Computing approximate blocking probabilities for a class of all-optical networks

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Abstract

We study a class of all-optical networks using wavelength division multiplexing and wavelength routing in which a connection between a pair of nodes in the network is assigned a path and a wavelength on that path. Moreover, on the links of that path no other connection can share the assigned wavelength. Using a generalized reduced load approximation scheme we calculate the blocking probabilities for the optical network model for two routing schemes: Fixed Routing and Least Loaded Routing.

1 Introduction

We study a class of all-optical networks using wavelength division multiplexing and wavelength routing [4] in which a connection between a pair of nodes in the network is assigned a path and a wavelength on that path. Moreover, on the links of that path no other connection can share the assigned wavelength. While we will refer to this type of network as the 'wavelength routing' model we should point out that a routing scheme for the connections through the the network is not implied, and in fact has to be specified.

The problem of routing and assignment of wavelength in such networks has previously been studied in [1, 2] where several heuristic algorithms have been proposed and their performance evaluated through simulation. In [7] a lower bound on the blocking probabilities for any routing and wavelength assignment algorithms was given, by first formulating the problem as an integer linear programming problem and then relaxing the integer constraint in order to obtain a linear programming problem from which the bound was derived.

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Our starting point is a generalized reduced load approximation scheme for circuit-switched networks given by Kelly [5] and further developed by Chung, Kashper and Ross [3]. We extend the method to the wavelength routing model for two routing scheme: Fixed Routing and Least Loaded Routing (LLR). For the fixed routing case we consider networks of arbitrary topology with the restriction that connections may be established only on paths with at most three hops. For the LLR case we restrict our network to fully connected networks and paths of one or two hops. While the restrictions on the number of hops can be relaxed at the expense of additional computational and storage complexity, it is doubtful whether paths with many hops are a good idea for this type of networks since, as will be shown, blocking probabilities grow with the number of hops much faster than for circuit-switched networks.

The paper is organized as follows. In Section 2 which follows blocking probabilities for the wavelength routing model are compared with those for circuit-switched networks for the simple case of links in tandem. An approximate method for calculating blocking probabilities for wavelength routing model with fixed routing is developed in Section 3, while Section 4 deals with LLR. In section 5 numerical results for the approximate method are compared with simulation. Section 6 consists of concluding remarks.

2 Wavelength routing vs. circuit switching

In circuit-switched networks with fixed routing an arriving call is accepted if on all links on its route there is at least one idle trunk (circuit). Otherwise the call is blocked. In the wavelength routing model each link has a number of wavelengths, the counterpart to trunks in circuit-switched networks. However, while channels on a link are indistinguishable, the wave-

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lengths on a link are distinct. In the wavelength routing model with fixed routing a call is accepted if there exists at least one wavelength which is idle on all links which make up the route of this call.

Clearly, blocking probabilities are higher in the wavelength routing model. We illustrate this by considering two networks identical in every respect except that one is circuit-switched and the other based on the wavelength routing model. There are J links in tandem, all links have C channels (trunks or wavelengths). Arrivals are Poisson while holding times are exponentially distributed with unit mean. There are J arrival streams such that arrival stream j, $j = 1, \ldots, J$, is associated with the nodes of link j. All these arrivals have rate λ. We refer to these traffic streams as 'local' since their routes consist of a single link. An additional arrival stream $(J+1)$ is associated with the two end nodes, i.e. its route includes all J links. The rate of this stream is λ_0, where $\lambda_0 \ll \lambda$.

Let B_{CS} denote the blocking probability for the end-to-end traffic in the circuit-switched case. Since $\lambda_0 \ll \lambda$ we ignore the contribution of the end-to-end traffic on the network state. We then have:

$$B_{CS} = 1 - [1 - B(C, \lambda)]^J$$

where $B(C, \lambda)$ is the Erlang loss formula.

For the wavelength routing model let B_{WR} denote the blocking probability for the end-to-end traffic. Let X_R be the random variable the number of idle wavelengths on route R. If the route consists of the single link j we may write X_j. Let $E = \{1, 2, \ldots, J\}$ denote the route for the end-to-end traffic. Then $B_{WR} = \Pr[X_E = 0]$. By conditioning $\Pr[X_E = 0]$ on the set of disjoint events $\{X_1 = m_1, \ldots, X_J = m_J | m_1 \geq 0, \ldots, m_J \geq 0\}$ we obtain:

$$\begin{aligned} B_{WR} &= \sum_{\mathbf{m} \geq 0} \Pr[X_E = 0 | X_1 = m_1, \ldots, X_J = m_J] \\ &\quad \times \Pr[X_1 = m_1, \ldots, X_J = m_J] \\ &= \sum_{\mathbf{m} \geq 0} p_0(\mathbf{m}) \prod_{j=1}^{J} \Pr[X_j = m_j] \qquad (1) \end{aligned}$$

where $\mathbf{m} = (m_1, \ldots, m_J)$. The second equality is based on $\{X_j\}$ being independent and here, again, we ignore the impact of the end-to-end traffic. We also used the notation

$$p_n(\mathbf{x}) = \Pr[X_R = n | X_1 = x_1, \ldots, X_N = x_N] \qquad (2)$$

where $R = \{1, \ldots, N\}$ is any route consisting of N links, $N \geq 2$, and $\mathbf{x} = (x_1, \ldots, x_N)$, Since the dimension of the vector argument may vary, p_n denotes not a single function, but a family of functions. Nevertheless, we use the same notation and will identify the specific function involved from the dimension of the vector argument. The other term under the summation sign in (1) is obtained from the solution of the Erlang loss system:

$$\Pr[X_j = m_j] = \frac{\lambda^{C-m_j}}{(C - m_j)!} \left(\sum_{k=0}^{C} \frac{\lambda^k}{k!} \right)^{-1}.$$

The probabilities $p_n(\mathbf{x})$ are computed on the assumption that the allocation of wavelengths is done randomly. The alternative is to assume that wavelengths are ordered, e.g. in order of increasing wavelength. Then, at call arrival time the wavelengths are scanned in this order and the first idle wavelength is allocated. While the ordered scheme leads to smaller blocking probabilities the random case is easier to analyze and it is the one we consider in the rest of the paper.

Let us first consider the case of a two-hop route $R = \{i, j\}$ and focus on

$$p_n(x, y) = \Pr[X_{i,j} = n | X_i = x; X_j = y].$$

We can think of the x wavelengths on link i as red balls which are distributed at random in C bins, not more than one per bin. The y wavelengths on link j are blue balls which are then randomly distributed in the same C bins. We calculate the probability that there are n bins which contain two balls, one red and one blue. Observe that $p_n(x, y) = p_n(y, x)$, by symmetry. We obtain:

$$\begin{aligned} p_n(x, y) &= \beta(x, y, n), \quad \text{if} \quad x \geq y \geq n, \\ &\qquad x + y - n \leq C, \quad 1 \leq x, y \leq C, \\ &= \beta(y, x, n), \quad \text{if} \quad y \geq x \geq n, \\ &\qquad x + y - n \leq C, \quad 1 \leq x, y \leq C, \qquad (3) \\ &= 0, \quad \text{otherwise,} \end{aligned}$$

where

$$\beta(x, y, n) = \binom{y}{n} \left(\prod_{i=1}^{n} \frac{n - i + 1}{C - i + 1} \right) \left(\prod_{i=1}^{y-n} \frac{C - x - i + 1}{C - n - i + 1} \right).$$

For the general case of an N-hop route, $N \geq 3$, let x_j be the number of idle wavelengths on the j-th hop, and assume without loss of generality that

$$x_1 \geq x_2 \geq \ldots \geq x_N.$$

Starting with (2), we condition the expression on the right on the set of disjoint events $\{X_R = k | k =$

$n, \ldots, x_{N-1}\}$, where $\bar{R} = \{1, \ldots, N-1\}$. We thus obtain the recursive relation:

$$p_n(x_1, \ldots, x_N) = \sum_{k=n}^{x_{N-1}} p_n(k, x_N) p_k(x_1, \ldots, x_{N-1}) \tag{4}$$

where $p_n(k, x_N)$ is given by (3).

Table 1 shows blocking probabilities for tandem networks with 1, 2 and 3 links under varying local traffic. The circuit-switching case B_{CS} is compared with the wavelength routing model B_{WR}. Not surprising, blocking probabilities for the wavelengths routing model are shown to grow much faster with the number of hops.

J	λ	$B_{CS}(\%)$	$B_{WR}(\%)$
1	1.0	0.31	0.31
	1.2	0.63	0.63
	1.5	1.42	1.42
2	1.0	0.61	1.53
	1.2	1.25	3.01
	1.5	2.82	6.41
3	1.0	0.92	4.48
	1.2	1.86	8.21
	1.5	4.19	15.92

Table 1: Network with J links in tandem: comparing blocking probabilities for circuit-switching B_{CS} vs. wavelength routing B_{WR}. $C = 5$, λ is the offered traffic.

The computational requirements of the method above for calculating blocking probabilities for the network of links in tandem with wavelength routing are significantly greater then for circuit-switching. The circuit-switched network requires $O(C)$ operations while the wavelength routing model requires $O(C^3)$ operations for two links and $O(C^4)$ for three links. It is plausible that the computational requirements for more general networks would have a similar behavior. For realistic networks, where C could be large, these computational requirements present a difficult challenge.

3 Fixed Routing

Consider a network of arbitrary topology with J links and C wavelengths on each link. A route R is a subset of links from $\{1, \ldots, J\}$. Calls arrive for route R as a Poisson stream with rate a_R. A call for route R is set up if there is a wavelength w_i such that w_i is idle on all links of route R. If such a wavelength is not available then the call is blocked and lost. If the call is accepted it simultaneously holds wavelength w_i on all links on route R. The holding times of all calls are assumed exponentially distributed with unit mean.

Let X_R be the random variable the number of idle wavelengths on route R. If $R = \{i, j, k\}$ then we may write $X_{i,j,k}$. Let $\mathbf{X} = (X_1, \ldots, X_J)$ and let

$$q_j(m) = \Pr[X_j = m], \quad m = 0, \ldots, C$$

be the idle capacity distribution on link j. We will assume as in [3] that random variables X_j are independent. Then

$$q(\mathbf{m}) = \prod_{j=1}^{J} q_j(m_j), \tag{5}$$

where $\mathbf{m} = (m_1, \ldots, m_J)$.

Following [3] we also assume, given m idle wavelengths on link j, that the time until the next call is set up on link j is exponentially distributed with parameter $\alpha_j(m)$. It follows that the number of idle wavelengths on link j can be viewed as a birth-and-death process and therefore we have

$$q_j(m) = \frac{C(C-1)\cdots(C-m+1)}{\alpha_j(1)\alpha_j(2)\cdots\alpha_j(m)} q_j(0), \tag{6}$$

where

$$q_j(0) = \left[1 + \sum_{m=1}^{C} \frac{C(C-1)\cdots(C-m+1)}{\alpha_j(1)\alpha_j(2)\cdots\alpha_j(m)}\right]^{-1}. \tag{7}$$

The call set up rate on link j when there are m idle wavelengths on link j, $\alpha_j(m)$, is obtained by combining the contributions from the request streams to routes which have link j as a member.

$$\begin{aligned} \alpha_j(m) &= 0, \quad \text{if } m = 0, \\ &= \sum_{R:j \in R} a_R \Pr[X_R > 0 | X_j = m], \\ &\qquad m = 1, \ldots, C. \end{aligned} \tag{8}$$

If the route consists of a single link then the probability term under the summation sign is seen to be $\Pr[X_j > 0 | X_j = m] = 1$. If the route consists of two links let $R = \{i, j\}$. The term $\Pr[X_R > 0 | X_j = m]$ under the summation sign can be further refined by conditioning it on the set of disjoint events $\{X_i = l | l = 0, \ldots, C\}$.

$$\Pr[X_{i,j} > 0 | X_j = m] \tag{9}$$

$$
\begin{aligned}
&= \sum_{l=0}^{C} \Pr[X_i = l | X_j = m] \\
&\quad \times \Pr[X_R > 0 | X_j = m; X_i = l] \\
&= \sum_{l=1}^{C} \Pr[X_i = l] (1 - \Pr[X_R = 0 | X_j = m; X_i = l]) \\
&= \sum_{l=1}^{C} q_i(l)(1 - p_0(m, l))
\end{aligned}
$$

where $p_0(m, l)$ is defined in (3). The second equality above is obtained by taking into account that the term for $l = 0$ is zero and the previously made assumption that random variables X_i are independent. Similarly, for a three hop route $R = \{i, j, k\}$ we obtain:

$$
\Pr[X_{i,j,k} > 0 | X_j = m] = \sum_{l=1}^{C} \sum_{n=1}^{C} q_i(l) q_k(n)[1 - p_0(l, m, n)] \quad (10)
$$

where $p_0(l, m, n)$ is obtained from (4) and (3).

The blocking probability for calls to route R is $L_R = \Pr[X_R = 0]$:

$$
\begin{aligned}
L_R &= q_j(0), \quad \text{if} \quad R = \{j\}, \\
&= \sum_{l=0}^{C} \sum_{m=0}^{C} q_i(l) q_j(m) p_0(l, m), \quad \text{if} \quad R = \{i, j\}, \\
&= \sum_{l=0}^{C} \sum_{m=0}^{C} \sum_{n=0}^{C} q_i(l) q_j(m) q_k(n) p_0(l, m, n), \\
&\qquad \text{if} \quad R = \{i, j, k\}.
\end{aligned}
$$

By separating the cases with $l = 0$, $m = 0$ or $n = 0$ we obtain:

$$
\begin{aligned}
L_R &= q_j(0), \quad \text{if} \quad R = \{j\}, \\
&= q_i(0) + q_j(0) - q_i(0) q_j(0) + \\
&\quad \sum_{l=1}^{C} \sum_{m=1}^{C} q_i(l) q_j(m) p_0(l, m), \quad \text{if} \quad R = \{i, j\}, \\
&= q_i(0) + q_j(0) + q_k(0) - q_i(0) q_j(0) \\
&\quad - q_i(0) q_k(0) - q_j(0) q_k(0) + q_i(0) q_j(0) q_k(0) + \\
&\quad \sum_{l=1}^{C} \sum_{m=1}^{C} \sum_{n=1}^{C} q_i(l) q_j(m) q_k(n) p_0(l, m, n), \\
&\qquad \text{if} \quad R = \{i, j, k\}. \qquad (11)
\end{aligned}
$$

The algorithm in Figure 1 below computes approximately the blocking probabilities for the traffic on all the routes.

In Section 5 numerical results are given and compared with simulation.

1. Initialization. For all routes R let $\hat{L}_R = 0$. For $j = 1, \ldots, J$: let $\alpha_j(0) = 0$, $\alpha_j(m) = \sum_{R: j \in R} a_R$, $m = 1, \ldots, C$.
2. Determine $q_j(.)$, $j = 1, \ldots, J$, using (6) and (7).
3. Calculate $\alpha_j(.)$, $j = 1, \ldots, J$, using (9), (10) and (10).
4. Calculate L_R, for all routes R, using (11). If $\max_R |L_R - \hat{L}_R| < \epsilon$ then terminate. Otherwise let $\hat{L}_R = L_R$, go to Step 2.

Figure 1: Calculation of L_R for fixed routing

4 Least Loaded Routing

We deal in this section with fully connected networks with Least Loaded Routing. Let N be the number of nodes. The number of links J is thus $J = N(N-1)/2$. Each pair of nodes has a direct route $\{j\}$ and a set of $N - 2$ alternate two-link routes denoted by $\mathcal{A}_j$. When a call arrives it is set up on the direct route $\{j\}$ if $m_j > 0$, where m_j is the number of idle wavelengths on link j. Otherwise call setup is attempted on the least loaded alternate route which is the route with the largest number of idle wavelengths. The routes in $\mathcal{A}_j$ are assumed ordered in some way, and in case of ties the first of the candidate routes is chosen. If m_R, the number of idle wavelengths on the alternate route R, is such that $m_R \leq r$, where r is the trunk reservation parameter, then the call is blocked and lost. For simplicity we assume all links have the same trunk reservation parameter r.

Denote by a_j the arrival rate of calls for the node pair connected by link j. $\mathcal{S}_j$ denotes the set of links adjacent to link j; there are $2(N-2)$ links in this set. If links j, k have a node in common we denote by $\tau(j, k)$ the link which closes the triangle. For a link j denote by $alt(j)$ the two-link alternate route to j according to the LLR scheme above.

Given m idle wavelengths on link j, the setup rate $\alpha_j(m)$ is:

$$
\begin{aligned}
\alpha_j(m) &= 0, \quad \text{if} \quad m = 0, \\
&= a_j, \quad \text{if} \quad 1 \leq m \leq r, \\
&= a_j + \sum_{k \in \mathcal{S}_j} a_k q_k(0) h(j, k, m), \quad (12) \\
&\qquad \text{if} \quad m > r,
\end{aligned}
$$

where

$$
h(j, k, m) = \qquad (13)
$$

$$\Pr[alt(k) = \{j, \tau(j,k)\}; X_{j,\tau(j,k)} > r | X_j = m], \quad m > r.$$

We condition $h(j,k,m)$ on the set of disjoint events $\{X_{j,\tau(j,k)} = l; l = r+1, \ldots, m\}$ and obtain

$$\begin{aligned} h(j,k,m) &= \sum_{l=r+1}^{m} \Pr[X_{j,\tau(j,k)} = l | X_j = m] \times \\ &\quad \Pr[alt(k) = \{j, \tau(j,k)\} | X_j = m; X_{j,\tau(j,k)} = l] \\ &= \sum_{l=r+1}^{m} f(j,k,m,l) g(j,k,l), \end{aligned} \tag{14}$$

where

$$f(j,k,m,l) = \Pr[X_{j,\tau(j,k)} = l | X_j = m] \tag{15}$$

and

$$g(j,k,l) = \Pr[alt(k) = \{j, \tau(j,k)\} | X_{j,\tau(j,k)} = l]. \tag{16}$$

The expression for $g(j,k,l)$ can be further developed as a product of probabilities in accordance with the meaning of the LLR scheme:

$$g(j,k,l) = \prod_{R \in \mathcal{A}_k^-(j)} \Pr[X_R < l] \prod_{R \in \mathcal{A}_k^+(j)} \Pr[X_R \le l] \tag{17}$$

where $\mathcal{A}_k^-(j)$ denotes the set of routes in $\mathcal{A}_k$ which precede, in the assumed ordering, the route in $\mathcal{A}_k$ to which j belongs, while $\mathcal{A}_k^+(j)$ denotes the set of routes in $\mathcal{A}_k$ which succeed that route.

By conditioning $f(j,k,m,l)$ on the set of disjoint events $\{X_{j,\tau(j,k)} = i; i = l, \ldots, m\}$ we obtain:

$$\begin{aligned} f(j,k,m,l) &= \sum_{i=l}^{m} \Pr[X_{\tau(j,k)} = i | X_j = m] \times \\ &\quad \Pr[X_{j,\tau(j,k)} = l | X_j = m; X_{\tau(j,k)} = i] \\ &= \sum_{i=l}^{m} q_{\tau(j,k)}(i) p_l(m,i), \end{aligned} \tag{18}$$

where $p_l(m,i)$ is given by (3). To compute terms of the form $\Pr[X_R < l]$ in (17) we first compute $\Pr[X_R = l]$, $l = r, \ldots, m$:

$$\Pr[X_{j,k} = l] = \sum_{x=l}^{C} \sum_{y=l}^{C} q_j(x) q_k(y) p_l(x,y). \tag{19}$$

Then for $l = r, \ldots, m$:

$$\Pr[X_{j,k} < l] = 1 - \sum_{i=l}^{C} \Pr[X_{j,k} = i], \tag{20}$$

$$\Pr[X_{j,k} \le l] = \Pr[X_{j,k} < l] + \Pr[X_{j,k} = l]. \tag{21}$$

The blocking probability for the traffic between the nodes of link j is given by:

$$L_j = \Pr[X_j = 0] \prod_{R \in \mathcal{A}_j} \Pr[X_R \le r]. \tag{22}$$

The algorithm in Figure 2 computes approximately the blocking probabilities for the traffic between all node pairs.

1. Initialization. Let $\hat{L}_j = 0$, $j = 1, \ldots, J$. For $j = 1, \ldots, J$: let $\alpha_j(0) = 0$, $\alpha_j(m) = a_j$, $m = 1, \ldots, C$.
2. Determine $q_j(.)$, $j = 1, \ldots, J$, using (6) and (7).
3. Calculate $\alpha_j(.)$, $j = 1, \ldots, J$, using (12) through (21).
4. Calculate L_R, for all routes R, using (22). If $\max_j |L_j - \hat{L}_j| < \epsilon$ then terminate. Otherwise let $\hat{L}_j = L_j$, go to Step 2.

Figure 2: Calculation of L_R for LLR

For circuit-switched networks Chung, Kashper and Ross [3] describe two algorithms for calculating $\alpha_j(.)$. The first requires $O(CN^4)$ operations and $O(CN^2)$ storage, the second, which trades some gain in computational efficiency for storage, requires $O(CN^3)$ operations and $O(CN^3)$ storage. The implementation in Figure 3 below is similar to their second algorithm. The required number of operations for this calculation of $\alpha_j(.)$ is $O(C^3N^3)+O(CN^4)$, significantly more than for the circuit-switched case. Let us assume for simplicity that C and N are of the same order (a possible value may be 30). Then the computational complexity is of the order $O(C^6)$, two orders of magnitude greater than the circuit-switched case.

In Section 5 numerical results are given and compared with simulation.

5 Numerical results

The analytical results of previous sections are used here to calculate approximate blocking probabilities for two networks: a network with fixed routing and a network with LLR. The results are then compared with blocking probabilities obtained by simulation.

Simulation results are given as 95% confidence intervals estimated by the method of batch means. The number of batches is 20 or more.

Do for $j = 1, \ldots, J$.
Do for $l = r, \ldots, C$.
Calculate $\Pr[X_R = l]$ for all $R \in \mathcal{A}_j$ using (19).
Do for $j = 1, \ldots, J$.
Do for $l = r, \ldots, C$.
Calculate $\Pr[X_R < l]$ for all $R \in \mathcal{A}_j$ using (20).
Do for $j = 1, \ldots, J$.
Do for $l = r, \ldots, C$.
Calculate $g(j, k, l)$ for all $k \in \mathcal{S}_j$ using (17).
Do for $j = 1, \ldots, J$.
Dor for all $k \in \mathcal{S}_j$.
Calculate $h(j, k, m)$ for $m = r + 1, \ldots, C$ using (14) and (18).
Calculate $\alpha_j(m)$ for $m = r + 1, \ldots, C$ using (12).

Figure 3: Calculation of $\alpha_j(.)$ for LLR

Tables 2, 3 and 4 below show numerical results for a network with fixed routing. There are seven links ($J = 7$) and fifteen source/destination pairs (or equivalently, routes). The number of wavelengths is $C = 12$. The routes are shown in column R where a route is a set of links. The offered traffic on a route appears in the column marked a_R. Blocking probabilities in light, moderate and heavy traffic are shown. The approximation results are generally close to the simulation results.

Table 5, below shows numerical results for a fully connected network with LLR. There are four nodes ($N = 4$) and six links ($J = 6$). The number of wavelengths is $C = 6$ and the trunk reservation parameter is $r = 2$. Blocking probabilities in light, moderate and heavy traffic are shown.

While the results are less accurate here than for the fixed routing case they are similar to the results in [3] for the circuit-switched networks. The accuracy is good for heavy and moderate traffic but less so for light traffic. We note that whenever the approximation deviates from the simulation results, the approximation usually overestimates the blocking probabilities, while in [3] the approximation often underestimates them.

R	a_R	$L_R^{sim}(\%)$	$L_R(\%)$
$\{1\}$	3.0	(0.02,0.03)	0.03
$\{2\}$	3.0	(0.02,0.03)	0.03
$\{3\}$	3.0	(0.03,0.03)	0.03
$\{4\}$	3.0	(0.02,0.03)	0.03
$\{5\}$	3.0	(0.00,0.00)	0.01
$\{6\}$	3.0	(0.01,0.02)	0.01
$\{7\}$	3.0	(0.01,0.02)	0.01
$\{4,7\}$	0.3	(0.15,0.19)	0.19
$\{2,3\}$	0.3	(0.23,0.27)	0.28
$\{1,6\}$	0.3	(0.19,0.23)	0.20
$\{1,2\}$	0.3	(0.23,0.28)	0.27
$\{3,4\}$	0.3	(0.23,0.28)	0.27
$\{2,3,6\}$	0.03	(1.14,1.42)	1.46
$\{3,4,7\}$	0.03	(1.19,1.48)	1.40
$\{1,2,6\}$	0.03	(1.03,1.29)	1.43

Table 2: Network with fixed routing and light traffic. R: routes, a_R: offered traffic, $J = 7$, $C = 12$. L_R^{sim}: obtained by simulation.

6 Concluding Remarks

For a class of all-optical networks using WDM and wavelength routing we presented an approximate method for calculating the blocked traffic. We studies two types of networks. First we studied networks with arbitrary topology, fixed routing and paths with three hops or less. We also considered fully connected networks, Least Loaded Routing and paths with one or two hops.

While the computational requirements of the generalized reduced load approximation scheme in [5, 3] are significant the problem is worse for the wavelength routing model. The technique of 'truncated distributions' in [3] could be applied here as well, and will alleviate the problem somewhat for moderate and heavy traffic.

The two types of network studied can be viewed as two extremes of a range of possible network types. While the fixed routing case has a single route for a given source/destination pair the fully connected network with LLR has many alternate routes. The accuracy of the method in our case study is good for the fixed routing case but it less so for the LLR case, especially for light traffic. We suspect that the method will perform well for in-between cases such as Fixed Alternate Routing (FAR) [6] in which a route may have one or two predetermined alternate routes. A scheme such as FAR will also have the advantage of

R	a_R	$L_R^{sim}(\%)$	$L_R(\%)$
$\{1\}$	3.6	(0.11,0.12)	0.11
$\{2\}$	3.6	(0.11,0.12)	0.12
$\{3\}$	3.6	(0.11,0.13)	0.12
$\{4\}$	3.6	(0.10,0.11)	0.11
$\{5\}$	3.6	(0.02,0.03)	0.03
$\{6\}$	3.6	(0.06,0.07)	0.06
$\{7\}$	3.6	(0.05,0.06)	0.06
$\{4,7\}$	0.36	(0.70,0.78)	0.78
$\{2,3\}$	0.36	(0.98,1.08)	1.10
$\{1,6\}$	0.36	(0.75,0.84)	0.80
$\{1,2\}$	0.36	(0.95,1.04)	1.07
$\{3,4\}$	0.36	(0.90,1.00)	1.07
$\{2,3,6\}$	0.036	(3.88,4.50)	4.71
$\{3,4,7\}$	0.036	(3.41,3.91)	4.56
$\{1,2,6\}$	0.036	(3.59,4.21)	4.64

Table 3: Network with fixed routing and moderate traffic. $J = 7$, $C = 12$.

R	a_R	$L_R^{sim}(\%)$	$L_R(\%)$
$\{1\}$	4.5	(0.50,0.53)	0.53
$\{2\}$	4.5	(0.53,0.56)	0.56
$\{3\}$	4.5	(0.54,0.57)	0.56
$\{4\}$	4.5	(0.51,0.55)	0.53
$\{5\}$	4.5	(0.16,0.18)	0.16
$\{6\}$	4.5	(0.31,0.33)	0.33
$\{7\}$	4.5	(0.29,0.32)	0.31
$\{4,7\}$	0.45	(3.14,3.32)	3.44
$\{2,3\}$	0.45	(4.19,4.40)	4.54
$\{1,6\}$	0.45	(3.19,3.38)	3.52
$\{1,2\}$	0.45	(4.06,4.26)	4.45
$\{3,4\}$	0.45	(4.03,4.22)	4.45
$\{2,3,6\}$	0.045	(13.75,14.71)	15.20
$\{3,4,7\}$	0.045	(13.05,13.97)	14.84
$\{1,2,6\}$	0.045	(12.68,13.49)	15.02

Table 4: Network with fixed routing and heavy traffic. $J = 7$, $C = 12$.

reduced computational complexity, which would allow the method to be applied to more realistic networks.

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j	a_j	$L_j^{sim}(\%)$	$L_j(\%)$
1,2	1.00	(0.00,0.01)	0.02
1,3	1.50	(0.02,0.04)	0.09
1,4	2.00	(0.10,0.15)	0.25
2,3	1.00	(0.00,0.01)	0.01
2,4	1.50	(0.02,0.05)	0.06
3,4	2.00	(0.11,0.15)	0.25
1,2	1.50	(0.14,0.20)	0.21
1,3	2.25	(0.56,0.71)	0.87
1,4	3.00	(2.12,2.33)	2.25
2,3	1.50	(0.11,0.20)	0.16
2,4	2.25	(0.74,0.90)	0.79
3,4	3.00	(2.00,2.27)	2.25
1,2	2.00	(0.96,1.15)	0.92
1,3	3.00	(3.00,3.34)	3.27
1,4	4.00	(7.93,8.42)	7.59
2,3	2.00	(0.79,1.02)	0.80
2,4	3.00	(3.82,4.24)	3.40
3,4	4.00	(7.63,8.18)	7.59

Table 5: Network with LLR in light, moderate and heavy traffic. Traffic stream j given as a node pair, a_j is the offered traffic. $J = 6$, $C = 6$, $r = 2$.