l(v'_{0j}, v'_{ij})$, and assign 0 as the link's cost.

Step 5. Add a node v'_{00}. For $j = s_1, s_2, \ldots, s_k$, add a directed link $l(v'_{00}, v'_{0j})$, and assign 0 as the link's cost.

Step 6. For the core node v_c, add a virtual node v'_{0c}. For $i = 1, 2, \ldots, w$, add a directed link $l(v'_{ic}, v'_{0c})$, assign 0 as the link's cost.

In the WDM network G, a lightpath starts from a source node v_{s_i} and ends at the core node v_c. Correspondingly, in the WG, only one node in column s_i is chosen as the starting point of the lightpath. Before we assign the wavelength for the lightpath, it is unknown which node in column s_i is suitable to be the starting point. Hence, it is necessary to construct a unique source node in WG to represent each source node in G, which is completed in step 4. Step 5 is to construct a (virtual) common source node which connects to all those source nodes constructed in step 4. Step 6 is to construct a unique destination node for the core node in G.

Clearly, k lightpaths in a WDM network G with wavelength collision free must be k link-disjoint paths in the transformed WG. By introducing a common source node v'_{00}, the k shortest lightpaths problem from k different sources to the core node v_c in graph G is converted to the k shortest paths problem between two nodes v'_{00} and v'_{0c} in the WG subject to the constraint of link disjoint.

Lemma 1. *Both the problem of finding a connected path from v'_{00} to v'_{0c} in WG, and the problem of finding a shortest path from v'_{00} to v'_{0c} in WG, can be solved in $O(nw(n + w))$ time.*

Proof. In [4], the special shortest path algorithm for the WG (SPAWG) which has $O(nw(n + w))$ complexity was proposed to find a shortest path between the source and destination columns. There are some additional nodes v'_{0j}s added in step 4 and v'_{00} added in step 5 in the WG in this paper than the WG in [4]. However, the extra computation time needed to handle these nodes, and their associated links when using the SPAWG algorithm [4], is in $O(kw)$, which can be ignored compared with $O(nw(n + w))$.

Since the shortest path problem can be solved in $O(nw(n + w))$ time, the problem of finding a connected path between two nodes can also be solved in $O(nw(n + w))$ time. □

3. Problem solutions

In this section, we address the solutions to the k shortest lightpaths problems proposed in the previous section.

3.1. Feasibility problem

As discussed in the previous section, the feasibility problem of k lightpaths in the WDM network G is equivalent to the feasibility problem of k link-disjoint paths on the WG. To solve the feasibility problem on the WG, we formulate a network flow model on the WG and solve a maximum flow problem from the source node v'_{00} to the destination node v'_{0c} as follows.

We first assign a capacity constraint for each link in the WG. For the links added in steps 2 and 3, we assign 1 as their capacity, implying only one wavelength can be used. For the links added in steps 4 and 5, we assign 1 as their capacity, implying each source node only needs one connection. Let all other links have a capacity of k. We next solve the maximum flow problem from node v'_{00} to v'_{0c} on the constructed network flow model. Clearly, in a solution of the maximum flow problem, at most one unit flow can pass node v'_{0s_i} and the flow corresponds to a lightpath from source node v_{s_i} to the core node v_c. Thus, the k lightpaths problem has a feasible solution if and only if the maximum flow problem on the WG has a solution with flow size k.

The maximum flow problem is a classical problem and can be readily solved by many existing algorithms [1]. Due to the special structure of the WG, the maximum flow problem on the WG can be solved by simplified classical maximum flow algorithms. We adopt the labeling algorithm [1], which repeatedly finds a feasible path from the source node to the destination node, augments flows along the path, and revises the network. In general, such an algorithm runs in $O(MNU)$ time, where M is the number of edges, N is the number of nodes, and U is the maximum capacity of any links. In the complexity of the labeling algorithm, the term M represents the complexity of finding a path from the source node to the destination node, and the term NU represents the maximum possible flow. In the WG, finding a path can be completed in $O(nw(n+w))$ time (from Lemma 1), and there are at most k flows. Therefore, the maximum flow problem in the WG can be solved in $O(knw(n+w))$ by a modified labeling algorithm, the pseudocode of which is given below. Details of updating the network in each iteration can be found in [1].

Theorem 1. *The feasibility problem of the k shortest lightpaths can be solved in $O(knw(n+w))$ time.*

The labeling algorithm is the simplest maximum flow algorithm, but not the most efficient one in the general network. Many more efficient maximum flow algorithms are described in [1]. The major drawback of the labeling algorithm is the number of iterations needed for augmentation may be as large as NU. However, when used in the WG, this number is reduced to k. Hence, the major drawback of the labeling algorithm is overcome by our modified labeling algorithm.

3.2. Minimum total cost problem

Similarly, the k shortest lightpaths problem of minimizing the total cost can also be solved using the network flow model. We first formulate a minimum cost network flow model by assigning demand and supply for each node in the WG in addition to the link capacity assigned in Section 3.1. Since there are k sources in the CBT problem, we assign k as the supply of node v'_{00} and k as the demand of node v'_{0c}. All other nodes have 0 demand and supply. We next find a minimum cost flow on the network which corresponds to the k lightpaths with minimum total cost.

One popular minimum cost flow algorithm is the successive shortest path algorithm [1], which is also an iterative algorithm. In each step, it finds the shortest path from a node with excess supplies to a node with unsatisfied demands, sends flows along this path, and revises the network for next step. The running time of this algorithm is in $O(NUS)$, where N is the number of nodes, U is the upper bound of the supply of a node, S is the complexity of a proper shortest path algorithm, and the term NU comes from the bound of the maximum number of iterations. In our problem, the number of iterations is no more than k, and the shortest path problem can be solved in $O(nw(n+w))$ time. Therefore, the k shortest lightpaths problem of minimizing the total cost can be solved in $O(knw(n+w))$ time by a modified successive shortest path algorithm in WG. The pseudocode of the algorithm is listed as follows. Please refer to [1] for the details of updating the network in each iteration.

Theorem 2. *The k shortest lightpaths problem of minimizing the total cost can be solved in $O(knw(n+w))$ time.*

In the above approach, we assume that there exist k lightpaths with wavelength collision free. Recall that when the feasibility is an issue, the problem is formally defined as minimizing the total cost subject to that the maximum number of lightpaths (not exceeding k) can be found. To solve this problem, we propose a two-step scheme, in which the first step is to solve a maximum flow problem which decides the maximum number of paths from node v'_{00} and node v'_{0c}. If there exist no less than k paths, then the above minimum cost flow model can be applied. Otherwise, assuming there are maximum k' paths found, $k' < k$, then we need to assign k' as the supply of node v'_{00} and k' as the demand of node v'_{0c}, and solve the minimum cost flow problem.

In conclusion, a maximum flow problem is solved at first, then a minimum cost flow problem is solved by specifying the supply and demand values for the source node and destination node respectively based on the result of the maximum flow algorithm. We call this algorithm as the maximum-flow-minimum-cost (MFMC) algorithm.

3.3. Minimum max-cost problem

We first prove that the decision version of the minimum max-cost problem is NP-complete by a polynomial-time reduction from the EVEN–ODD PARTITION problem, a known NP-complete problem [6]. We only need to consider the case of $k = 2$. The statement of the EVEN–ODD PARTITION problem in as follows [9].

EVEN–ODD PARTITION problem: Given a set $S = \{1, 2, \ldots, 2d\}$ of $2d$ objects and a positive integer size a_i for each object i, $i = 1, 2, \ldots, 2d$, such that $\sum_{i=1}^{2d} a_i = 2B$. Does there exist a subset $S' \subseteq S$ such that $\sum_{i \in S'} a_i = \sum_{i \in S \setminus S'} a_i$ holds and $|S' \cap \{2i-1,\ 2i\}| = 1$ holds for all $i = 1, 2, \ldots, d$?

Theorem 3. *The decision version of the k shortest lightpaths problem of minimizing the max-cost is NP-complete.*

Proof. We show that for each instance of the EVEN–ODD PARTITION problem, we can construct an instance of the decision version of the k shortest lightpaths problem of minimizing the max-cost for the case of $k = 2$. □

For each object i, $i = 1, 2, \ldots, 2d$, in the EVEN–ODD PARTITION problem, we construct two nodes connected by one link, in which the link is called link i, one node is called the starting point of link i and the other node is called the end point of link i. We assume that each link i only has one available wavelength λ_1 and its cost is a_i. In addition, for each i, $i = 1, 2, \ldots, d-1$, we add 4 links, namely from the end point of link $2i-1$ to the starting point of link $2i+1$, from the end point of link $2i-1$ to the starting point of link $2(i+1)$, from the end point of link $2i$ to the starting point of link $2i+1$, and from the end point of link $2i$ to the starting point of link $2(i+1)$. These links are called inter-connecting links. We also assume that the inter-connecting links also have only wavelength λ_1 and they have identical cost of 1. One can verify that totally there are $2d$ inter-connecting links in the network. At last, we add one node v_c as the core node and add two links ending at v_c, one of which starts from the end point of link $2d-1$, and the other starts from the end point of link $2d$. Assume that the cost of each of these two links is 1 too. Let the starting points of links 1 and 2 be the two source nodes v_{s_1} and v_{s_2}. Fig. 1 illustrates the construction of such a network. Now the problem is to find two lightpaths starting from nodes v_{s_1} and v_{s_2} to node v_c such that the maximum cost of the two lightpaths is no more than $B + d$. Since there is only one wavelength available on each link, this problem is equivalent to finding two link-disjoint paths from nodes v_{s_1} and v_{s_2} to node v_c such that the maximum cost of the two paths is no more than $B + d$.

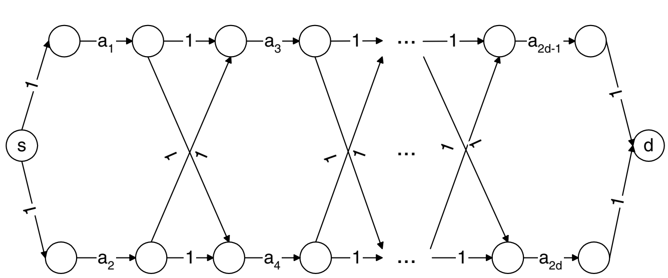

Fig. 1. NP-completeness proof for the minimum max-cost problem.

(1) If the EVEN–ODD PARTITION has a YES solution, then we can simply set up one lightpath by using all links i for $a_i \in S'$ and d inter-connecting unit cost links, and set up another lightpath by using all links i for $a_i \notin S'$ and the rest d inter-connecting links. It can be verified that such two lightpaths have the same cost of $B + d$, hence a feasible solution to the two shortest lightpaths problem is found.
(2) Assume that we have a feasible solution to the 2 shortest path problem of which the maximum cost is no more than $B + d$. It can be verified that any two feasible lightpaths in the above instance must have the total cost of $2B + 2d$, where all links i with cost a_i are used, and other $2d$ inter-connecting links are used. Thus we must have both lightpaths with the same cost of $B + d$, where the term d comes from the total cost of inter-connecting links, and the term B comes from links i for $i = 1, 2, \ldots, 2d$. From the construction of the problem, we see that each path can only use one of the link j for $j \in \{2i-1, 2i\}$, $i = 1, 2, \ldots, d$. Therefore, we have a YES solution to the EVEN–ODD PARTITION problem.

Because of the NP-completeness, the problem of minimizing the max-cost of k shortest lightpaths is unlikely to be solved in polynomial time unless $P = NP$. We have to switch to efficient heuristic algorithms. A simple heuristic algorithm, called the maximum-shortest-path (MSP) algorithm, is proposed as follows. The MSP algorithm is based on the idea of greediness, i.e., whenever a wavelength collision occurs, we keep the lightpath that has the largest cost and then re-route other lightpaths. Specifically, for each source node v_{s_i}, we calculate the shortest lightpath to the core node v_c individually, which can be solved by finding the shortest path from node v_{0s_i} to node v_{0c} on the WG using the SPAWG algorithm [4]. These k lightpaths do not necessarily satisfy the constraint of wavelength collision free. We then sort these k lightpaths according to their costs and pick up the lightpath $L_{s_{\max}}$ with the maximum cost. It is obvious that the cost of other lightpaths from source node $v_{s_{\max}}$ to the core node cannot be smaller than the cost along $L_{s_{\max}}$. We take $L_{s_{\max}}$ as the lightpath from source node $v_{s_{\max}}$ to the core node and remove wavelengths it uses from the corresponding links in $L_{s_{\max}}$. In other words, we fix this lightpath and re-route lightpaths from other $k-1$ source nodes to the core node in the residual network. This process is repeated until k lightpaths without wavelength collision are found or no lightpath exists in the residual network. The pseudocode of the MSP algorithm is listed as follows.

The MSP algorithm may fail to find k feasible lightpaths even though there exists a feasible solution that has k lightpaths without wavelength collision. To ensure a feasible solution when there exists one, we can use the MFMC algorithm described in the previous subsection as another heuristic algorithm.

The MSP algorithm has k iteration at most. In each iteration, we need to solve at most k shortest lightpath problems, each of which takes $O(nw(n+w))$ time. Therefore, the total complexity of the MSP algorithm is $O(k^2nw(n+w))$, which is higher than the MFMC algorithm. The effectiveness comparison of these two algorithms is given in Section 4.

3.4. Constrained feasibility problem

We first show that the constrained feasibility problem of the k shortest lightpaths is also NP-complete. Given an instance of the EVEN–ODD PARTITION problem, we can construct an instance of the constrained feasibility problem as in the proof of the NP-completeness of the minimum max-cost problem. Recall that in the constrained feasibility problem, each source node v_{s_i} has two parameters, the cost of its shortest path $F^*_{s_i}$ and the cost tolerance Δ_{s_i}. In constructing the instance of the constrained feasibility problem, let $x = \sum_{i=2}^{d} \min\{a_{2i-1}, a_{2i}\}$. Then $F^*_{s_1} = a_1 + x + d$ is the cost of the shortest path from v_{s_1} to v_c, and $F^*_{s_2} = a_2 + x + d$ is the cost of the shortest path from v_{s_2} to v_c. Let $\Delta_{s_1} = B + d - F^*_{s_1}$, $\Delta_{s_2} = B + d - F^*_{s_2}$. Then we have an instance of the constrained feasibility problem with two shortest lightpaths, to which a feasible solution corresponds to a solution to the minimum max-cost problem in which the maximum cost is no more than $B + d$. As we showed in Section 3.3, such a solution is corresponding to a solution to the EVEN–ODD PARTITION problem. Therefore, we have the following theorem:

Theorem 4. *The decision version of the constrained feasibility problem of the k shortest lightpaths is NP-complete.*

The above procedure also illustrates that the constrained feasibility problem is closely related to the minimum max-cost problem. Based on this observation, we can convert a constrained feasibility problem, denoted by P, to a minimum max-cost problem, denoted by $\overline{P}$, as follows. Let $F^*_{\max} = \max_{1\leqslant i\leqslant k}\{F^*_{s_i} + \Delta_{s_i}\}$. For each source node v_{s_i}, we create a node $\overline{v}_{s_i}$, a link from $\overline{v}_{s_i}$ to v_{s_i}, and assign the cost of this link as $F^*_{\max} - (F^*_{s_i} + \Delta_{s_i})$. In $\overline{P}$, we need to find k shortest lightpaths from $\overline{v}_{s_i}$ to the core node v_c such that the maximum cost is minimized.

Theorem 5. *The constrained feasibility problem P of the k shortest lightpaths has a feasible solution if and only if the minimum max-cost problem $\overline{P}$ has a solution in which the maximum cost is no more than $F^*_{\max}$.*

Proof. From the construction of $\overline{P}$, we know that a lightpath from $\overline{v}_{s_i}$ in $\overline{P}$ corresponds to a lightpath from v_{s_i} in P, and the cost difference is $F^*_{\max} - (F^*_{s_i} + \Delta_{s_i})$. Therefore, the maximum cost for all lightpaths in $\overline{P}$ is no more than $F^*_{\max}$ if and only if each corresponding lightpath from v_{s_i} has a cost no more than $F^*_{s_i} + \Delta_{s_i}$. □

By the above theorem, the constrained k shortest lightpaths problem can be solved by a corresponding minimum max-cost problem. Thus, we can directly apply the algorithms for the minimum max-cost problem to solve the constrained k shortest lightpaths problem.

3.5. Lower bounds

Several lower bounds with respect to the minimum max-cost are developed for the case where there exist k lightpaths.

First in the MSP algorithm, the $L_{s_{\max}}$ obtained in the first iteration is obviously a lower bound because it is obtained by assuming no other lightpaths exist in the network. We denoted this lower bound as LB_1.

In the MFMC algorithm, assuming that the total cost obtained is x, then $LB_k = x/k$ gives another lower bound of the max-cost because x is the minimum total cost of any k feasible lightpaths.

More lower bounds can be found by generalizing the above approach. For example, arbitrarily selecting two source nodes v_{s_i} and v_{s_j} defines a two shortest paths problem of minimum total cost, which can be solved by the MFMC algorithm with the total cost as $x^{(2)}_{ij}$. Then $x^{(2)}_{ij}/2$ is a lower bound of the minimum max-cost because the minimum max-cost with 2 source nodes cannot be more than the minimum max-cost with $k \geqslant 2$ source nodes. We can try all possible combinations of i and j, and define the lower bound LB_2 as

$$LB_2 = \max_{i\neq j}\left\{\frac{x^{(2)}_{ij}}{2}\,\middle|\,\text{2 shortest lightpaths are found for the problem with nodes } v_{s_i} \text{ and } v_{s_j}\right\}.$$

In general, assuming that Θ_l is a subset of l, $l \leqslant k$, source nodes, and $x^{(l)}_{\Theta_l}$ is the total cost obtained from the MFMC algorithm for an l shortest lightpath problem with the source nodes in Θ_l, we can define a lower bound LB_l as

$$LB_l = \max_{\Theta_l}\left\{\frac{x^{(l)}_{\Theta_l}}{l}\,\middle|\, l \text{ lightpaths are found for the problem with nodes in } \Theta_l\right\}.$$

Clearly, the previously defined LB_1 and LB_k are special cases of the general definition. In the experiments, we have computed lower bounds LB_1, LB_2, and LB_k, and used the largest one as the final lower bound.

4. Experimental results

In this section, we report the experimental results for the MFMC algorithm and the MSP algorithm when they are used as heuristic algorithms to solve the minimum max-cost problem.

4.1. Experiment settings

Experiments have been done for the minimum max-cost problem on the topology of the Arpanet shown in Fig. 2 [13], which has 47 nodes and 68 links. We then randomly select k nodes as source nodes and one other node as the core node. Let w be a parameter indicating the number of wavelengths in the entire wavelength set. We randomly assign available wavelengths from λ_1 to λ_w to each link such that the number of available wavelengths on each link is uniformly distributed in $\{w/2, w/2+1, \ldots, w\}$. For each wavelength on a link, we randomly assign an integral cost which is uniformly distributed in $\{1, 2, \ldots, 50\}$. The wavelength conversion cost is fixed at 10.

In our experiments, we vary two parameters, w and k, which represent the number of wavelengths and the number of sources, respectively. For each combination of parameters w and k, we generate 200 instances and solve each instance using the MFMC algorithm and the MSP algorithm, respectively. We compare the average performance of these two algorithms in terms of their ability of finding k feasible lightpaths and the maximum cost of lightpaths if both algorithms find the same number of lightpaths. The reason of deciding these two measurements is explained as follows. Recall that the MFMC algorithm finds k feasible lightpaths if there exist, but it does not make any effort to minimize the max-cost. On the other hand, the MSP algorithm minimizes the max-cost, but it may fail to find k feasible lightpaths when there exist such k lightpaths. As stated in the definition of the k shortest lightpaths problem, finding k feasible lightpaths is more important than minimizing the max-cost among k lightpaths. Therefore, we take the ability of finding k feasible lightpaths as the major measurement and the max-cost as the second measurement when they can find the same number of lightpaths.

We first compare the performance of these two algorithms in terms of their ability of finding k feasible lightpaths. Given an instance of the minimum max-cost problem, the MSP algorithm is said to be better than the MFMC algorithm, denoted as MSP-better, if the two algorithms find the same number of feasible lightpaths and the max-cost obtained by the MSP algorithm is smaller than that obtained by the MFMC algorithm; the MSP algorithm is said to be equal to the MFMC algorithm, denoted as MSP=MFMC, if the two algorithms find the same number of feasible lightpaths with the same max-cost; the MFMC algorithm is said to be better than the MSP algorithm, denoted as MFMC-better, for other cases, including the case that if the two algorithms find the same number of feasible lightpaths and the max-cost obtained by the MSP algorithm is greater than that obtained by the MFMC algorithm, and the case that the MSP algorithm finds fewer number of feasible lightpaths than the MFMC algorithm.

We then compare their performance in terms of the relative error of the max-cost found by each algorithm compared with the lower bound. Let C_{MFMC} denote the max-cost found by MFMC, C_{MSP} denote the max-cost found by MSP, and $C_{\mathrm{Lower\ bound}}$ denote the lower bound. The relative error of MFMC, denoted by e_{MFMC}, is defined as:

$$e_{\mathrm{MFMC}} = \frac{C_{\mathrm{MFMC}} - C_{\mathrm{Lower\ bound}}}{C_{\mathrm{Lower\ bound}}},$$

and the relative error of MSP, denoted by e_{MSP}, is defined as:

$$e_{\mathrm{MSP}} = \frac{C_{\mathrm{MSP}} - C_{\mathrm{Lower\ bound}}}{C_{\mathrm{Lower\ bound}}}.$$

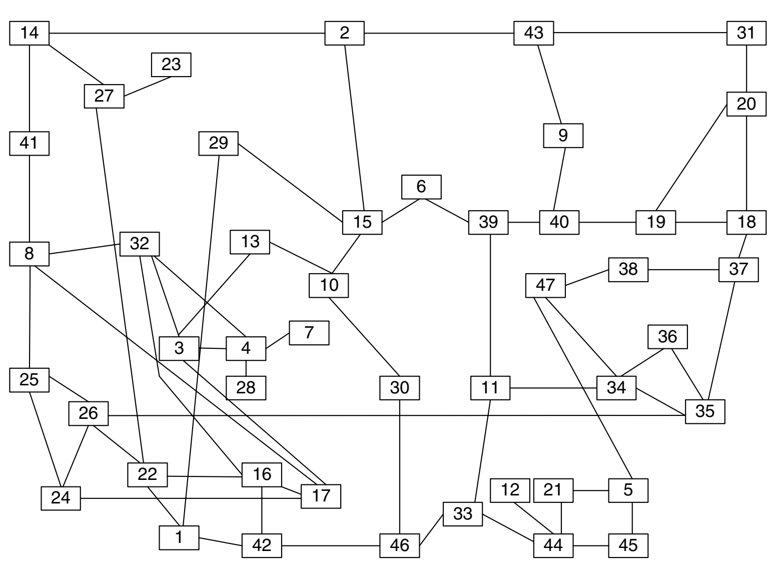

Fig. 2. Arpanet.

4.2. Impact of the number of sources

First, we evaluate the performance of MFMC and MSP by fixing the wavelength parameter w at 8 and varying the number of sources k in $\{4, 5, 6, 7, 8, 9, 10\}$. Table 1 lists the percentage of instances for the cases of MSP-better, MSP=MFMC, and MFMC-better. It shows that the MSP algorithm is more suitable for cases with smaller k, while the MFMC is more suitable for cases with larger k. This is consistent with our intuition. When k is small, the wavelength resource is relatively redundant, hence, the MSP algorithm has much space to minimize the cost of each lightpath. When k is large, the wavelength resource becomes sparse, therefore, the MFMC algorithm which optimizes the total cost of lightpaths in a global view is more effective.

Fig. 3 shows the performance of MFMC and MSP in terms of their relative error. The effectiveness of both algorithms is evidenced by the relative errors. From the figure, we find that with the number of sources increasing, the relative errors of both algorithms increase. We also find that relative error of the MSP algorithm is less than that of the MFMC algorithm when k is small. This is consistent with our conclusion from Table 1 that the MSP algorithm is more suitable for cases with fewer number of sources.

4.3. Impact of the number of available wavelengths

Next we evaluate the performance of MFMC and MSP by fixing the number of sources k at 8 and changing the wavelength parameter w in $\{6, 7, 8, 9, 10, 11, 12\}$. The results are listed in Table 2 and Fig. 4. Table 2 lists the percentage of instances for the cases of MSP-better, MSP=MFMC, and MFMC-better. It shows that the MSP algorithm works better than the MFMC algorithm, when w is large; and the MFMC algorithm outperforms the MSP algorithm, when w is small. Consistent with the observation in the previous subsection, the MSP algorithm is more appropriate for cases with more wavelength resource, and the MFMC algorithm is more appropriate for cases with less wavelength resource.

Table 1
Comparison of MFMC and MSP algorithms with different number of sources

k	4	5	6	7	8	9	10
MSP-better (%)	11.5	17	15.5	22	25	0	0.5
MSP=MFMC (%)	88.5	79	79.5	69	57	1	1
MFMC-better (%)	0	4	5	9	18	99	98.5

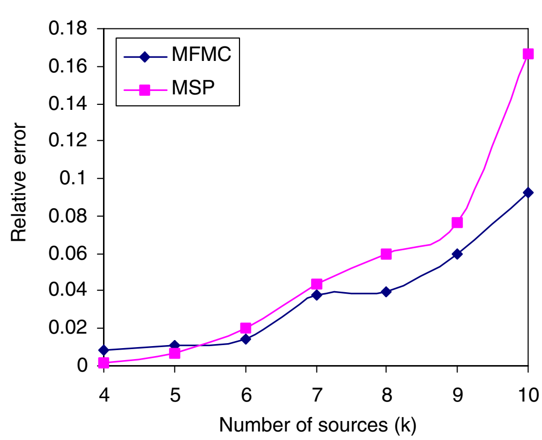

Fig. 3. Relative errors for MFMC and MSP with different number of sources.

Table 2
Comparison of MFMC and MSP algorithms with different number of wavelengths

w	6	7	8	9	10	11	12
MSP-better (%)	1.5	1.5	25	27	20.5	19	11.5
MSP=MFMC (%)	2.5	3	57	61	73	77	86
MFMC-better (%)	96	95.5	18	12	6.5	4	2.5

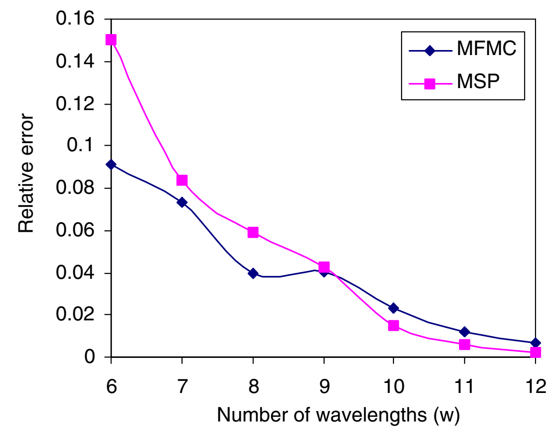

Fig. 4. Relative errors for MFMC and MSP with different number of wavelengths.

The relative errors of both heuristic algorithms with different number of wavelengths are given in Fig. 4. With wavelength parameter w increasing, the relative errors of both heuristic algorithms decrease. Again, we find that the MFMC algorithm works better when the wavelength resource is relatively sparse (w is small), and the MSP algorithm works better when the wavelength resource is sparse (w is large).

5. Summary

In this paper, we addressed the routing and wavelength assignment problem for the CBT service in a WDM network where k sources need to send data to a common core node. We modeled the problem as a problem of finding k shortest lightpaths from sources to the core with the constraint of wavelength collision free. To address different objectives, we defined and studied several subproblems, including the feasibility problem, the minimum total cost problem, the minimum max-cost problem, and the constrained feasibility problem. The major contributions of this paper include: (1) To the best of our knowledge, this is the first work in the literature addressing the CBT service in a WDM network.

(2) We showed how the classical network flow algorithms can be modified and efficiently applied on the wavelength graph to solve the feasibility and minimum total cost problems. These modified network flow algorithms generalize the shortest path algorithm reported several years ago. (3) We proved the NP-completeness of the minimum max-cost problem and the constrained feasibility problem. (4) We proposed two heuristic algorithms for the last two problems. Simulation results show that the proposed two heuristic algorithms achieve performance very close to the calculated lower bounds.

core-based tree in WDM networks is a promising research area. In this paper, we separate the routing and wavelength assignment for the CBT into two parts, routing for the core (multicast routing) and routing for the sources (k shortest lightpaths problem). Future work includes studying the correlation between these two problems.

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