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Network Coding Based Schemes for Imperfect Wireless Packet Retransmission Problems: A Divide and Conquer Approach

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Abstract NC (Network Coding) provides a new approach to packet retransmission problems in wireless networks, which are named as WPRTPs (Wireless Packet Retransmission Problems) in this paper. Some research has been conducted on P-WPRTPs (Perfect WPRTPs) where, for one receiver, a packet is either being requested by or already known to it. However, very few efforts are focused on IP-WPRTPs (Imperfect WPRTPs) where, for one receiver, a packet can be neither requested by nor already known to it. In this paper, we focus on IP-WPRTPs. Firstly, a WPRTP reduction theorem for simplifying WPRTPs is proposed and

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proved. Then, the upper and lower bounds of the number of necessary packet transmissions in optimal NC-based solutions to IP-WPRTPs are analyzed. Next, a scheme named as IP-WPRTP-DC (Divide and Conquer based scheme for IP-WPRTPs) is proposed based on the WPRTP reduction theorem using a divide and conquer approach. Extensive simulations show that the IP-WPRTP-DC scheme is effective in saving the number of packet transmissions for solving IP-WPRTPs.

Keywords Imperfect wireless packet retransmission problems · Divide and conquer approach · Problem reduction · Random network coding

1 Introduction

Compared with wired links, wireless links are more erroneous. Although FEC (Forward Error Correction) technologies [1,2] can provide some error detection and correction capacity at the expense of some redundant bits, packet retransmission is still necessary in wireless communications. One typical problem related to packet retransmission in 1-hop wireless broadcast/multicast applications, named as WPRTP (Wireless Packet ReTransmission Problem) in this paper, has attracted some research efforts in recent years. A typical scenario for WPRTPs is as follows [3]. One sender and several receivers in a wireless network form a sub system, and all receivers are in the radio range of the sender. The sender has a set of packets that need to be transmitted to all receivers. However, each receiver has already obtained a subset of the packets through some ways, such as through previous communications. Thus, the receivers request the sender to retransmit a subset of the packets to them. The set of packets requested by a receiver is called as its *Want* set, meanwhile the set of packets already known to the receiver is called as its *Has* set. In order to reduce communication overhead, the number of packets retransmitted by the sender should be minimized. Thus, a WPRTP emerges: how to schedule packet retransmissions so as to minimize the total number of retransmitted packets.

First coined in 2000, NC (Network Coding) [4] provides an interesting approach to many problems in wireless networks. Its core notation is to allow and encourage mixing of packets at intermediate nodes [5]. Previous research [6] has showed that NC can increase throughput, enhance robustness, improve fairness, reduce complexity of techniques to wireless networks by exploiting the intrinsic characteristics of wireless networks, such as data redundancy, broadcast nature of wireless transmissions, and spatial diversity, etc. Till now, NC has been adopted in many research topics of wireless networks, such as multicast [7], unicast [8,9], broadcast [10], fault tolerance [11], network security [12], etc. RNC (Random Network Coding) [13] makes it applicable for distributed applications.

NC provides a promising approach to WPRTPs. By using NC, the sender can combine original packets into several coded packets and then transmit these coded packets instead of the original packets to the receivers. If properly designed, each receiver could obtain all its wanted packets by decoding from the received coded packets. Constructed based on Galois Field Theory, coded packets are usually of the same size to the original packets. Thus, if the number of the coded packets transmitted by the sender is smaller than that of the original packets requested by the receivers, communication overhead in the metric of packet transmissions is reduced.

In recent years, researchers of [3,14–16] have adopted NC approaches to solve WPRTPs. The schemes proposed in [3,14,15] only search for network coding solutions on Galois Field GF(2). Since that finding optimal NC-based solutions on GF(2) to WPRTPs is a NP-Complete problem [3], the solutions are usually not optimal. Hence, in [16], we relieve the restriction on Galois Field GF(2) and show that finding optimal solutions to WPRTPs in such cases is no longer a NP-Complete problem. Then, we propose an optimal NC-based scheme for WPRTPs based on RNC. However, the solutions found by the scheme in [16] are only optimal for P-WPRTPs (Perfect Wireless Packet ReTransmission Problems) where, for one receiver, a packet can be neither requested by nor already known to it. All other WPRTPs are named as IP-WPRTPs (ImPerfect Wireless Packet ReTransmission Problems). As a matter of fact, most WPRTPs are IP-WPRTPs. Hence, it is important to study efficient schemes for IP-WPRTPs.

In this paper, our study is focused on IP-WPRTPs. Firstly, a WPRTP reduction theorem for simplifying WPRTPs is proposed and proved. Then, the upper and lower bounds of the number of necessary packet transmissions in optimal NC-based solutions to IP-WPRTPs are analyzed. Next, a scheme named as IP-WPRTP-DC (Divide and Conquer based scheme for IP-WPRTPs) is proposed based on the WPRTP reduction theorem using a divide and conquer approach. Extensive simulations show that the IP-WPRTP-DC scheme is effective in saving the number of packet transmissions for solving IP-WPRTPs.

The rest of the paper is organized as follows. Section 2 gives an overview on related work. Section 3 provides the preliminaries of this work including the definitions related to WPRTPs and the operating processes of the sender and the receivers in general NC-based schemes to WPRTPs. Section 4 proposes and proves the WPRTP reduction theorem and gives the analysis. In Sect. 5, the IP-WPRTP-DC (Divide and Conquer based scheme for IP-WPRTPs) is proposed. In Sect. 6, performance evaluation of the IP-WPRTP-DC scheme and some other typical schemes are presented. Section 7 concludes the paper.

2 Related Work

Mathematical background of NC is the Galois Field Theory, and all calculations in NC are performed in a Galois Field. In computer science, a Galois Field is usually notated as GF(2^q), which means that the number of elements of the Field is 2^q. In [3,14–16], WPRTPs are studied using NC-based approaches, but most of these works are focused on solutions in Galois Field GF(2), where the coding and decoding calculations are both bitwise XOR operations (denoted as "$\oplus$"). Since that finding optimal NC-based solutions on GF(2) to WPRTPs is a NP-Complete problem[3], the solutions found on GF(2) are usually not optimal, thus, in many cases, solutions with fewer packet transmissions may be highly appreciated.

El Rouayheb et al. [3] prove that the WPRTP is a NP-Complete problem when Galois Field GF(2) is used, they then propose a heuristic scheme for solving IP-WPRTP s on GF(2). In their scheme, a WPRTP is firstly transformed into a new WPRTP where the size of the *Want* set of each receiver is just one. The transformation is performed by substituting a receiver that wants multiple packets with a set of new receivers such that: (1) the *Has* set of each new receiver is equal to that of the original receiver; (2) the *Want* set of each receiver contains just one packet in the *Want* set of the original receiver; (3) the union of all the *Want* sets of the new receivers equals to the *Want* set of the original receiver. Then, an undirected graph $G(V, E)$ is constructed according to the new WPRTP such that: (1) for each receiver, there is a corresponding vertex in $G(V, E)$; (2) an edge exists between a pair of vertexes in $G(V, E)$ if and only if one of the following two conditions hold: (a) the two receivers of the pair have identical *Want* set; (b) the *Want* set of each one of the two receivers is a subset of the *Has* set of the other receiver. At last, a heuristic algorithm is used to find a solution to the graph coloring problem of the complimentary graph of $G(V, E)$ and then the solution is transformed into a final solution as follows: the *Want* sets of all the receivers that correspond to the set of vertexes with the same color are combined into a coded packet, and all such coded packets corresponding to the node sets with different colors make up a solution to the original WPRTP. In the following text, this scheme is designated as ColorNC. As the authors have mentioned, solutions found by ColorNC are usually suboptimal, which is also confirmed by our simulations in Sect. 6.

Xu et al. [14] also adopt graph theory to determine solutions for P-WPRTPs by transforming P-WPRTPs to clique partition problems of the corresponding graphs. Hence, this scheme is designated as CliqueNC in the following text. In CliqueNC, the graph $G(V, E)$ of a WPRTP is constructed in the following two steps: (1) create a node in $G(V, E)$ for each packet; (2) create edges in $G(V, E)$ such that, for any pair of packets, if no receiver whose *Want* set includes the both packets; Otherwise, no edge should exist between the two nodes. The edge creation criteria in CliqueNC assure that, if a pair of nodes in $G(V, E)$ is connected by an edge (denotes the two packets corresponding to the two nodes as p_1 and p_2, respectively), the sender can broadcast a coded packet $p_1 \oplus p_2$. When receiving the coded packet, all the corresponding receivers could obtain the packets in their *Want* sets by decoding. Based on this property, a clique in $G(V, E)$ is translated into a coded packet, and the set of all coded packets translated from the cliques in $G(V, E)$ make up a valid solution. Thus, the number of coded packets in a CliqueNC solution is equal to the number of cliques in the graph $G(V, E)$. However, CliqueNC is proposed particularly for P-WPRTPs, and the solutions found by it are also usually suboptimal.

Nguyen et al. [15] propose two NC-based packet retransmission methods to improve spectrum utilization efficiency in reliable broadcast applications. Bandwidth efficiency of the proposed NC-based schemes as well as two traditional ARQ (automatic repeat-request) schemes are analyzed and verified through simulations. The NC schemes proposed in [15] are similar to ColorNC. However, since these schemes in [15] run on-the-fly, usually partial information of the WPRTP is used when searching for solutions. The solutions found by these schemes are usually not better than those of ColorNC.

In [16], using Galois Field GF(2^q) where $q > 1$, optimal schemes for P-WPRTPs are studied, where the number of retransmitted packets in optimal valid NC-based solutions to P-WPRTPs is analyzed and proved. Then, based on RNC, a scheme that is optimal in the number of retransmitted packets is proposed for P-WPRTPs. However, the scheme is not suitable for IP-WPRPTs.

3 Operation Process of NC-based Schemes for WPRTPs

3.1 Preliminaries

Def. 1: WPRTP (Wireless Packet ReTransmission Problem). A WPRTP can be described as a 4-element tuple:

$$\text{WPRTP} = \{P, R, \{H_i\}, \{W_i\}\}$$

where (1) $P = \{p_i | i \in \{1, 2, \ldots, |P|\}\}$ represents the set of packets considered in the problem; (2) $R = \{r_i | i \in \{1, 2, \ldots, |R|\}$ represents the set of receivers; (3)H_i is receiver r_i' *Has* set, i.e., the set of packets already known to receiver r_i; $\{H_i\}$ represents the set of the *Has* sets of all the receivers; (4) W_i is receiver r_i' *Want* set, i.e., the set of packets requested by receiver r_i.

Def. 2: Valid Solutions to WPRTPs. A solution to a WPRTP is valid if all receivers can obtain their wanted packets by decoding after receiving all the retransmitted packets in the solution.

Def. 3: Optimal Solutions to WPRTPs. An optimal solution to a WPRTP is a valid solution with minimum number of retransmitted packets.

Def. 4: P-WPRTP (Perfect WPRTP). A WPRTP is a P-WPRTP if for $\forall r_i \in R$, $P = H_i \cup W_i$.

Def. 5: IP-WPRTP (Imperfect WPRTP). A WPRTP is an IP-WPRTP if $P \supset H_i \cup W_i$ holds for at least one receiver r_i.

3.2 Coding and Decoding Calculations in NC

Calculations in NC are all performed on a particular Galois Field GF(2^q). In NC, a packet of length L is considered as a list of symbols (or called as elements) on GF(2^q), and list size is $\lceil L/q \rceil$.Several bits will be appended to the packet if L is not exactly dividable by q. Coding calculation is performed following Eq. 1 on GF(2^q).

$$P_Y = M \cdot P_X \tag{1}$$

where:

$$P_Y = \begin{bmatrix} P_{Y,1} \\ P_{Y,2} \\ \vdots \\ P_{Y,k} \end{bmatrix} = \begin{bmatrix} y_{1,1} & y_{1,2} & \cdots & y_{1,\lceil L/q \rceil} \\ y_{2,1} & y_{2,2} & \cdots & y_{2,\lceil L/q \rceil} \\ \vdots & \vdots & \ddots & \vdots \\ y_{k,1} & y_{k,2} & \cdots & y_{k,\lceil L/q \rceil} \end{bmatrix}, \quad M = \begin{bmatrix} m_{1,1} & m_{1,2} & \cdots & m_{1,n} \\ m_{2,1} & m_{2,2} & \cdots & m_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ m_{k,1} & m_{k,2} & \cdots & m_{k,n} \end{bmatrix},$$

$$P_X = \begin{bmatrix} P_{X,1} \\ P_{X,2} \\ \vdots \\ P_{X,n} \end{bmatrix} = \begin{bmatrix} x_{1,1} & y_{1,2} & \cdots & x_{1,\lceil L/q \rceil} \\ x_{2,1} & y_{2,2} & \cdots & x_{2,\lceil L/q \rceil} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n,1} & x_{n,2} & \cdots & x_{n,\lceil L/q \rceil} \end{bmatrix}$$

In Eq. 1, k coded packets $P_{Y,i}$ (i={1, 2, ..., k}) are created from n original packets $P_{X,i}$ (i={1, 2, ..., n}) using network coding coefficient matrix M(called as coding matrix).

When transmitting packets using NC, the coded packet and some auxiliary information for decoding such as the coding vector used for creating the coded packet are usually enclosed in an assembled packet and then broadcasted to the receivers. After receiving enough assembled packets, receivers can construct an equation system similar to Eq. 1. Usually some elimination methods (such as the Gauss-Jordan elimination method [17]) are performed on Eq.1 to decode out all original packets.

Coding and decoding calculations are performed on GF(2^q) symbols. However, since that all symbols in a packet are all treated using the same coding matrix, for simplicity, we will say that coding calculations are performed on packets.

3.3 Operation Process of General NC-based Schemes for WPRTPs

NC-based schemes for WPRTPs usually employ similar operation processes. The major differences between different schemes lie in the parameters of the solutions, such as the number of coded packets, coding matrix, etc. Coding and decoding process of general NC-based schemes for WPRTPs are as follows.

3.3.1 General Operation Process of the Sender

Suppose that a solution is determined by a NC-based scheme to a WPRTP, and basic parameters of the solution are as follows: (1) the number of coded packets to be retransmitted is

k; (2) coding matrix is M. Using this solution, the sender will send packets in the following steps (suppose that $|P| = n$):

General operation process of the sender:

(1) Calculate the vector of k coded packets $P_Y = [P_{Y,1}, P_{Y,2}, \ldots, P_{Y,k}]^T$ by using $P_Y = M_{k\times n} \times P_X$, here $P_X = [P_{X,1}, P_{X,2}, \ldots, P_{X,n}]^T = [p_1, p_2, \ldots, p_n]^T$;
(2) Construct k assembled packets $P_{Z,i}=\{D_i, P_{Y,i}\}$, $i \in\{1, 2, \ldots, k\}$. Here D_i represents auxiliary information for decoding, such as the coding vector used for creating the corresponding coded packet. Actual information in D_i depends on the scheme used to solve the WPRTP;
(3) Broadcast all assembled packets $P_{Z,i}$, $i \in\{1, 2, \ldots, k\}$.

3.3.2 General Operation of the Receivers

When receiving assembled packets retransmitted by the sender, all receivers perform the same process to obtain their wanted packets. The operations performed by receiver r_i is described below.

General operation process of receiver r_i:

(1) For each packet $H_{i,j}$ in H_i, determine its order $x(i, j)$ in P, and create a row vector r_j of length n according to Eq. 2, i.e., r_j's $x(i, j)$-th element is 1, and other elements are all 0. Let $n_i = |H_i|$;

$$r_{j,t} = \begin{cases} 1 & t = x(i, j) \\ 0 & t \in \{1, 2, \ldots, n\} \;\&\; t \neq x(i, j) \end{cases} \tag{2}$$

(2) Assemble all row vectors r_i ($i=\{1, 2, \ldots, |n_i|\}$) created in the step (2) into a matrix $R_{n_i\times n}$ with size $n_i \times n$, as shown in Eq. 3;

$$R_{n_i\times n} = \begin{bmatrix} r_1 \\ \vdots \\ r_{n_i} \end{bmatrix} = \begin{bmatrix} r_{1,1} & \cdots & r_{1,j} & \cdots & r_{1,n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{n_i,1} & \cdots & r_{n_i,j} & \cdots & r_{n_i,n} \end{bmatrix} \tag{3}$$

(3) Construct all packets in H_i into a column vector $P_R = [P_{R,1}, P_{R,2}, \ldots, P_{R,n_i}]^T$;
(4) After receiving all assembled packets $P_{Z,i} = \{D_i, P_{Y,i}\}$ ($i = \{1, 2, \ldots, k\}$), extract decoding information from D_i, such as coding vector m_i ($i = \{1, 2, \ldots, k\}$), and assemble them into a matrix $M_{k\times n}$ of size $k \times n$;
(5) Assemble $R_{n_i\times n}$ and $M_{k\times n}$ into a hybrid matrix $M'_{(k+n_i)\times n}$ as shown in Eq. 4. $M'_{(k+n_i)\times n}$ is called Decoding Matrix;

$$M'_{(k+n_i)\times n} = \begin{bmatrix} R_{n_i\times n} \\ M_{k\times n} \end{bmatrix} \tag{4}$$

(6) Extract all coded packets $P_{Y,i}$ ($i = \{1, 2, \ldots, k\}$) from assembled packets $P_{Z,i}$ ($i = \{1, 2, \ldots, k\}$), and construct a column vector $P_Y = [P_{Y,1}, P_{Y,2}, \ldots, P_{Y,k}]^T$.
(7) Assemble P_R and P_Y into a longer vector $[P'_R]_{(k+n_i)\times 1}$ according to Eq. 5.

$$[P'_R]_{(k+n_i)\times 1} = \begin{bmatrix} [P_R]_{n_i\times 1} \\ [P_Y]_{k\times 1} \end{bmatrix} \tag{5}$$

(8) Create an equation system as shown in Eq. 6 using $[P_X]_{n\times 1} = [P_1, P_2, \ldots, P_n]^T$, $[P'_R]_{(k+n_i)\times 1}$, and $M'_{(k+n_i)\times n}$;

$$M'_{(k+n_i)\times n}[P_X]_{n\times 1} = [P'_R]_{(k+n_i)\times 1} \tag{6}$$

(9) Solve Eq. 6 in Galois Field GF(2^q). So long as $rank(M'_{(k+n_i)\times n}) = n$, the receiver will surely be able to obtain all packets in W_i;

(10) End.

3.4 Optimal Network Coding based Schemes for P-WPRTPs

The Theorem about the number of retransmitted packets in optimal NC based solutions to P-WPRTPs in [16] is cited as the following Theorem 1.

Theorem 1 [16]*: Given that the Galois Field GF(2^q) to be used is not constrained, the number of retransmitted packets in an optimal valid NC-based solution to a P-WPRTP(P, R, $\{H_i\}$, $\{W_i\}$) is* $\max_{r_i\in R}|W_i|$.

4 WPRTP Reduction and Upper and Lower Bounds of the Number of Packet Retransmissions

RNCOPT is only optimal for P-WPRTPs. In this section, we focus on the background for deriving efficient schemes for IP-WPRTPs. Firstly, a WPRTP reduction theorem is proposed and proved, and then the upper and lower bounds of the number of packet retransmissions in optimal NC-based solutions for IP-WPRTPs are analyzed. The results in this section provide the foundation of the divide and conquer based scheme proposed in the next section.

4.1 WPRTP Reduction

Def. 6: WPRTP Reduction. A WPRTP may be transformed into a simpler WPRTP. If any valid/optimal solution of the simpler WPRTP is also a valid/optimal solution to the original WPRTP, the transformation from the original WPRTP to the simpler WPRTP is called WPRTP reduction, or called reduction for short.

About WPRTP reduction, we have the following Theorem 2. The Theorem is valid for all WPRTPs, including both P-WPRTPs and IP-WPRTPs.

Theorem 2 *WPRTP Reduction Theorem.WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$ can be reduced to WPRTP$\{P', R', \{H'_i\}, \{W'_i\}\}$ where $R' = \{r_i|W_i \neq \varnothing, r_i \in R\}$, $W'_i = W_i$, $P' = \cup_{r_i\in R}W_i$, and $H'_i = H_i \cap (\cup_{r_i\in R}W_i)$.*

Proof We will prove it in two steps: (1) to prove that any valid solution to WPRTP$\{P', R', \{H'_i\}, \{W'_i\}\}$ must be a valid solution to WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$; (2) to prove that any optimal solution to WPRTP$\{P', R', \{H'_i\}, \{W'_i\}\}$ must be an optimal solution to WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$.

(1) To prove that any valid solution to WPRTP$\{P', R', \{H'_i\}, \{W'_i\}\}$ must also be a valid solution to WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$.

For $\forall r_i \in R - R' = \{r_i|W_i = \varnothing, r_i \in R\}$, since that r_i does not request any packets, any valid solution to WPRTP$\{P', R', \{H'_i\}, \{W'_i\}\}$ must also be valid for r_i . Therefore, any valid solution to WPRTP$\{P', R', \{H'_i\}, \{W'_i\}\}$ must also be valid for all receivers in $R - R'$.

For $\forall r_i \in R'$, according the definition of valid solutions to WPRTPs, for any valid solution to WPRTP$\{P', R', \{H_i'\}, \{W_i'\}\}$, r_i must be able to decode out all packets in W_i'. Additionally, for any receiver r_i, the set of available information in WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$ is a superset of that in WPRTP$\{P', R', \{H_i'\}, \{W_i'\}\}$. In other words, any information available to $r_i \in R'$ in WPRTP$\{P', R', \{H_i'\}, \{W_i'\}\}$ must also be available to $r_i \in R'$ in WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$. Therefore, $r_i \in R'$ must also be able to decode out all packets in $W_i = W_i'$ in WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$.

As a conclusion, any valid solution to WPRTP$\{P', R', \{H_i'\}, \{W_i'\}\}$ must also be a valid solution to WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$.

(2) To prove that any optimal solution to WPRTP$\{P', R', \{H_i'\}, \{W_i'\}\}$ must also be an optimal solution to WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$.

Now we will prove it by contradiction.

According to the definition of optimal solutions to WPRTPs, optimal solutions are all valid solutions, and the numbers of retransmitted packets in all optimal solutions to a WPRTP are of the same value.

Suppose that a solution s_1 is an optimal solution to WPRTP$\{P', R', \{H_i'\}, \{W_i'\}\}$, but it is not an optimal solution to WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$. Suppose that the number of retransmitted packets in the solution s_1 is $|s_1| = k$.

Since that any optimal solution must also be a valid solution to the same problem, the solution s_1 must be a valid solution to WPRTP$\{P', R', \{H_i'\}, \{W_i'\}\}$. Basing on the results obtained in the previous step, the solution s_1 must also be a valid solution to WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$.

The numbers of retransmitted packets of all optimal solutions to a certain WPRTP are the smallest among those of all valid solutions. Since that the solution s_1 is not an optimal solution to WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$, the number of retransmitted packets in an optimal solution to WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$, denoted as k', must meet that $k' < |s_1| = k$, otherwise the solution s_1 will be an optimal solution to WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$, which is contradict to the previous assumption.

Suppose that a solution s_2 is an optimal solution to WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$, and the number of retransmitted packets in the solution s_2 is $|s_2| = k' < k$. Being an optimal solution to WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$, the solution s_2 must also be a valid solution to WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$.

Now, basing on the solution s_2, we will construct a valid solution to WPRTP$\{P', R', \{H_i'\}, \{W_i'\}\}$.

Suppose that the coding operation in solution s_2 can be expressed as Eq. 7 (It is identical to Eq. 1, but it was re-shown here for convenient).

$$P_Y = M \cdot P_X \tag{7}$$

where:

$$P_Y = \begin{bmatrix} P_{Y,1} \\ P_{Y,2} \\ \vdots \\ P_{Y,k'} \end{bmatrix} = \begin{bmatrix} y_{1,1} & y_{1,2} & \cdots & y_{1,\lceil L/m \rceil} \\ y_{2,1} & y_{2,2} & \cdots & y_{2,\lceil L/m \rceil} \\ \vdots & \vdots & \ddots & \vdots \\ y_{k',1} & y_{k',2} & \cdots & y_{k',\lceil L/m \rceil} \end{bmatrix}, \quad M = \begin{bmatrix} m_{1,1} & m_{1,2} & \cdots & m_{1,n} \\ m_{2,1} & m_{2,2} & \cdots & m_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ m_{k',1} & m_{k',2} & \cdots & m_{k',n} \end{bmatrix},$$

$$P_X = \begin{bmatrix} P_{X,1} \\ P_{X,2} \\ \vdots \\ P_{X,n} \end{bmatrix} = \begin{bmatrix} x_{1,1} & y_{1,2} & \cdots & x_{1,\lceil L/m \rceil} \\ x_{2,1} & y_{2,2} & \cdots & x_{2,\lceil L/m \rceil} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n,1} & x_{n,2} & \cdots & x_{n,\lceil L/m \rceil} \end{bmatrix}$$

Transforms Eq. 7 as follows: (1) partitions $P_X = [P_{X,1}, P_{X,2}, \ldots, P_{X,n}]^T$ into two smaller vectors: $P'_X = [P'_{X,1}, P'_{X,2}, \ldots, P'_{X,n'}]^T$ and $P''_X = [P''_{X,1}, P''_{X,2}, \ldots, P''_{X,n''}]^T$. Elements in P'_X are all belong to set P' and elements in P''_X are all belong to $P'' = P - P'$. Relative orders of different packets in P'_X and P''_X are kept unchanged respect to those in vector P_X; (2) partitions the coding matrix M into M' and M'' following the same rule used in partitioning the vector P_X.

After finishing the transformation described above, Eq. 7 can be transformed into Eq. 8:

$$P_Y = M' \cdot P'_X + M'' \cdot P''_X \tag{8}$$

Equation 8 can be re-written as Eq. 9:

$$P_Y - M'' \cdot P''_X = P'_Y = M' \cdot P'_X \tag{9}$$

Equation 9 can be regarded as the coding operation of a solution to WPRTP$\{P', R', \{H'_i\}, \{W'_i\}\}$. We denote the solution as s_3. Obviously, number of retransmitted packets in solution s_3 is $|s_3| = k'$. It is obvious that the solution s_3 is a valid solution to WPRTP$\{P', R', \{H'_i\}, \{W'_i\}\}$.

Till now, we get a result that $|s_3| = k' < k = |s_1|$, it means that number of retransmitted packets in a valid solution s_3 to WPRTP$\{P', R', \{H'_i\}, \{W'_i\}\}$ is smaller than another valid solution s_1, which is contrary to the assumption that s_1 an optimal solution to WPRTP$\{P', R', \{H'_i\}, \{W'_i\}\}$. Therefore, with the contradiction, a conclusion can be made that any optimal solution to WPRTP$\{P', R', \{H'_i\}, \{W'_i\}\}$ must also be an optimal solution to WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$.

To make a conclusion basing on the previous two steps, the Theorem follows. □

4.2 Upper and Lower Bounds of the Number of Retransmitted Packets in Optimal NC-based Solutions to IP-WPRTPs

We project that IP-WPRTP is a NP-Complete problem although having not found the proof yet. Thus, the exact number of retransmitted packets in an optimal NC-based solution to an IP-WPRTP may be hard to determine. However, the upper and lower bounds can be determined easily by using the following Theorems 3 and 4.

Theorem 3 *An upper bound of the number of retransmitted packets in any optimal NC-based solution to an IP-WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$ is* $\max_{r_i \in R} |P - H_i|$.

Proof Following the transformation shown in Eq. 10, IP-WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$ can be transformed to P-WPRTP$\{P', R', \{H'_i\}, \{W'_i\}\}$:

$$P' = P, \quad R' = R, \quad H'_i = H_i, \quad W'_i = P - H_i \tag{10}$$

According to Theorem 1, the number of retransmitted packets in an optimal NC-based solution (denoted as s_1) to P-WPRTP$\{P', R', \{H'_i\}, \{W'_i\}\}$ is $|s_1| = \max_{r_i \in R'} |W'_i| = \max_{r_i \in R} |W'_i| = \max_{r_i \in R} |P - H_i|$.

The transformation in Eq. 10 guarantees that, any optimal solution to P-WPRTP$\{P', R', \{H'_i\}, \{W'_i\}\}$ must also be a valid solution to IP-WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$, but the vice versa may not hold. Hence, the number of retransmitted packets in an optimal solution s_2 to IP-WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$ must not be greater than that of the solution s_1, which is a valid solution to IP-WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$, i.e., $|s_2| \le |s_1| = \max_{r_i \in R} |P - H_i|$.

Therefore, the Theorem follows. □

Theorem 4 *A lower bound of the number of retransmitted packets in any optimal NC-based solution to IP-WPRTP{P, R, {H_i}, {W_i}} is* $\max_{r_i \in R} |W_i|$.

Proof Following the transformation shown in Eq. 11, IP-WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$ can be transformed into P-WPRTP$\{P', R', \{H_i'\}, \{W_i'\}\}$:

$$P' = P, \quad R' = R, \quad H_i' = P - W_i, \quad W_i' = W_i \tag{11}$$

According to Theorem 1, the number of retransmitted packets in an optimal NC-based solution (denoted as s_1) to P-WPRTP$\{P', R', \{H_i'\}, \{W_i'\}\}$ is $|s_1| = \max_{r_i \in R'} |W_i'| = \max_{r_i \in R} |W_i'| = \max_{r_i \in R} |W_i|$.

The transformation in Eq. 11 guarantees that, any valid solution to IP-WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$ must also be a valid solution to P-WPRTP$\{P', R', \{H_i'\}, \{W_i'\}\}$, but the vice versa may not hold. Suppose that a solution s_2 is an optimal solution to IP-WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$, it must also be a valid solution to P-WPRTP$\{P', R', \{H_i'\}, \{W_i'\}\}$. Hence, the number of retransmitted packets in s_2 must not be smaller than that of the optimal solution s_1 to P-WPRTP$\{P', R', \{H_i'\}, \{W_i'\}\}$, i.e., $|s_2| \geq |s_1| = \max_{r_i \in R} |W_i|$.

Therefore, the Theorem follows. □

5 A Divide and Conquer Based Scheme for IP-WPRTPs

Based on the WPRTP reduction theorem, and using a divide and conquer approach, we propose a scheme for solving IP-WPRTPs.

5.1 Divide and Conquer based Algorithm for Determining Coding Matrix

The basic idea of the divide and conquer based algorithm for determining a valid coding matrix for IP-WPRTPs can be expressed in the following three steps.

(1) Partition the set P in the original IP-WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$ into several subsets $\{P^1, P^2, \ldots, P^k, \ldots, P^n\}$ (here n represents the number of subsets) according to some predefined rules (called as Packet Set Partition Rules).

(2) Following the transformation shown in Eq. 12, transform the original IP-WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$ into n smaller sub-problems WPRTP$\{P^k, R^k, \{H_i^k\}, \{W_i^k\}\}$, $k = \{1, 2, \ldots, n\}$.

$$R^k = R, \quad H_i^k = P^k \cap H_i, \quad W_i^k = P^k \cap W_i \tag{12}$$

(3) Recursively partition all sub-problems into sub-sub-problems following the above step (1) and step (2). The upper and lower bounds of the problems are used to determine whether a partition is necessary. The combination of valid solutions to all sub-problems composes a valid solution to the parent problem.

Obviously, since that: (1) the number of retransmitted packets of an optimal solution to a P-WPRTP is easy to determine, and (2) an optimal solution to a P-WPRTP can be constructed easily by using RNC, partition to P-WPRTPs is unnecessary.

The core function of the divide and conquer based algorithm is named as WPRTP_DivideAndConquer(…). This function returns a special matrix called Coding Matrix Template, whose elements are either 0 or 1. Each row in the coding matrix template is called Coding Vector Template. Length of the coding vector template equals packet number $|P|$ in the problem, and each bit in the coding vector template corresponds to a packet in $|P|$. A coding matrix will be constructed basing on the coding matrix template.

The core pseudo code of the function WPRTP_DivideAndConquer(…) is as follows.

WPRTP_DivideAndConquer (WPRTP{P, R, {H_i}, {W_i}})
Input: P, R, {H_i}, {W_i};   **Output:** pktCodeNum, coding_matrix_template;   **Descriptions:** pktCodeNum is the number of retransmitted packets of a solution determined by the function; coding_matrix_template is a matrix template obtained by the function.

```
   //Reduce the WPRTP according to the WPRTP reduction theorem (Theorem 2).
1. WPRTP=WPRTP_Reduce(WPRTP);
   //Determine the upper bound and lower bound of the WPRTP according to Theorem 3 and Theorem 4. //The input
   WPRPT is reduced before calculating the bounds.
2. [upper_bound, lower_bound]=WPRTP_getBounds(WPRTP);
3. pktCodeNum=upper_bound;
4. if (upper_bound ≤ lower_bound) {
       4.1 [coding_matrix_template]=WPRTP_createCodingMatrixTemplate(WPRTP);
       4.2 return [pktCodeNum, coding_matrix_template]; }
   //Partition the WPRTP according to Eqn. (12);
5. [WPRTP_1, …, WPRTP_i, …, WPRTP_n]=WPRTP_Partition(WPRPT);
6. for i=1 to n{
      6.1 [upper_bound_i, lower_bound_i]=WPRTP_getBounds(WPRTP_i); }
7. if pktCodeNum>∑_i (upper_bound_i) {
      7.1 pktCodeNum=∑_i (upper_bound_i);
      7.2 coding_matrix_template=WPRTP_createCodingMatrixTemplateFromSubs(∪_i(WPRTP_i));}
8. if pktCodeNum>∑_i (lower_bound_i) {
     8.1 for each i {
           8.1.1 if (upper_bound_i>lower_bound_i){
                 8.1.1.1 [pktCodeNum_i, coding_matrix_template_i]=WPRTP_DivideAndConquer (WPRTP_i);}
           8.1.2 else{
                 8.1.2.1 pktCodeNum_i=upper_bound_i;
                 8.1.2.2 [coding_matrix_template_i]=WPRTP_createCodingMatrixTemplate(WPRTP_i);} }
9. if pktCodeNum>∑_i (pktCodeNum_i ){
     9.1 pktCodeNum=∑_i (pktCodeNum_i );
     //Construct coding_matrix_template by combining all coding_matrix_template_i;
     9.2 coding_matrix_template=WPRTP_combineCodingMatrixTemplate(∪_i(coding_matrix_template_i)); }
10 Return [pktCodeNum, coding_matrix_template];
```

The function WPRTP_createCodingMatrixTemplate(WPRTP) in line 4.1 constructs coding matrix template for the WPRTP as follows: (1) the number of rows of the matrix template equals the upper bound; (2) the columns corresponding to the packets in the input WPRTP are all set to 1,and all other columns are set to 0.

The function WPRTP_combineCodingMatrixTemplate($\cup_i$(coding_matrix_template$_i$)) in line 9.2 combines all input coding_matrix_template$_i$ ($i = \{1, 2, \ldots, n\}$) into one coding matrix template by collecting all coding vector templates in the inputted templates into one bigger matrix.

The function WPRTP_createCodingMatrixTemplateFromSubs($\cup_i$(WPRTP$_i$)) in line 7.2 works as follows: (1) creates coding matrix templates for all input WPRTP$_i$ by calling function WPRTP_createCodingMatrixTemplate(WPRTP$_i$); (2) combines all created coding matrix templates in the first step by calling function WPRTP_combineCodingMatrixTemplate(…).

Table 1 Example of operation process of the divide and conquer based algorithm

Depth	Step operations	WPRTP description	Results
1	WPRTP	$P = \{p_1, p_2, p_3, p_4, p_5, p_6\}$; $R = \{r_1, r_2, r_3, r_4, r_5\}$; $\{H_1, H_2, H_3, H_4, H_5\} = \{\{p_1, p_3, p_6\}, \{p_1, p_4\},$ $\{p_2, p_3, p_4, p_6\}, \{p_1, p_5, p_6\}, \{p_2, p_3\}\}$ $\{W_1, W_2, W_3, W_4, W_5\} = \{\{p_4\}, \{p_3\}, \{p_1, p_5\}, \{p_2\}, \{\}\}$	
1	Reduced WPRTP	$P' = \{p_1, p_2, p_3, p_4, p_5\}$; $R' = \{r_1, r_2, r_3, r_4\}$; $\{H_1', H_2', H_3', H_4'\} = \{\{p_1, p_3\}, \{p_1, p_4\}, \{p_2, p_3, p_4\},$ $\{p_1, p_5\}\}$ $\{W_1', W_2', W_3', W_4'\} = \{\{p_4\}, \{p_3\}, \{p_1, p_5\}, \{p_2\}\}$	Upper_bound=3 Lower_bound=2
1	Packet partition	$P^1 = \{p_1, p_3, p_4\}$; $P^2 = \{p_2, p_5\}$;	
2	WPRTP_1	$P^1 = \{p_1, p_3, p_4\}$; $R^1 = \{r_1, r_2, r_3, r_4\}$; $\{H_1^1, H_2^1, H_3^1, H_4^1\}$=$\{\{p_1, p_3\}, \{p_1, p_4\}, \{p_3, p_4\}, \{p_1\}\}$ $\{W_1^1, W_2^1, W_3^1, W_4^1\} = \{\{p_4\}, \{p_3\}, \{p_1\}, \{\}\}$	
2	WPRTP_2	$P^2 = \{p_2, p_5\}$; $R^2 = \{r_1, r_2, r_3, r_4\}$; $\{H_1^2, H_2^2, H_3^2, H_4^2\} = \{\{\}, \{\}, \{p_2\}, \{p_5\}\}$ $\{W_1^2, W_2^2, W_3^2, W_4^2\} = \{\{\}, \{\}, \{p_5\}, \{p_2\}\}$	
2	Reduced WPRTP_1	$P^1 = \{p_1, p_3, p_4\}$; $R^1 = \{r_1, r_2, r_3\}$; $\{H_1^1, H_2^1, H_3^1\} = \{\{p_1, p_3\}, \{p_1, p_4\}, \{p_3, p_4\}\}$ $\{W_1^1, W_2^1, W_3^1\} = \{\{p_4\}, \{p_3\}, \{p_1\}\}$	$\text{Upper\_bound}_1 = 1$ $\text{Lower\_bound}_1 = 1$ $\text{pktCodeNum}_1 = 1$ $T_1 = [1, 0, 1, 1, 0]$
2	Reduced WPRTP_2	$P^2 = \{p_2, p_5\}$; $R^2 = \{r_3, r_4\}$; $\{H_3^2, H_4^2\} = \{\{p_2\}, \{p_5\}\}$ $\{W_3^2, W_4^2\} = \{\{p_5\}, \{p_2\}\}$	$\text{Upper\_bound}_2 = 1$ $\text{Lower\_bound}_2 = 1$ $\text{pktCodeNum}_2 = 1$ T_2=[0, 1, 0, 0, 1]
1	Combine template	WPRTP_combineCodingMatrixTemplate(T_1, T_2)	$T = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$

What the function WPRTP_DivideAndConquer(…) obtained is a coding matrix template (denoted as T), which is used by a function called as coding matrix construction function to construct coding matrix M according to Eq. 13.

$$m_{i,j} = \begin{cases} \text{r and } (\text{GF}(2^q)/0) & t_{i,j} = 1 \\ 0 & t_{i,j} = 0 \end{cases} \tag{13}$$

Table 1 shows an example of the operation process of the divide and conquer algorithm. The 1st column indicates the calling depth of the recursive function WPRTP_DivideAndConquer(…). The 2nd column lists the main operation in the current step. The 3rd column shows the content of main data structures of the WPRTP in the current step. The 4th column gives the main results for the corresponding WPRTPs shown in the 3rd column, such as the upper bound, lower bound, optimal number of coded packets, and coding matrix template. The packet set partition rule used in the 3rd row is as follows: partition the packet set Pinto two subsets: $W_j \cup H_j$, $P - W_j \cup H_j$, where $j = \arg\max_{r_i \in R} |W_i \cup H_i|$. Results of simulations for determining more preferable packet set partition rules in the following section showed that this partition rule is the most preferable packet set partition rule among several candidate rules. Hence, this partition rule is adopted in our Divide and Conquer based Algorithm.

5.2 Divide and Conquer Based Scheme for IP-WPRTPs

Based on the divide and conquer algorithm proposed in the previous section for searching for coding matrix templates for IP-WPRTPs, and combining with the coding matrix construction function which constructs the final coding matrix according to the coding matrix template according to Eq. 13, we propose a scheme named as IP-WPRTP-DC (Divide and Conquer based scheme for IP-WPRTPs) for solving IP-WPRTPs.

When transmitting packets using NC, auxiliary information and coded packets are usually enclosed into one packet called as assembled packet. Assembled packets in IP-WPRTP-DC consist of three fields: *PacketBitFlag*, *CodingCoefVector*, and *CodedPacket*. Field *PacketBitFlag* indicates which packets are used in creating the enclosed coded packet in the assembled packet. It is a vector of bits where each bit corresponds to one packet in P. If one bit is '1', the corresponding packet is selected. Otherwise, the corresponding packet is not selected. Field *CodingCoefVector* stores the coding vector used to create the coded packet. The size of the vector equals the number of '1' bits in field *PacketBitFlag*. Length of each element in the vector equals code coefficient bit length q. Field *CodedPacket* stores the corresponding coded packet.

In IP-WPRTP-DC, all receivers use the previously described general process to obtain their wanted packets. Most modifications in IP-WPRTP-DC are made to the operation process of the sender. The operation process of the sender in IP-WPRTP-DC is as follows:

Operation process of the sender in IP-WPRTP-DC:

(1) Obtain coding matrix template T and the number of coded packets (denoted as k) to be retransmitted by calling the function WPRTP_DivideAndConquer(…);
(2) Create coding matrix $M_{k\times n}$ according to Eq. 13 basing on the coding matrix template T;
(3) Check whether $M_{k\times n}$ makes up a valid solution. If yes, go to step (4); otherwise go to step (2); (The sender has enough information to check whether the coding matrix makes up a valid solution);
(4) Calculate the vector of coded packets $P_Y = [P_{Y,1}, P_{Y,2}, \ldots, P_{Y,k}]^T$ according to $P_Y = M_{k\times n} \times P_X$, Here $P_X = [P_{X,1}, P_{X,2}, \ldots, P_{X,n}]^T = [p_1, p_2, \ldots, p_n]^T$;
(5) Construct k assembled packets $P_{Z,i} = \{flag_i, m_i, P_{Y,i}\}$ ($i \in \{1, 2, \ldots, k\}$). Here, $flag_i$ is a vector created according to Eq. 14 basing on the coding matrix template $T = \{t_{i,j}\}_{k\times n}$; $m_i = (m_{i,1}, m_{i,2}, \ldots, m_{i,n})$; $P_{Y,i}$ is the i-th coded packet in P_Y;

$$flag_{i,j} = \begin{cases} 1 & t_{i,j} = 1 \\ 0 & t_{i,j} = 0 \end{cases} \tag{14}$$

(6) Broadcast all assembled packets $P_{Z,i}$ ($i \in \{1, 2, \ldots, k\}$);
(7) End;

5.3 Determine the Packet Set Partition Rule for the Divide and Conquer Algorithm

One key task in the partition of IP-WPRTPs is the partition of the problem's packet set P. To determine a more preferable packet set partition rule, we compare the performance of several typical packet set partition rules by simulation in Matlab. The partition rules tested and the simulation results are shown in Table 2.

Table 2 Performance comparison of different packet set partition rules for the Divide and Conquer Algorithm

Test	Partition rule	Description of the packet set partition rule	Performance
1	(1)	Two sets: $W_j \cup H_j$, $P - W_j \cup H_j$, where $j = \arg\max_{r_i \in R} \|W_i \cup H_i\|$;	Good
	(2)	Two sets, but elements in the set are determined randomly;	Worse
2	(1)	Two sets: $W_j \cup H_j$, $P - W_j \cup H_j$, where $j = \arg\max_{r_i \in R} \|W_i \cup H_i\|$;	Good
	(2)	Three sets: W_j, H_j, $P - W_j \cup H_j$, where $j = \arg\max_{r_i \in R} \|W_i \cup H_i\|$;	Worse
3	(1)	Two sets: $W_j \cup H_j$, $P - W_j \cup H_j$, where $j = \arg\max_{r_i \in R} \|W_i \cup H_i\|$;	good
	(2)	Two sets: $W_j \cup H_j$, $P - W_j \cup H_j$, where $j = \arg\max_{r_i \in R} \|W_i\|$;	Worse
	(3)	Two sets: $W_j \cup H_j$, $P - W_j \cup H_j$, where $j = \arg\min_{r_i \in R} \|W_i \cup H_i\|$;	Worse
	(4)	Two sets: $W_j \cup H_j$, $P - W_j \cup H_j$, where $j = \arg\min_{r_i \in R} \|W_i\|$;	Worse
	(5)	Two sets: $W_j \cup H_j$, $P - W_j \cup H_j$, where j is selected randomly;	Worse
4	(1)	Two sets: $W_j \cup H_j$, $P - W_j \cup H_j$, where$j = \arg\max_{r_i \in R} \|W_i \cup H_i\|$;	Good
	(2)	Two sets: W_j, $P - W_j$, where$j = \arg\max_{r_i \in R} \|W_i \cup H_i\|$;	Worse
	(3)	Two sets: H_j, $P - H_j$, where$j = \arg\max_{r_i \in R} \|W_i \cup H_i\|$;	Worse

Results in Table 2 indicate that the most preferable packet set partition rule is as follows: partitions the packet set P into two subsets: $W_j \cup H_j$, $P - W_j \cup H_j$, where $j = \arg\max_{r_i \in R} |W_i \cup H_i|$.

6 Performance Evaluation

We evaluated the performance of our scheme and compared it with some other typical schemes by simulations using Matlab.

6.1 Performance Metrics

(1) Number of Retransmitted Packets

This metric represents the number of retransmitted packets in a solution to a WPRTP. This is the main metric for measuring the performance of the schemes for WPRTPs.

For NC-based schemes, this metric represents the number of coded packets (assembled packets) in a solution. In a traditional scheme (designated as NoNC) where NC is not used, each requested packets should be sent once. Hence, this metric of NoNC represents the number of requested packets, i.e., $| \cup_{r_i \in R} W_i |$.

In fact, length of coded packets will be much longer than auxiliary bits in the assembled packets. Hence, number Of retransmitted packets can partly reflect the differences between these schemes in the metric of total bit length. If the total bit length of retransmitted packets is preferred, the result as a product of the number of retransmitted packets and the bit length of a retransmitted packet can easily be obtained.

(2) Relative Number of Retransmitted Packets

Relative number of retransmitted packets of *scheme1/scheme2* represents the ratio of the number of retransmitted packets of *scheme1* to that of *scheme2*. For example, relative number of retransmitted packets of IP-WPRTP-DC/CliqueNC is the ratio of the number of retransmitted packets of IP-WPRTP-DC to that of CliqueNC. This metric directly reveals the performance gains of a scheme over another.

6.2 Simulation Configuration

Basic simulation parameters of IP-WPRTP$\{P, R, \{H_i\}, \{W_i\}\}$ include: packet number $|P|$, receiver number $|R|$, coding coefficient bit length q, packet consideration level c_{Level} (which means the probability that a packet belongs to $H_i \cup W_i$), and packet request level p_{Level} (the probability that a packet in $H_i \cup W_i$ belongs to W_i). We assume that symbol bit length m of the used Galois Field GF(2^m) meets that $m \geq q$. A profile $(|P|, |R|, q, c_{\text{Level}}, p_{\text{Level}})$ is called a simulation configuration. In our simulations, q is fixed to 8. Simulation results in [16] showed that $q = 8$ is generally acceptable.

For each simulation configuration, 100 IP-WPRTP instances are generated and treated using the tested schemes one by one. The performance metrics are averaged over these instances and their 95% confidence intervals were also calculated. In some of the following figures, confidence intervals are also shown.

Each IP-WPRTP instance is represented as a 2-dimentional matrix $M_P$of size $|R| \times |P|$. Each element $a_{i,j}$in $M_P$has three possible values: 0, 1, and 2. The value of $a_{i,j}$ indicates the status of packet p_j respect to receiver r_i. If $a_{i,j} = 0$, then $p_j \in W_{i,}$; if $a_{i,j} = 1$, then $p_j \in H_{i,}$; if $a_{i,j} = 2$, then $p \in P - W_i - H_i$. Each element $a_{i,j}$ was generated as follows: (1) randomly selects a variable x which is uniformly distributed in range [0, 1]; (2) if $x < c_{\text{Level}} \times p_{\text{Level}}$, then $a_{i,j} = 0$; If $c_{\text{Level}} \times p_{\text{Level}} \leq x < c_{\text{Level}}$, then $a_{i,j} = 1$; if $x \geq c_{\text{Level}}$, then $a_{i,j} = 2$.

6.3 Simulation Results of Packet Set Partition Rule Selection Experiment

Simulation results of the packet set partition rule selection experiment are shown in this section. Simulation parameters in these simulations were set as follows: $|P| = 10$, $|R| = 10$, $q = 8$, $p_{\text{Level}} = 0.2$, $c_{\text{Level}} \in [0.2, 0.9]$.

Simulation results of Test 1 in Table 2 are shown in Fig. 1a. In In Fig. 1a, the curve designated as "NodeBased" represents the results of partition rule (1) in Test 1, and the curve designated as "Random" represents the results of partition rule (2) in Test 1. Simulation results of Test 2 in Table 2 are shown in Fig. 1b. In Fig. 1b, the curve designated as "TwoGroup" represents the results of partition rule (1) in Test 2, and the curve designated as "ThreeGroup" represents the results of partition rule (2) in Test 2. Simulation results of Test 3 in Table 2 are shown in Fig. 1c. In Fig. 1c, the curves designated as "Rule1" to "Rule5" represent the results of partition rule (1) to (5) in Test 3, respectively. Simulation results of Test 4 in Table 2 are shown in Fig. 1d. In Fig. 1b, the curve designated as "WantHas" represents the results of partition rule (1) in Test 4, the curve designated as "Want" represents the results of partition rule (2) in Test 4, and the curve designated as "Has" represents the results of partition rule (3) in Test 4.

Simulation results in Fig. 1 show that, the most preferable packet set partition rule of IP-WPRTP-DC is as follows: partitions the packet set P into two sets: $W_j \cup H_j$, $P - W_j \cup H_j$, where $j = \arg\max_{r_i \in R} |W_i \cup H_i|$. Hence, in the simulations in the following section, this packet set partition rule is used in IP-WPRTP-DC.

6.4 Performance Simulation

Performances of several typical schemes for solving IP-WPRTPs were tested though simulations using Matlab. The tested schemes include: IP-WPRTP-DC, ColorNC [3], CliqueNC [14], and NoNC. Additionally, the upper and lower bounds of IP-WPRTPs are also obtained by using Theorems 3 and 4, respectively. The results of the upper and lower bounds are designated as IP-WPRTP-MAX and IP-WPRTP-MIN, respectively.

The NC schemes proposed in [15] are similar to CliqueNC. However, since that these schemes in [15] runs on-the-fly and thus usually use partial information of the WPRTP when

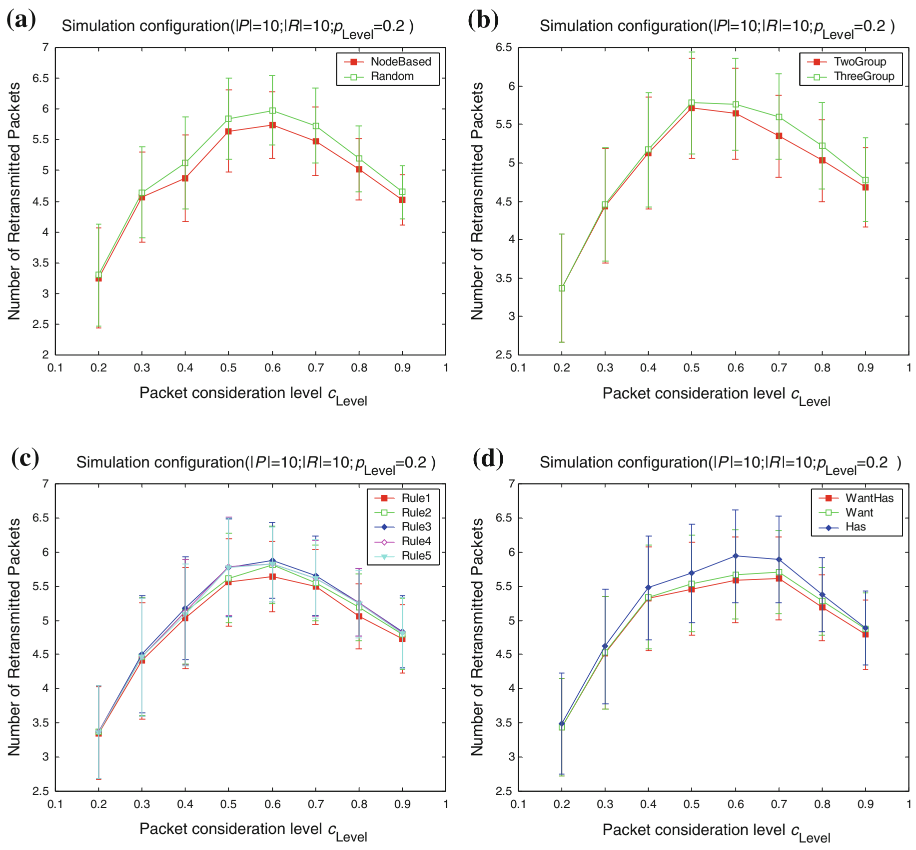

Fig. 1 Results of packet set partition rule selection experiment

searching for solutions, the solutions found by their schemes are usually not better than those of CliqueNC. Hence, the schemes proposed in [15] are not tested in our simulation.

Before being treated with the selected schemes, each WPRTP instance was firstly treated following the WPRTP reduction theorem. CliqueNC [14] is originally proposed for P-WPRTPs. In order to treat IP-WPRTPs using CliqueNC, each IP-WPRTP instance was pre-treated using Eq. 15 in our simulations. Although this treatment will definitely worsen the performance metrics of CliqueNC, this treatment is used in our simulation since that no suitable and simple treatment available.

$$W_i = P - H_i \quad \forall i \in \{1, 2, \ldots, |R|\} \tag{15}$$

6.4.1 Effects of Packet Number and Receiver Number

A set of simulations is performed to test the effects of packet number $|P|$ and receiver number $|R|$ on the performances of the schemes. Simulation configurations are set as follows: packet consideration level $c_{\text{Level}} = 0.8$, packet request level $p_{\text{Level}} = 0.7$, coefficient bit length $q = 8$, packet number $|P|$ increases from 3 to 30 with step size 3, and receiver number $|R|$ also increases from 2 to 10 with step size 1. Results of these simulations are shown in Fig. 2 to Fig. 3.

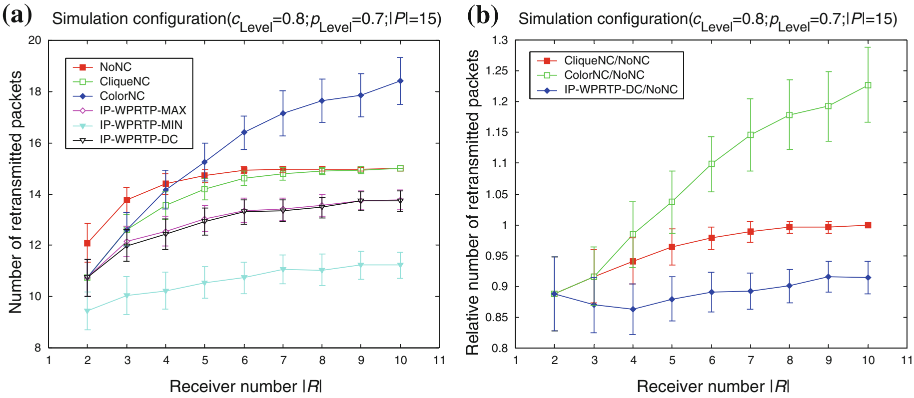

Fig. 2 Effect of receiver number on tested schemes. **a** Number of retransmitted packets. **b** Relative number of retransmitted packets

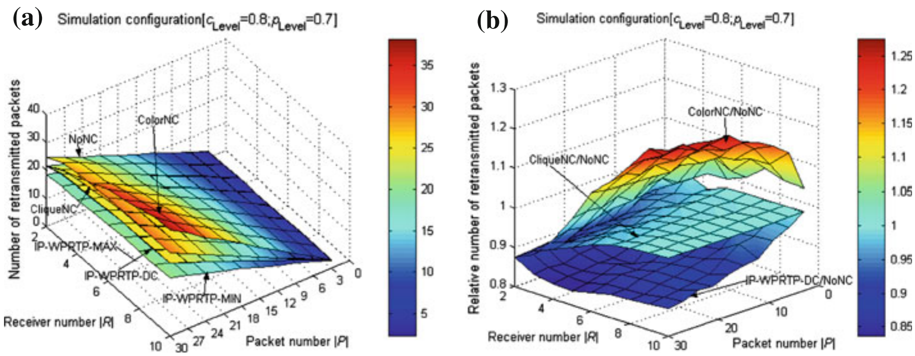

Fig. 3 Effects of packet number and receiver number on tested schemes. **a** Number of retransmitted packets. **b** Relative number of retransmitted packets

Figure 2a shows the variation of the number of retransmitted packets when $|R|$ increases from 2 to 10 while $|P| = 15$. When $|R| = 2$, the number of retransmitted packets in the solutions determined by using NoNC is about 12. However, those in solutions determined by using CliqueNC, ColorNC, and IP-WPRTP-DC are all about 10.6. Thus, 12% packet transmissions are saved by using NC. As $|R|$ increases, the numbers of retransmitted packets of the schemes increase quickly, but those of NoNC, ColorNC, and CliqueNC increase more quickly than that of IP-WPRTP-DC. As the number of retransmitted packets approaches to packet number $|P|$, the increasing speed of the metric of NoNC and CliqueNC slow down. This metric of NoNC approaches to $|P|$ when $|R| = 6$, whereas that of CliqueNC increases to about $|P|$ when $|R| = 9$. Contrastively, this metric of ColorNC increases continuously to much more than $|P|$. This may be partly because that the greedy heuristic algorithm used to solve the clique partition problems in ColorNC becomes much less efficient as the problems size increases. By substituting an original receiver with multiple new receivers, the new problems in ColorNC are usually much larger than the original WPRTPs. Contrastively, the number of retransmitted packets of IP-WPRTP-DC increases more slowly than those of others. Even when $|R| = 10$, the number of retransmitted packets of IP-WPRTP-DC is still fewer than 90% of that of NoNC. The relative performances among pairs of the tested

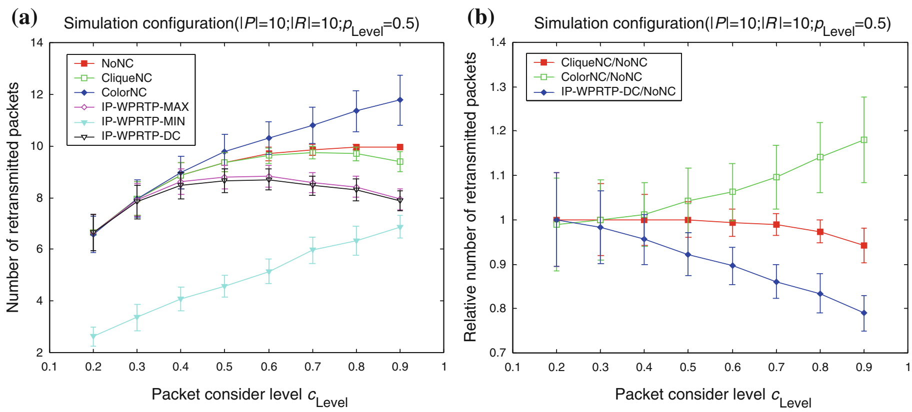

Fig. 4 Effect of packet consideration level on tested schemes. **a** Number of retransmitted packets. **b** Relative number of retransmitted packets

schemes are more clearly shown in Fig. 2b. The percent of retransmitted packets saved by IP-WPRTP-DC is always larger than 10% in the simulations. When $|R| = 4$, about 14% packets are saved.

Compared with IP-WPRTP-MAX, the performance gain of IP-WPRTP-DC is not very distinctive; meanwhile, the differences between IP-WPRTP-MIN and IP-WPRTP-DC are quite large. However, it does not definitely mean that IP-WPRTP-DC scheme is much inefficient. One reason for the larger gap between IP-WPRTP-MIN and IP-WPRTP-DC may be that the lower bound may be much looser than the upper bound.

Figure 3a shows the effects of $|P|$ and $|R|$ on the numbers of retransmitted packets of all the tested schemes when $|P| \in [3, 30]$ and $|R| \in [2, 10]$, whereas Fig. 3b shows the effects of $|P|$ and $|R|$ on the relative numbers of retransmitted packets. The results show that, as $|R|$ increases, the performance gain of CliqueNC over NoNC becomes more trivial. Contrastively, the performance gain of IP-WPRTP-DC over NoNC is more distinctive with larger $|P|$. As $|R|$ increases, ColorNC becomes much worse than NoNC. Packet number $|P|$ shows little effect on the relative metrics, but $|P|$ has much more distinctive effects on the relative number of retransmitted packets of ColorNC and IP-WPRTP-DC. As $|P|$ increases, the performance gain of IP-WPRTP-DC becomes more distinctive, whereas ColorNC becomes even worse.

6.4.2 Effects of Packet Consideration Level and Packet Request Level

Another set of simulations is performed to test the effects of packet consideration level c_{Level} and packet request level p_{Level} on the performances of the schemes. Simulation configurations were set as follows: coding coefficient bit length $q = 8$, packet number $|P| = 10$, receiver number $|R| = 10$. Packet consideration level c_{Level} increases from 0.2 to 0.9 with step size 0.1; and packet request level p_{Level} also increases from 0.2 to 0.9 with step size 0.1. Results of these simulations are shown in Fig. 4 to Fig. 5.

Figure 4a shows the variation of the number of retransmitted packets of the tested schemes as c_{Level} increases from 0.2 to 0.9 when $p_{\text{Level}} = 0.5$. The results show that, when $c_{\text{Level}} = 0.2$, the number of retransmitted packets of the schemes except for IP-WPRTP-MIN are almost the same, and no performance gain is obtained by the schemes using NC. As c_{Level} increases,

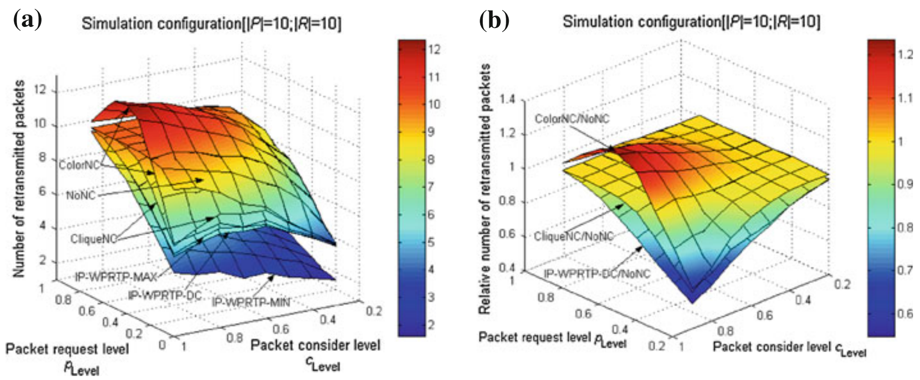

Fig. 5 Effects of packet request level and packet consideration level on tested schemes. **a** Number of retransmitted packets. **b** Relative number of retransmitted packets

the performance gains of IP-WPRTP-DC and CliqueNC increases linearly. The performance gain in saving packet transmissions of IP-WPRTP-DC over NoNC is much larger than that of CliqueNC over NoNC. Compared with NoNC, when $c_{\mathrm{Level}} = 0.9$, IP-WPRTP-DC reduces about 20% packet transmissions, meanwhile CliqueNC saves only about 5% packet transmissions.

Figure. 5(a) shows the effects of p_{Level} and c_{Level} on the numbers of retransmitted packets of all the tested schemes when $p_{\mathrm{Level}} \in [0.2, 0.9]$ and $c_{\mathrm{Level}} \in [0.2, 0.9]$, whereas Fig. 5b shows the effects of p_{Level} and c_{Level} on the relative numbers of retransmitted packets. The results show that, the smaller the p_{Level}, and the larger the c_{Level}, the greater the performance gains of IP-WPRTP-DC and CliqueNC over NoNC. When $p_{\mathrm{Level}} = 0.2$ and $c_{\mathrm{Level}} = 1$, IP-WPRTP-DC can even save 45% packet transmissions of that of NoNC.

7 Conclusions

In this paper, our study is focused on schemes for solving IP-WPRTPs. Firstly, a WPRTP reduction theorem for reducing the size of IP-WPRTPs is proposed and proved. Then, the upper bound and the lower bound of the number of retransmitted packets in optimal network coding based solutions to IP-WPRTPs are analyzed. Next, based on the WPRTP reduction theorem, a scheme for solving IP-WPRTPs is proposed using a divide and conquer approach, namely IP-WPRTP-DC. Extensive simulations show that, IP-WPRTP-DC can save up to 45% of retransmitted packets when compared with NoNC (a traditional scheme not using NC) when $p_{\mathrm{Level}} = 0.2$ and $c_{\mathrm{Level}} = 1$. Possible future work includes: (1) finding better algorithms for solutions with fewer retransmitted packets to IP-WPRTPs; (2) finding tighter lower and upper bounds of the number of packet retransmissions of optimal NC-based solutions to IP-WPRTPs.

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