# BASIC CONCEPTS


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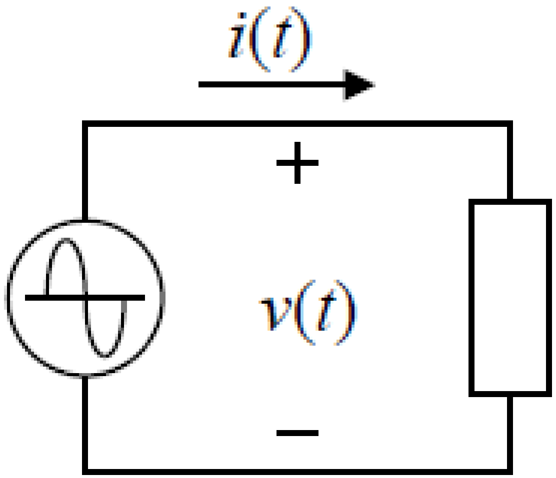
# INSTANTANEOUS VOLTAGE, CURRENT AND POWER, RMS VALUES

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$p(t) = v(t)\, i(t)$$




$$\theta = \theta_v - \theta_i \quad V_m = \sqrt{2}\,|V| \quad I_m = \sqrt{2}\,|I|$$

$$p(t) = \underbrace{|V|\,|I| \cos\theta \,\{1 + \cos 2(\omega t + \theta_v)\}}_{\text{energy flow into the circuit}} + \underbrace{|V|\,|I| \sin\theta \sin 2(\omega t + \theta_v)}_{\text{energy borrowed and returned by the circuit}}$$

# AVERAGE (REAL) POWER, REACTIVE POWER, APPARENT POWER, POWER FACTOR

$$p(t) = |V|\,|I|\,\{1 + \cos 2(\omega t + \theta_v)\}\cos\theta + |V|\,|I|\,\sin 2(\omega t + \theta_v)\sin\theta$$

$$p_R(t) = |V|\,|I|\,\{1 + \cos 2(\omega t + \theta_v)\}\cos\theta = \overline{P}\{1 + \cos 2(\omega t + \theta_v)\}$$

$$p_X(t) = |V|\,|I|\,\sin 2(\omega t + \theta_v)\sin\theta = S\sin\theta\sin 2(\omega t + \theta_v)$$

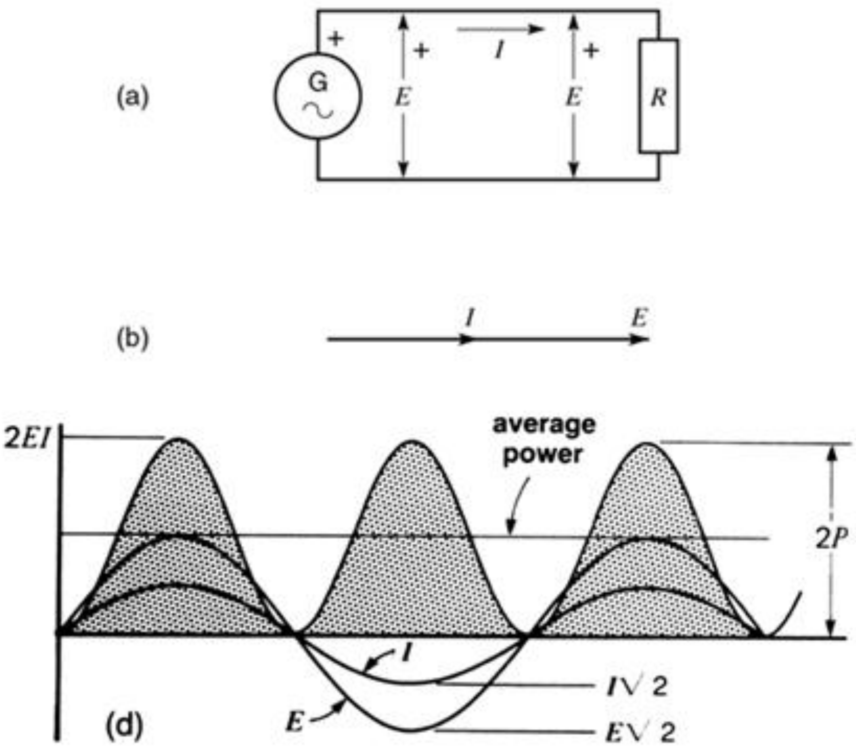
$$\overline{P} = |V|\,|I|\,\cos\theta$$

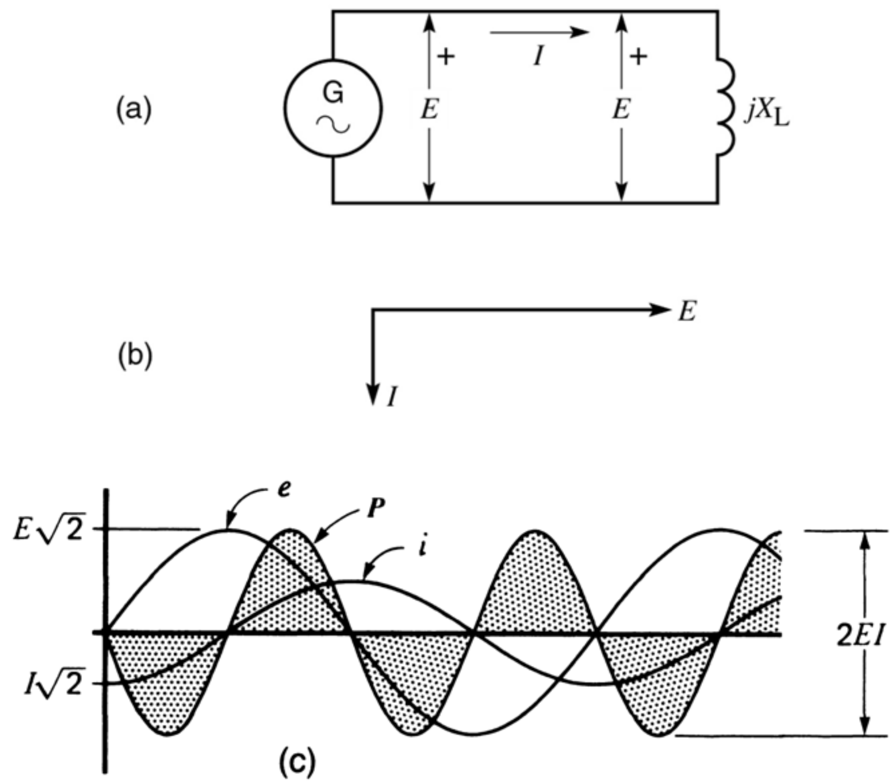
$$S = |V|\,|I|$$

$$Q \equiv S\sin\theta = |V|\,|I|\sin\theta$$

$$pf = \cos\theta = \frac{\overline{P}}{|V|\,|I|}$$

# INSTANTANEOUS POWER IN PURE RESISTIVE AND INDUCTIVE CIRCUITS



# PHASOR NOTATION, IMPEDANCE AND ADMITTANCE

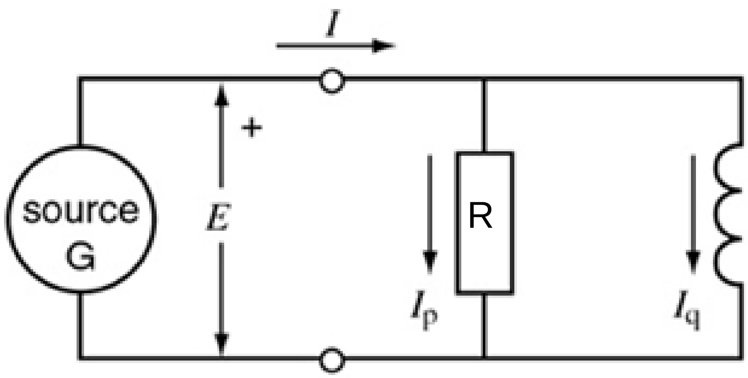
Transformation of a sinusoidal signal to and from the time domain to the phasor domain:

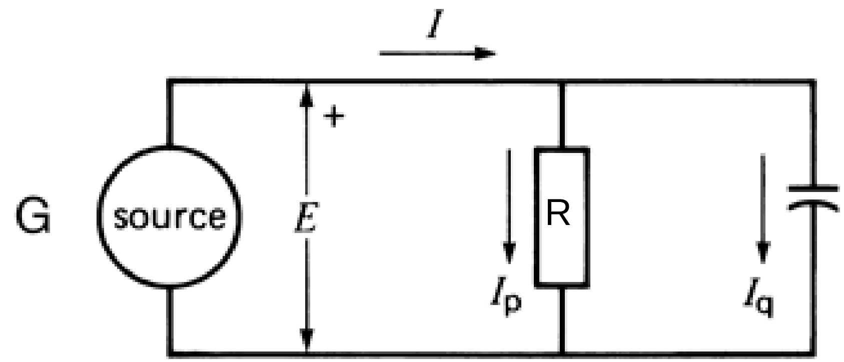
$$v(t) = \sqrt{2}\left|V\right|\cos(\omega t + \theta_v) \longleftrightarrow V = \left|V\right|\angle\theta_v$$

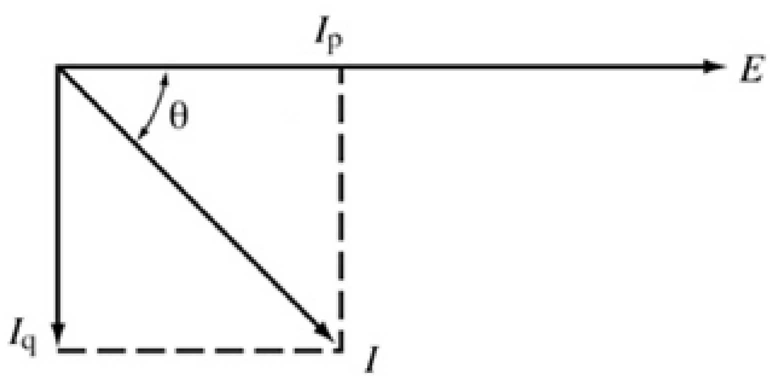
(time domain) (phasor domain)

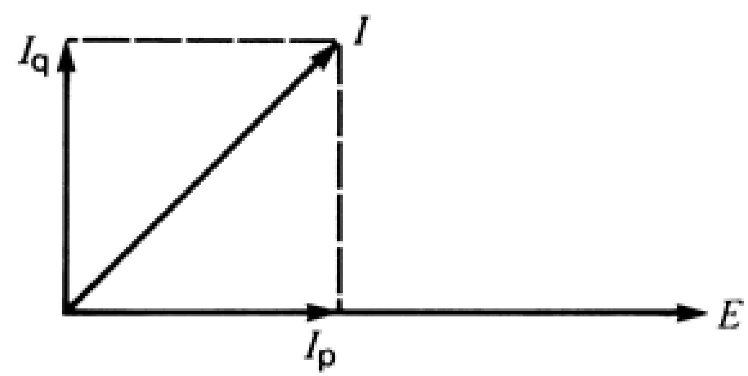
| Element | Impedance | Admittance |
|---|---|---|
| R | $Z = R$ | $Y = \frac{1}{R}$ |
| L | $Z = j\omega L$ | $Y = \frac{1}{j\omega L}$ |
| C | $Z = -j\frac{1}{\omega C}$ | $Y = j\omega C$ |

# RESISTIVE-INDUCTIVE, RESISTIVE-CAPACITIVE LOAD

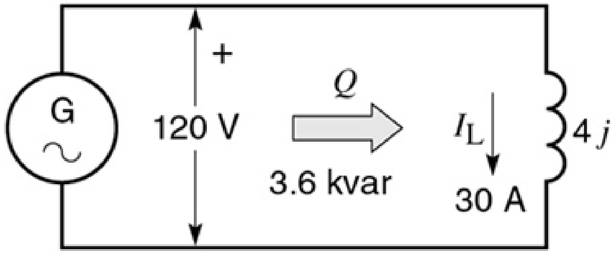


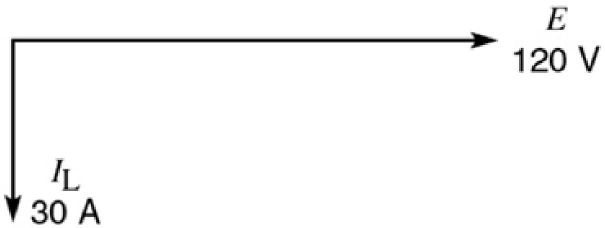



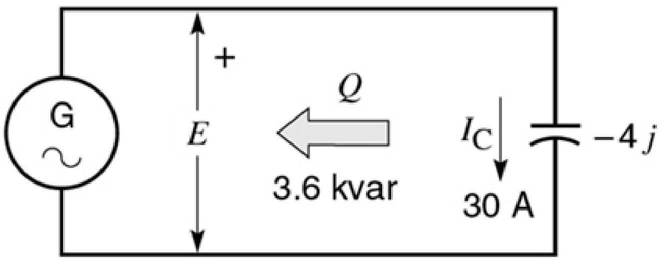



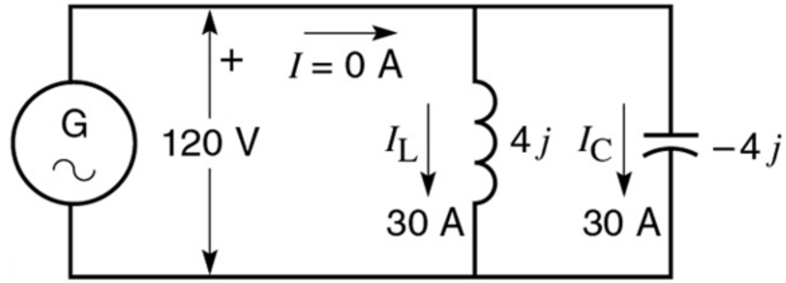
# POWER IN INDUCTIVE AND CAPACITIVE CIRCUITS

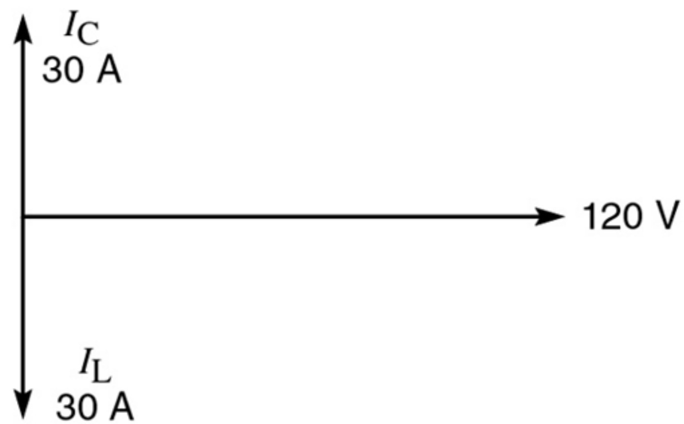






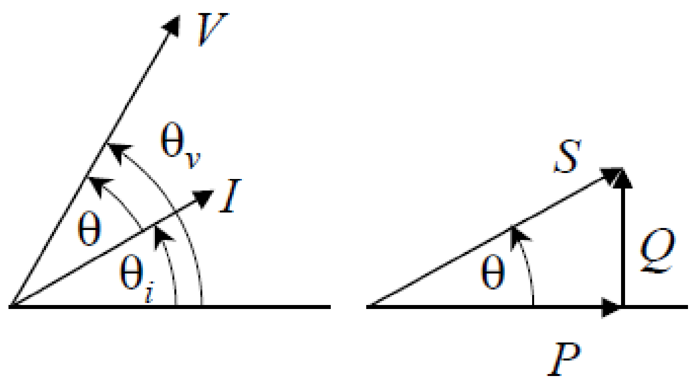



# COMPLEX POWER, POWER TRIANGLE

$$V\,I^{*} = |V|\,|I|\angle(\theta_v - \theta_i) = |V|\,|I|\angle\theta = S$$

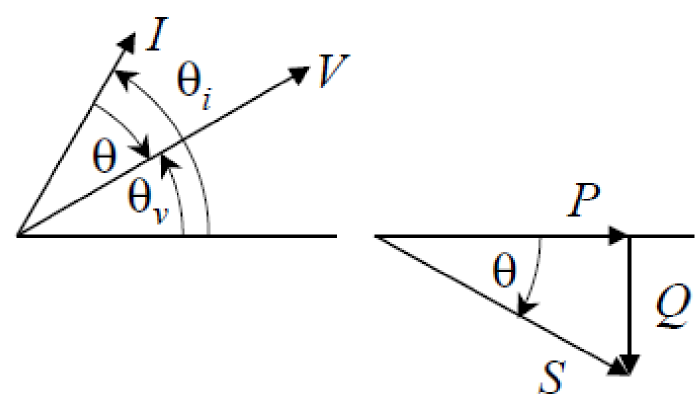
$$S = |V|\,|I|\cos\theta + j\,|V|\,|I|\sin\theta = P + jQ$$

$$|S| = \sqrt{P^2 + Q^2}$$



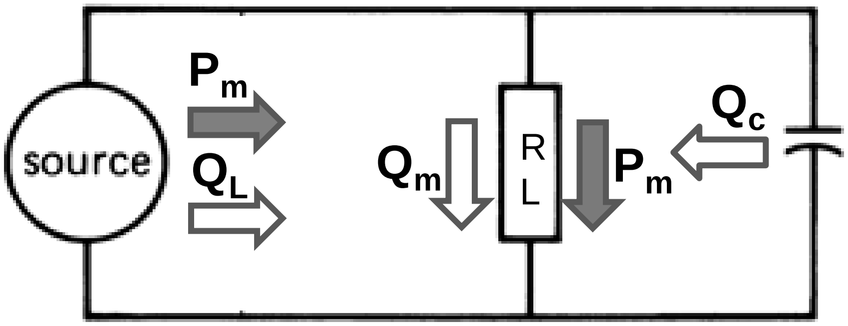

**Lagging Power Factor**

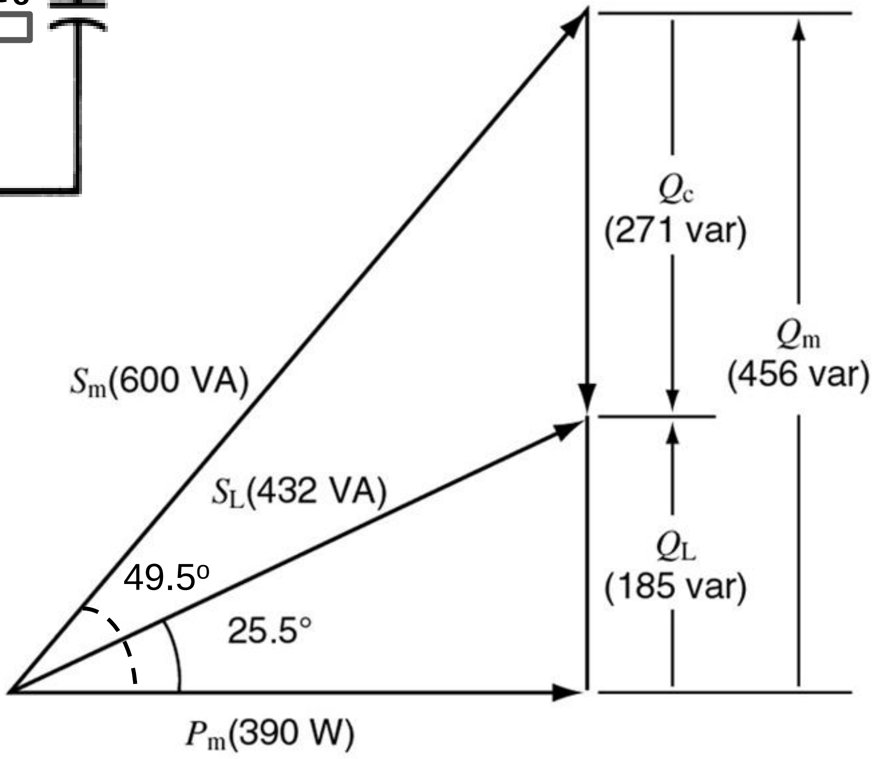
**Leading Power Factor**



# EXAMPLE: POWER FACTOR CORRECTION

The power triangle below shows that the power factor is corrected by a shunt capacitor from 65% to 90% (lag).



# CONSERVATION OF POWER

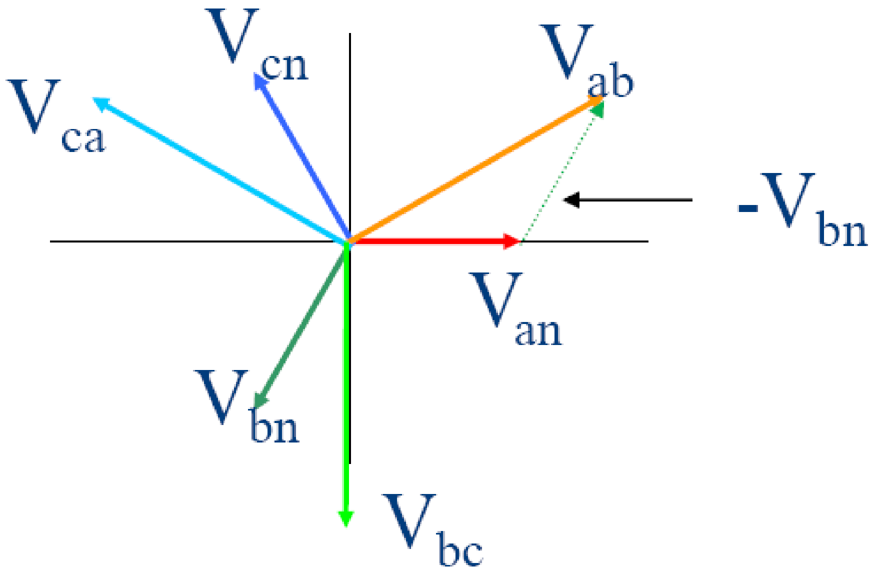
- At every node (bus) in the system,
  - the sum of real powers entering the node must be equal to the sum of real powers leaving that node.
  - The same applies for reactive power,
  - The same applies for complex power
  - The same **does not apply** for apparent power

- The above is a direct consequence of Kirchhoff's current law, which states that the sum of the currents flowing into a node must equal the sum of the currents flowing out of that node.

# BALANCED 3 PHASE CIRCUITS

- Bulk power systems are almost exclusively 3-phase. Single phase is used primarily only in low voltage, low power settings, such as residential and some commercial customers.
- Some advantages of three-phase system:
  - Can transmit more power for the same amount of wire (twice as much as single phase)
  - Torque produced by 3$\phi$ machines is constant, easy start.
  - Three phase machines use less material for same power rating

# PHASE AND LINE VOLTAGES



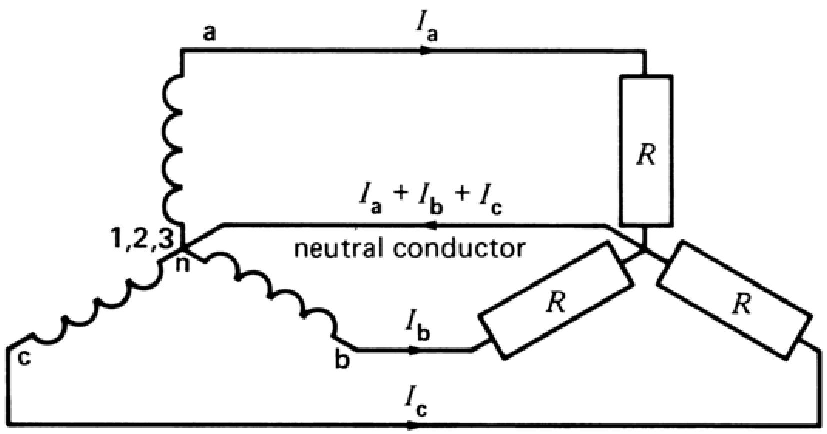

$$V_{an} = |V|\angle\alpha°$$
$$V_{bn} = |V|\angle\alpha° - 120°$$
$$V_{cn} = |V|\angle\alpha° + 120°$$

(α = 0 in this case)

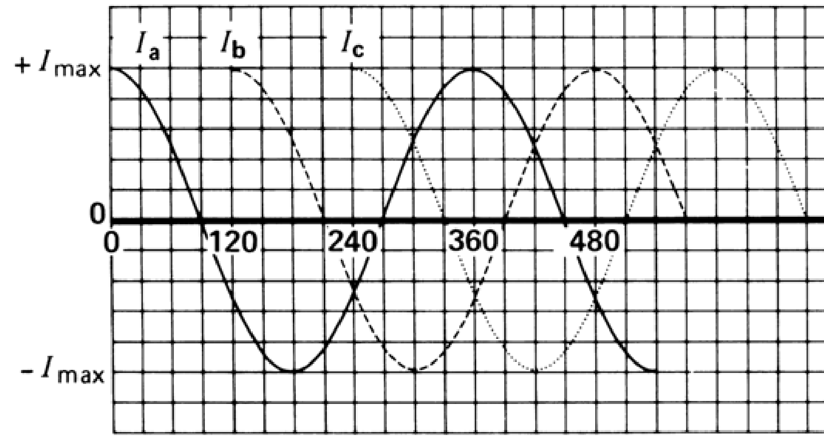
$$V_{ab} = V_{an} - V_{bn} = |V|(1\angle\alpha - 1\angle\alpha + 120°)$$
$$= \sqrt{3}\,|V|\angle\alpha + 30°$$
$$V_{bc} = \sqrt{3}\,|V|\angle\alpha - 90°$$
$$V_{ca} = \sqrt{3}\,|V|\angle\alpha + 150°$$

Line to line voltages are also balanced

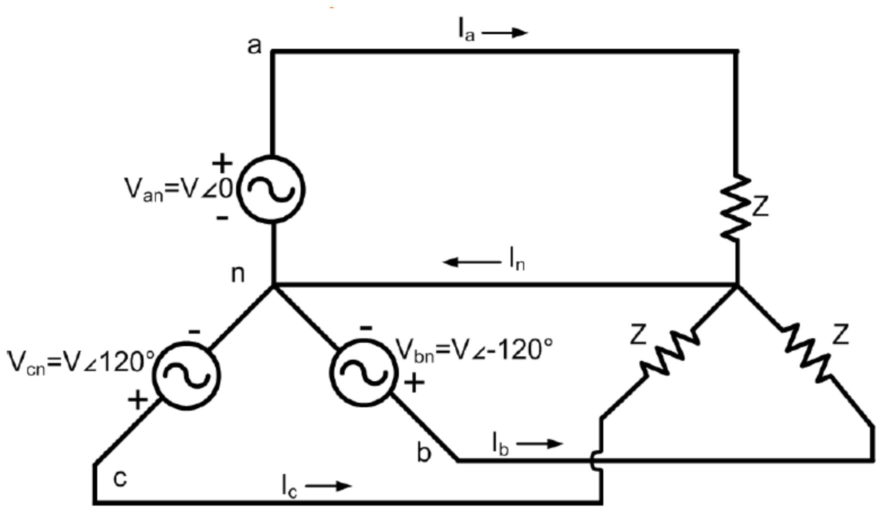
# NEUTRAL WIRE




(a)



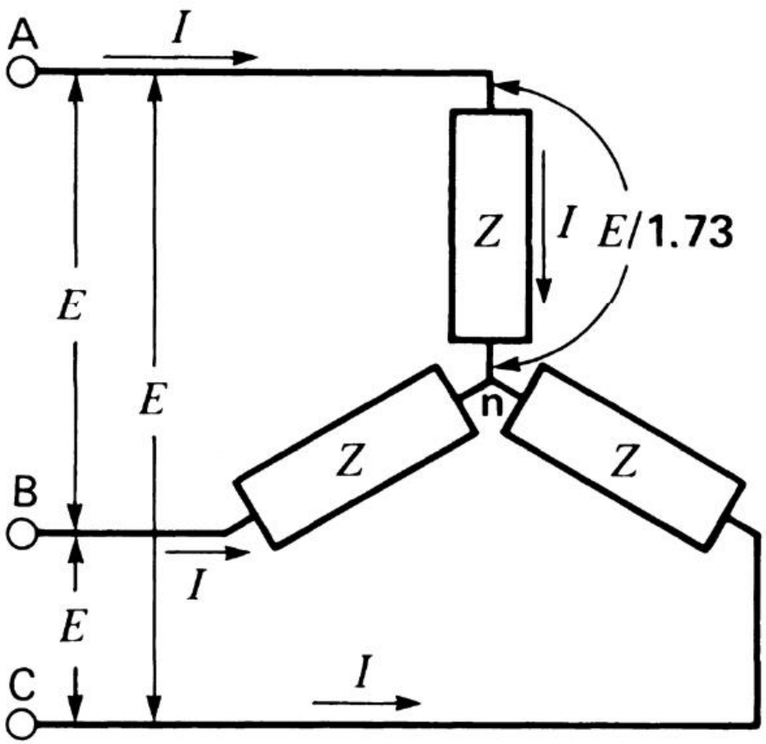

(b)

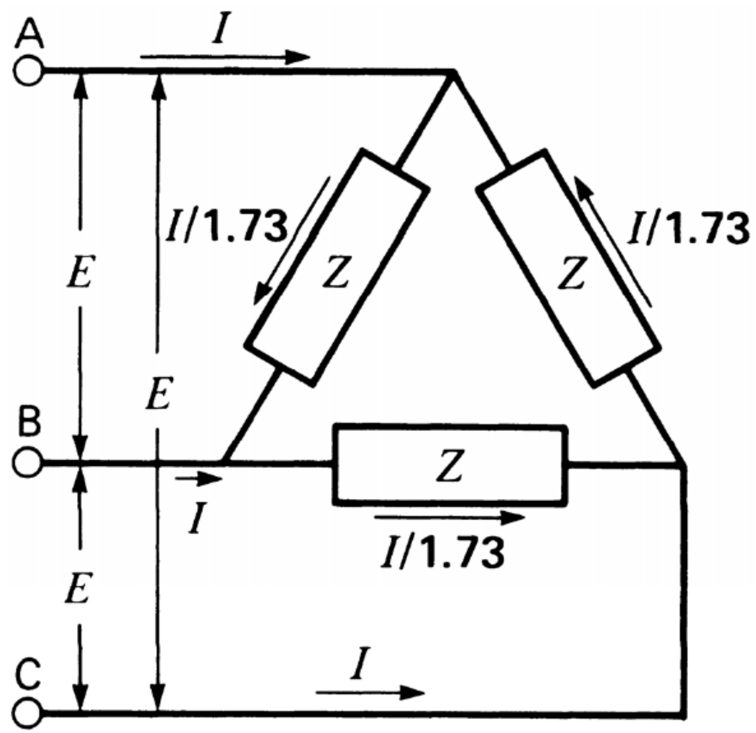



$$I_n = I_a + I_b + I_c$$

$$I_n = \frac{V}{Z}(1\angle 0° + 1\angle -120° + 1\angle 120°) = 0$$

# Y- AND Δ-CONNECTED LOADS



# POWER IN BALANCED 3-PHASE CIRCUITS

The real power, reactive power, apparent power, complex power and power factor are the same in each phase.

$$P = 3V_p I \cos(\theta) = \sqrt{3} V_L I \cos(\theta)$$

$$Q = 3V_p I \sin(\theta) = \sqrt{3} V_L I \sin(\theta)$$

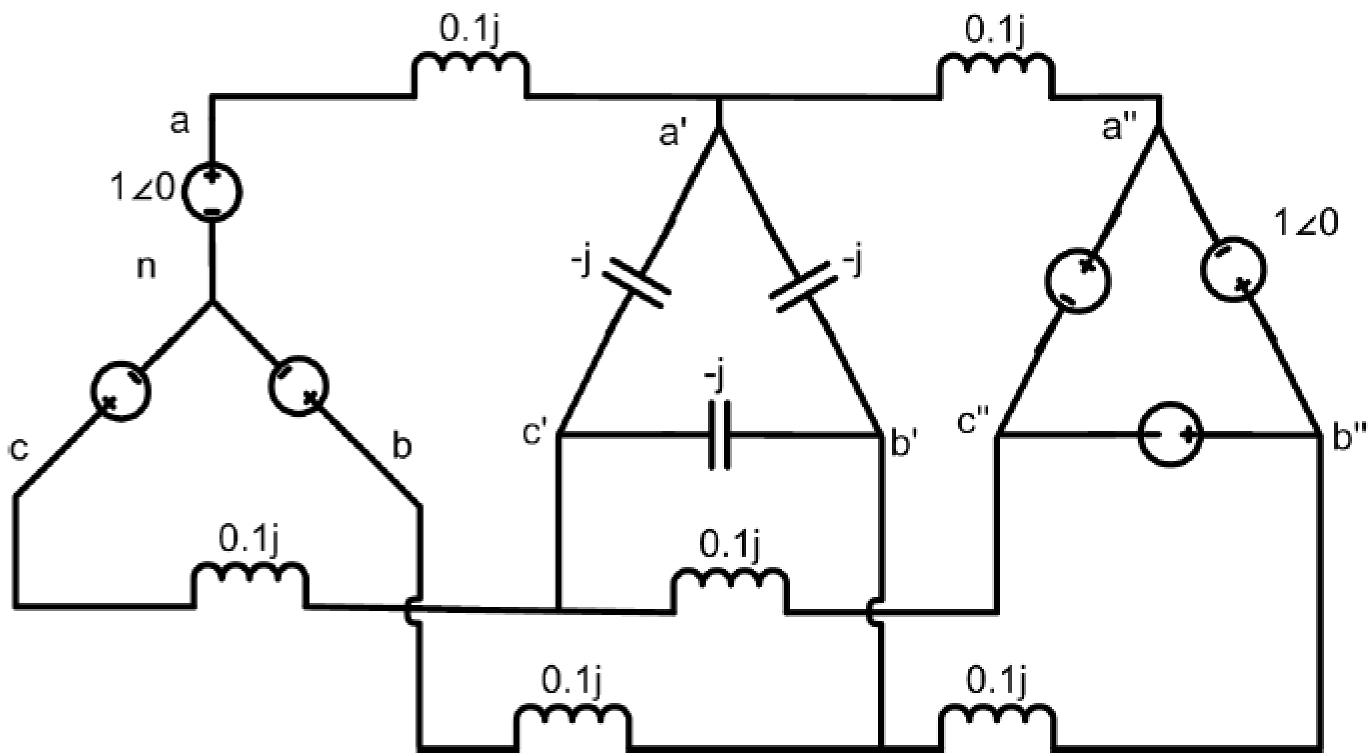
$$S = 3V_p I = \sqrt{3} V_L$$

# PER-PHASE ANALYSIS IN BALANCED 3-PHASE CIRCUITS

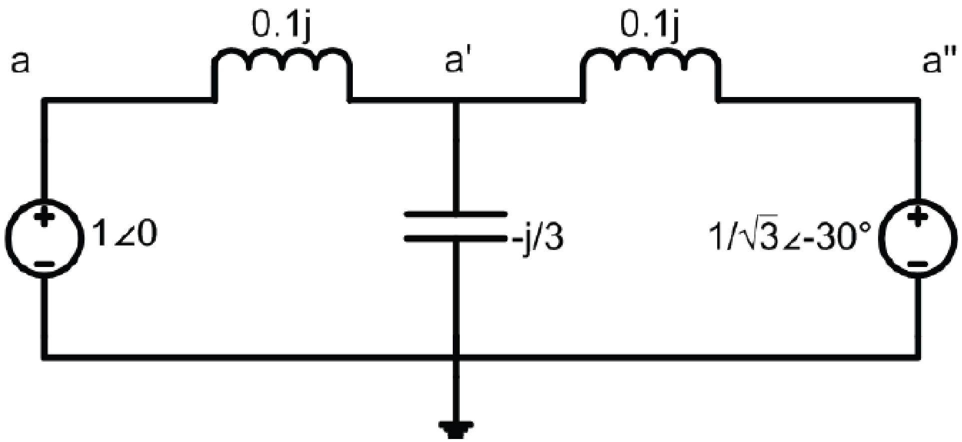
- Per phase analysis allows analysis of balanced 3ϕ systems with the same effort as for a single phase system
- **To do per phase analysis**
  1. Convert all 3ϕ load/sources to equivalent Y's
  2. Solve phase "a" independent of the other phases
  3. Total system power $S = 3\ V_a I_a^*$
  4. If desired, phase "b" and "c" values can be determined by inspection (i.e., ±120° degree phase shifts)
  5. If necessary, go back to original circuit to determine line-line values or internal 3ϕ values.

# EXAMPLE OF PER-PHASE ANALYSIS

Find the complex power supplied by each of the two sources.



# SOLUTION




To solve the circuit, write the KCL equation at a'

$$(V_a^{'} - 1\angle 0)(-10j) + V_a^{'}(3j) + (V_a^{'} - \frac{1}{\sqrt{3}}\angle -30°)(-10j) = 0$$

$$(10j + \frac{10}{\sqrt{3}}\angle 60°) = V_a^{'}(10j - 3j + 10j)$$

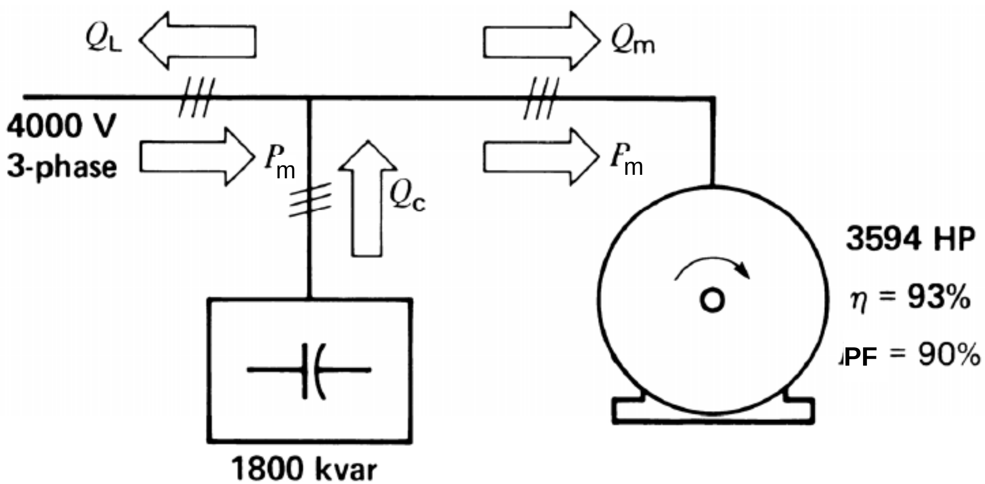
$$V_a^{'} = 0.9\angle -10.9° \text{ volts} \qquad V_b^{'} = 0.9\angle -130.9° \text{ volts}$$

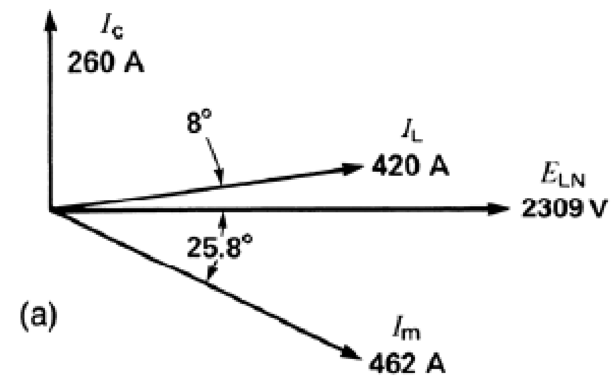
$$V_c^{'} = 0.9\angle 109.1° \text{ volts} \qquad V_{ab}^{'} = 1.56\angle 19.1° \text{ volts}$$

$$S_{Ygen} = 3V_a I_a^{*} = V_a\left(\frac{V_a - V_a^{'}}{j0.1}\right)^{*} = 5.1 + j3.5 \text{ VA}$$

$$S_{\Delta gen} = 3V_a^{''}\left(\frac{V_a^{''} - V_a^{'}}{j0.1}\right)^{*} = -5.1 - j4.7 \text{ VA}$$

# EXAMPLE: POWER FACT0R CORRECTION IN THREE-PHASE CIRCUIT.



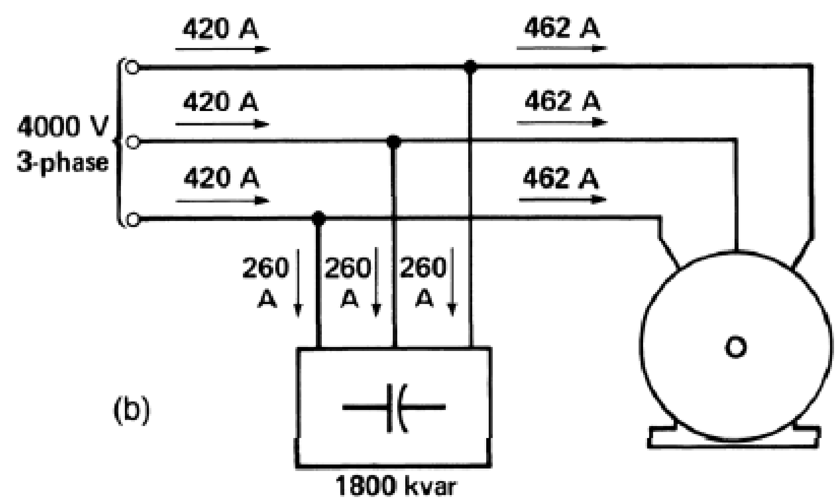




$P_m = \sqrt{3} x 4 x 0.462 x \cos(25.8^o) = 2.88$ MW

$Q_m = \sqrt{3} x 4 x 0.462 x \sin(25.8^o) = 1.39$ MVAR

$Q_c = 1.8$ MVAR

$Q_L = Q_m - Q_c = -0.41$ MVAR



# THE PER-UNIT SYSTEM

The voltages, currents, powers, impedances, and other electrical quantities are measured as fractions of some base level instead of conventional units.

$$Quantity\ per\ unit = \frac{actual\ value}{base\ value\ of\ quantity}$$

Usually, two base quantities are selected to define a given per-unit system. Often, such quantities are voltage and apparent power . In a single-phase circuit, once the base values of *S* and *V* are selected, all other base values can be computed form

$$P_{base},\ Q_{base},\ or\ S_{base} = V_{base} I_{base}$$

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{(V_{base})^2}{S_{base}}$$

$$Y_{base} = \frac{I_{base}}{V_{base}}$$

# PER-UNIT SYSTEM

**In a 3-phase circuit, given the base apparent power (3—phase) and base voltage (line-to-line), the base current and base impedance are given by**

$$I_{base} = \frac{S_{3\phi,base}}{\sqrt{3}V_{LL,base}}$$

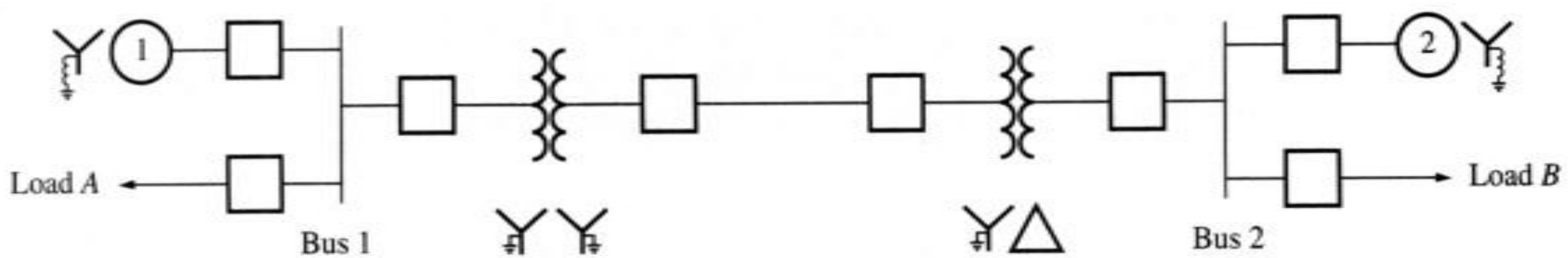
$$Z_{base} = \frac{V_{LL,base}}{\sqrt{3}I_{base}} = \frac{\left(V_{LL,base}\right)^2}{S_{3\phi,base}}$$

# PER-UNIT SYSTEM

**The per-unit impedance may be transformed from one base to another as**

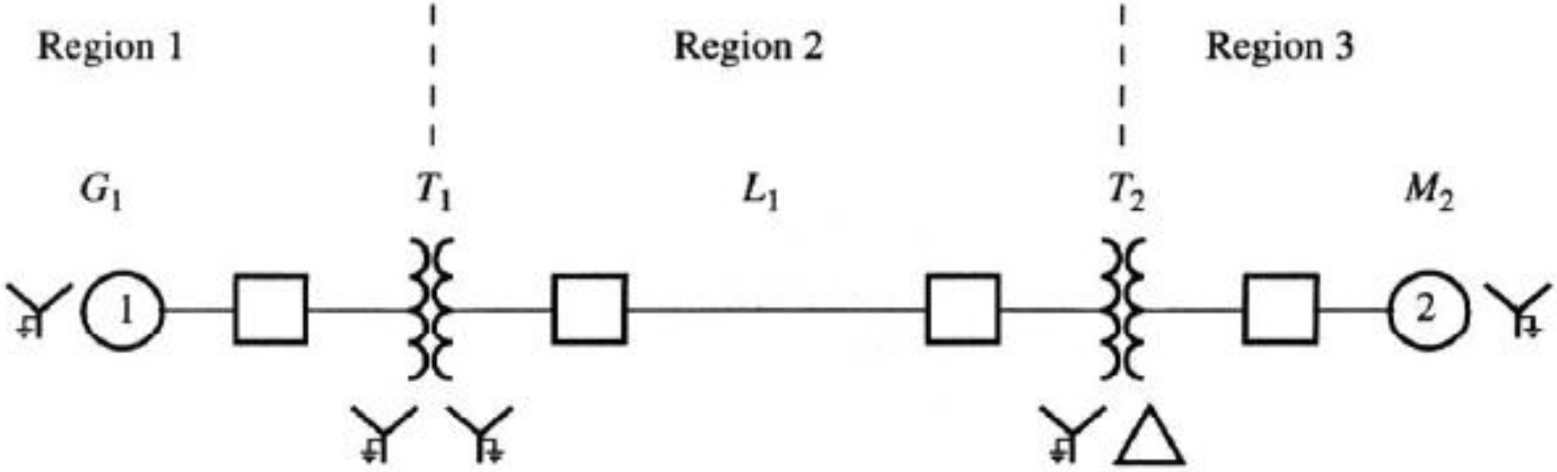
$$Per-unit\ Z_{new} = per-unit\ Z_{old}\left(\frac{V_{old}}{V_{new}}\right)^2\left(\frac{S_{new}}{S_{old}}\right)$$

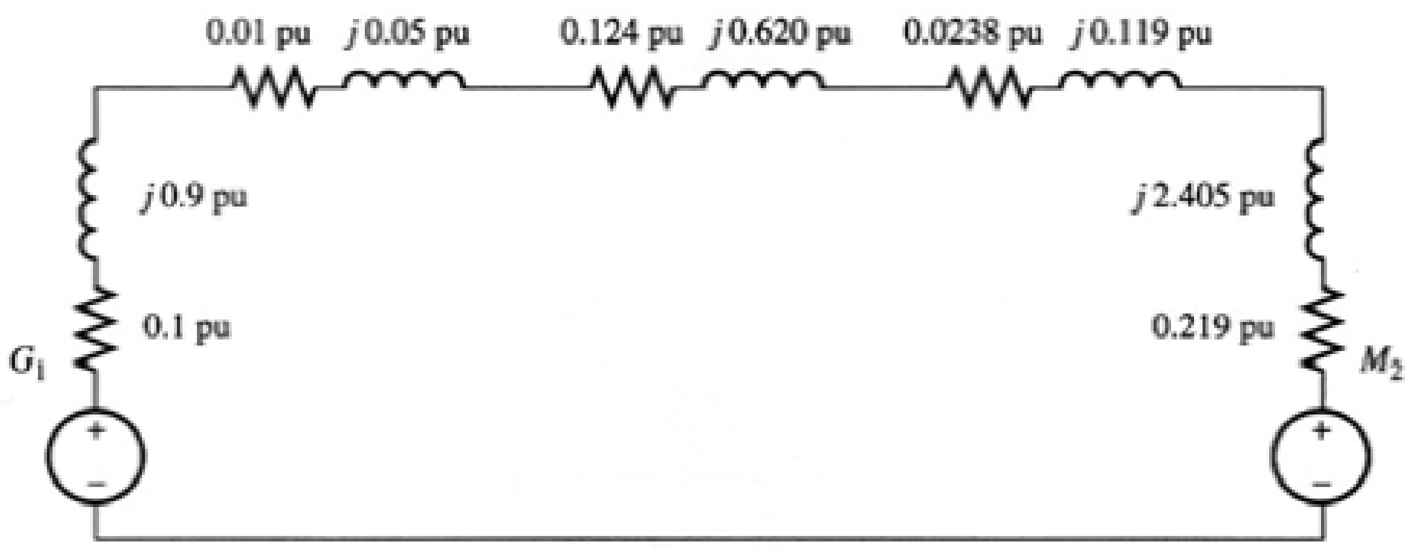
# ONE-LINE DIAGRAM (SIMPLE POWER SYSTEM)




Machine ratings, impedances, consumed and/or supplied powers are usually included in the diagrams

# EXAMPLE OF CONVERSION OF ONE-LINE DIAGRAM TO IMPEDANCE DIAGRAM

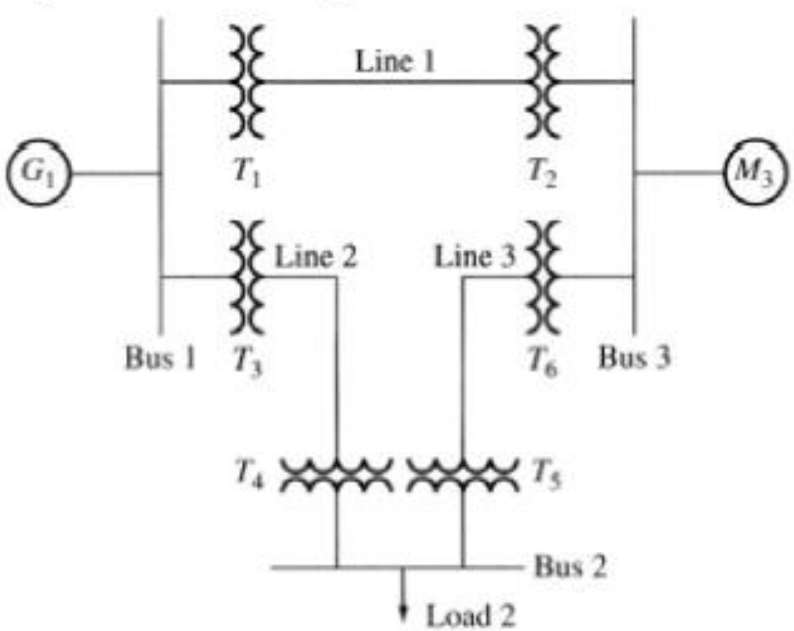


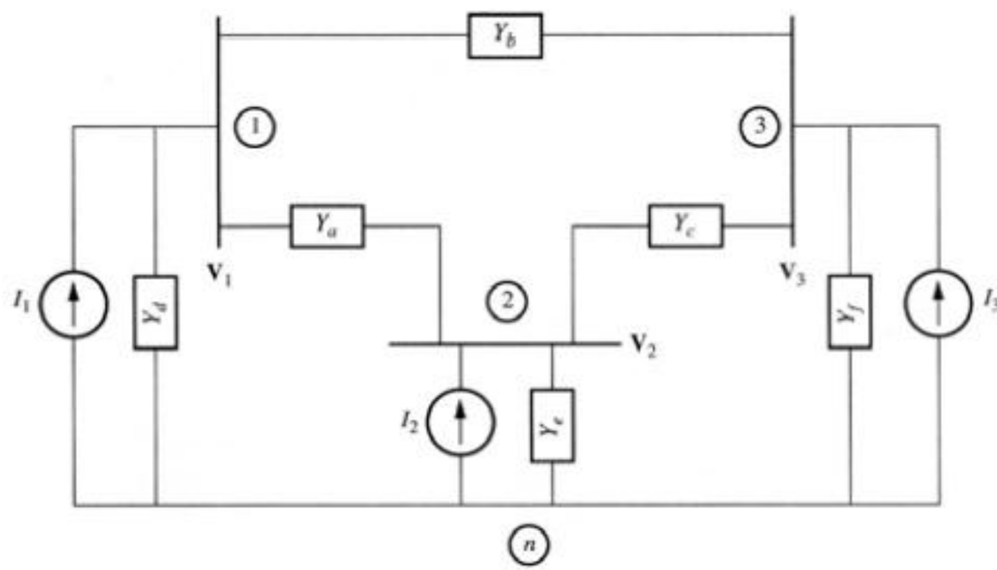



# NODE EQUATIONS

**The most common technique used to solve circuit problems is nodal analysis. To simplify the equations,**

- Replace the generators by their Norton equivalent circuits
- Replace the impedances by their equivalent admittances
- Represent the loads by the current they draw (for now)



# NODE EQUATIONS

KCL is used to establish and solve a system of simultaneous equations with the unknown node voltages:

$$(V_1 - V_2)Y_a + (V_1 - V_3)Y_b + V_1 Y_d = I_1$$

$$(V_2 - V_1)Y_a + (V_2 - V_3)Y_c + V_2 Y_e = I_2$$

$$(V_3 - V_1)Y_b + (V_3 - V_2)Y_c + V_3 Y_f = I_3$$

# NODE EQUATIONS – THE $Y_{BUS}$ MATRIX

In matrix from,

$$\begin{bmatrix} Y_a + Y_b + Y_d & -Y_a & -Y_b \\ -Y_a & Y_a + Y_c + Y_e & -Y_c \\ -Y_b & -Y_c & Y_b + Y_c + Y_f \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

Which is an equation of the form:

$$Y_{bus} V = I$$

where $Y_{bus}$ is the bus admittance matrix of a system, which has the form:

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

$Y_{bus}$ has a regular form that is easy to calculate:

1) The diagonal elements $Y_{ii}$ equal the sum of all admittances connected to node *i*.
2) Other elements $Y_{ij}$ equal to the negative admittances connected to nodes *I* and *j*.

The diagonal elements of $Y_{bus}$ are called the self-admittance or driving-point admittances of the nodes; the off-diagonal elements are called the mutual admittances or transfer admittances of the nodes.

# $Y_{BUS}$ AND $Z_{BUS}$ MATRICES

Inverting the bus admittance matrix $Y_{bus}$ yields the bus impedance mat

$$Z_{bus} = Y_{bus}^{-1}$$

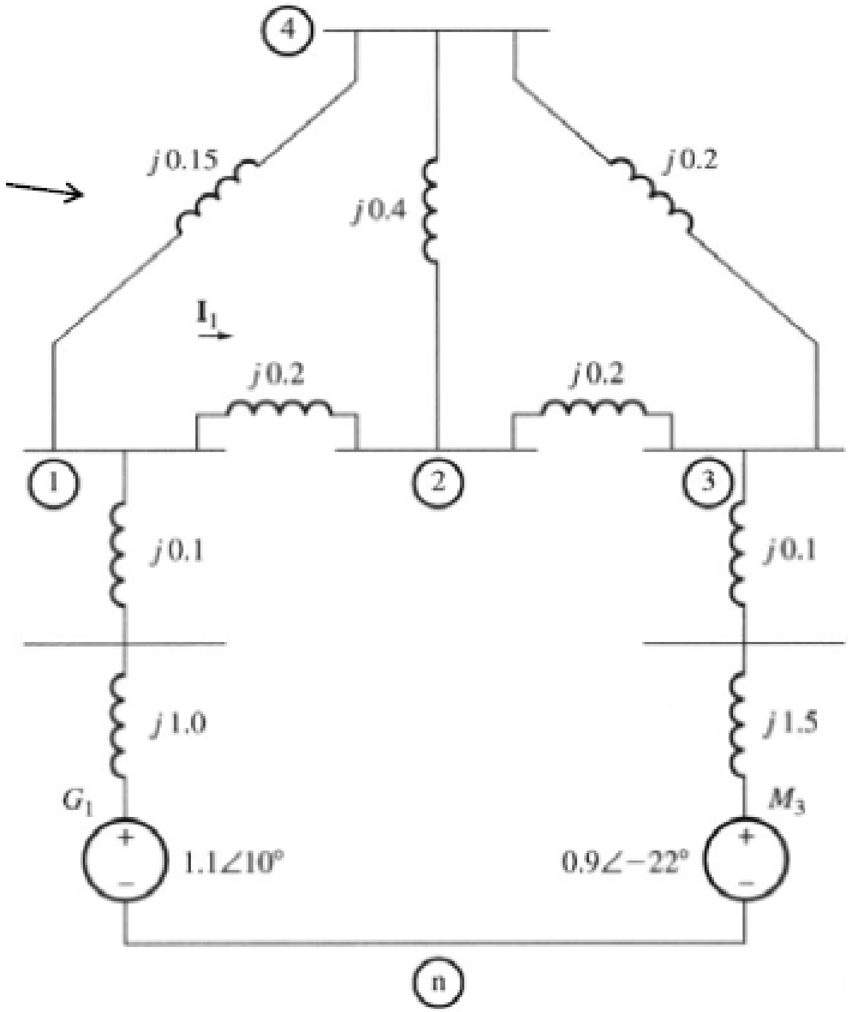
Then,

$$V = Y_{bus}^{-1} I$$

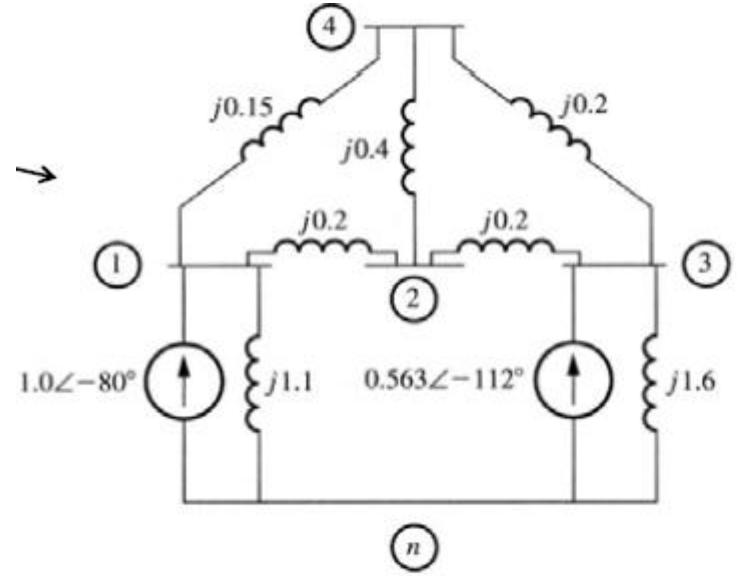
or

$$V = Z_{bus} I$$

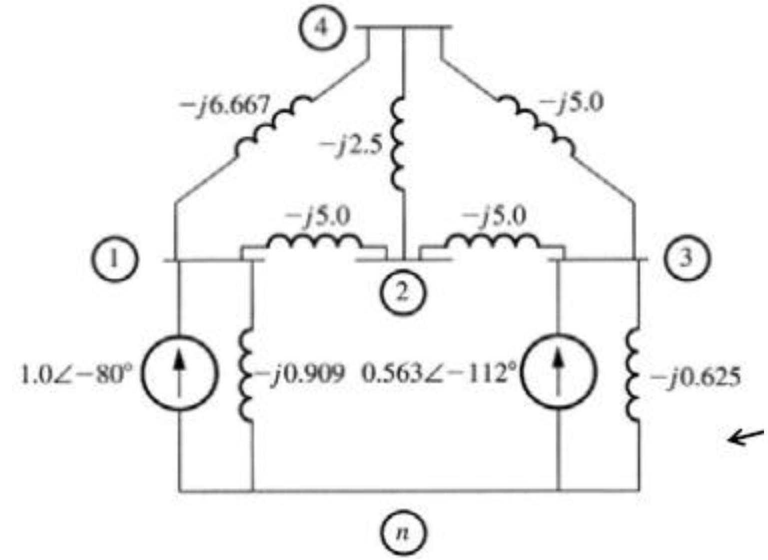
# EXAMPLE



4
j 0.15
j 0.4
j 0.2
I1
j 0.2
j 0.2
1
2
3
j 0.1
j 0.1
j 1.0
j 1.5
G1
M3
1.1∠10°
0.9∠−22°
n

4
j0.15
j0.4
j0.2
j0.2
j0.2
1
2
3
1.0∠−80°
j1.1
0.563∠−112°
j1.6
n

4
−j6.667
−j2.5
−j5.0
−j5.0
−j5.0
1
2
3
1.0∠−80°
−j0.909
0.563∠−112°
−j0.625
n

# EXAMPLE (CONT.)

The resulting admittance matrix is:

$$Y_{bus} = \begin{bmatrix} -j12.576 & j5.0 & 0 & j6.667 \\ j5.0 & -j12.5 & j5.0 & j2.5 \\ 0 & j5.0 & -10.625 & j5.0 \\ j6.667 & j2.5 & j5.0 & -j14.167 \end{bmatrix}$$

The current vector for this circuit is:

$$I = \begin{bmatrix} 1.0\angle -80^\circ \\ 0 \\ 0.563\angle -112^\circ \\ 0 \end{bmatrix}$$

The solution to the system of equations will be

$$V = Y_{bus}^{-1} I = \begin{bmatrix} 0.989\angle -0.60^\circ \\ 0.981\angle -1.58^\circ \\ 0.974\angle -2.62^\circ \\ 0.982\angle -1.48^\circ \end{bmatrix} V$$

# PROBLEMS FROM CHAP. 1:
# 7, 15, 19, 21, 26

# END!