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EE482: Digital Signal Processing Applications

FIR Design

http://www.ee.unlv.edu/~b1morris/ee482/

Outline

- Filter Characteristics
- Filter Types
- Linear Phase Filters
- Design of FIR Filters
- Window Functions

Why FIR Filters?

- Always stable
  - Finite length
- Linear phase property is guaranteed
  - Even/odd symmetry
- Finite precision errors are less severe
  - No feedback
- FIR filtering is efficient for implementation

- Modern filter design is FIR design

Filter Characteristics

- Remember the LTI system

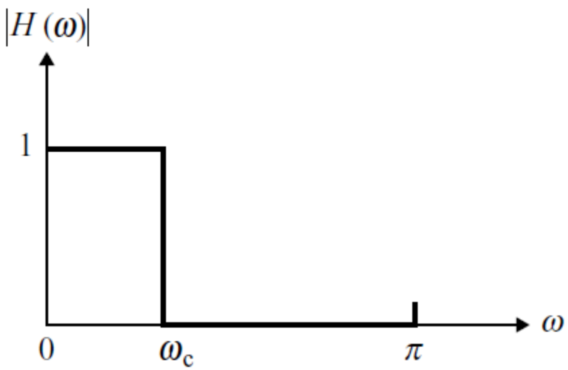
$$x(n) \longrightarrow \boxed{h(n)} \longrightarrow y(n)$$

- Transient response
  - Rising-time – how fast output can change (changing rate)
  - Settling-time – how long to settle to stable value
  - Overshoot – if output goes over the desired value

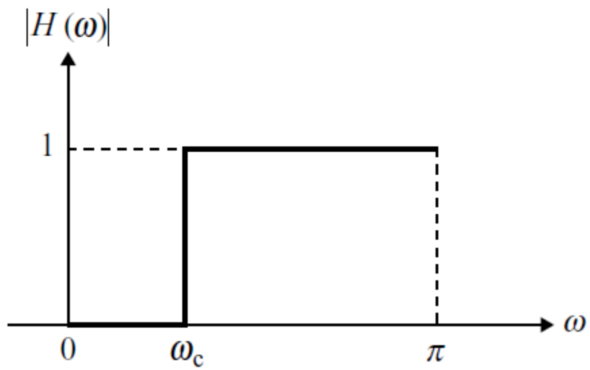
- Steady-state response
  - $Y(\omega) = H(\omega)X(\omega)$
  - Magnitude response
    - $|Y(\omega)| = |H(\omega)||X(\omega)|$
  - Phase response
    - $\Phi_Y(\omega) = \Phi_H(\omega) + \Phi_X(\omega)$
  - Group delay
    - $T_d(\omega) = -\frac{d\Phi_H(\omega)}{d\omega}$
    - Constant group delay for linear phase → no phase distortion
  - Linear phase filters
    - $\Phi_H(\omega) = -\alpha\omega$, or $\pi - \alpha\omega$
    - All frequencies delayed by same amount
    - Simple phase relationship indicates a time shift by α
      - $y(n) = x(n - \alpha)$

Filter Types

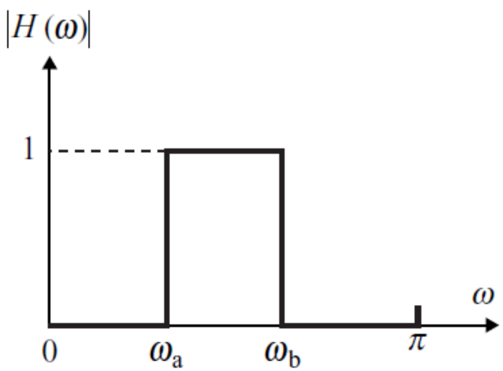
- Ideal filters



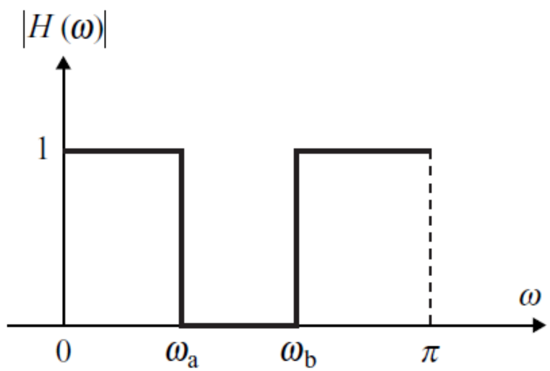
(a) Lowpass filter



(b) Highpass filter



(c) Bandpass filter



(d) Bandstop filter

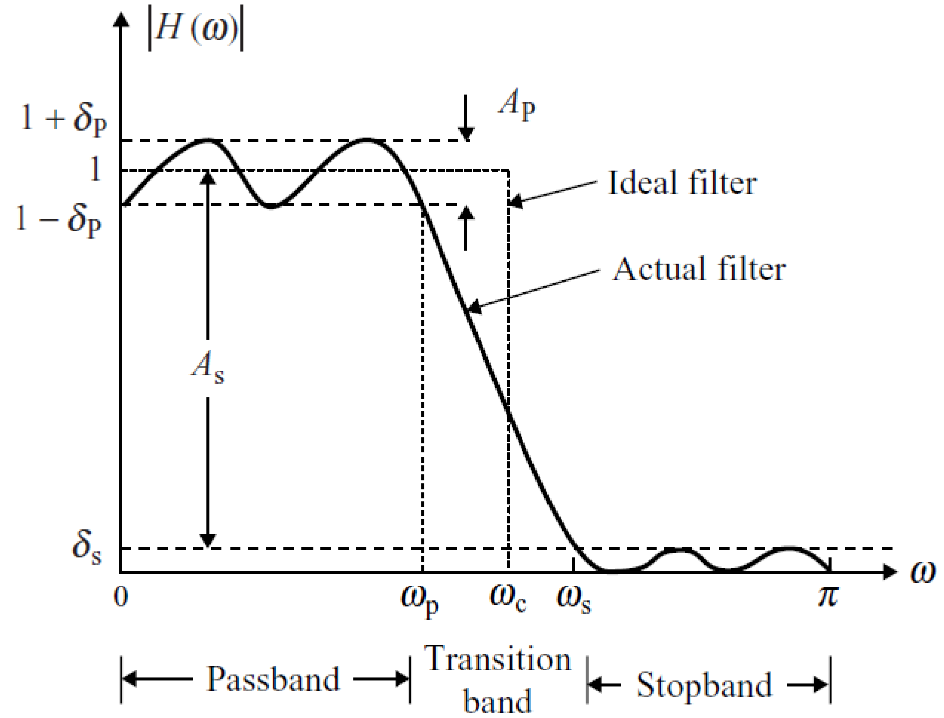
- Defined in terms of magnitude response
- Note: only $[0, \pi]$ given because with real filter coefficients $H(\omega)$ is even symmetric across $\omega = 0$
- Remember this is 2π periodic
- Bandstop with a narrow band is called a notch filter
- Allpass filter has $|H(\omega)| = 1, \ \forall\omega$

Filter Specifications

- Defined by magnitude response

- Must give a tolerance scheme
  - Cannot practically make ideal filters with sharp transitions

- ω_p- passband edge frequency
- ω_s- stopband edge frequency
- δ_p - passband ripple
  - $A_p = 20\log_{10}\left(\frac{1+\delta_p}{1-\delta_p}\right) dB$
- δ_s - stopband attenuation
  - $A_s = -20\log_{10}\delta_s\ dB$



- $1-\delta_p \le |H(\omega)| \le 1+\delta_p \quad 0 \le \omega \le \omega_p$
- $|H(\omega)| \le \delta_s \quad \omega_s \le \omega \le \pi$

Linear Phase FIR Filters

- Systems have symmetry which can be exploited
- Even
  - $b_l = b_{L-1-l}, \quad l = 0, 1, \dots, L-1$
- Odd
  - $b_l = -b_{L-1-l}, \; l = 0, 1, \dots, L-1$

- Group delay is constant
  - $T_d(\omega) = M = \begin{cases} L/2 & L \text{ even} \\ \frac{L-1}{2} & L \text{ odd} \end{cases}$

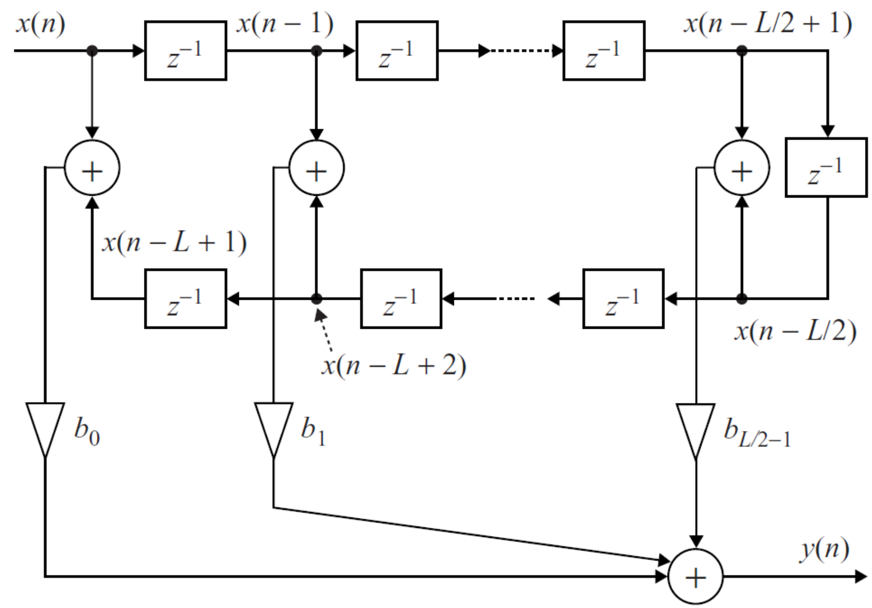


Figure 3.5 Signal-flow diagram of symmetric FIR filter; L is an even number

- Less multiplications are required because coefficients are shared

Design of FIR Filters

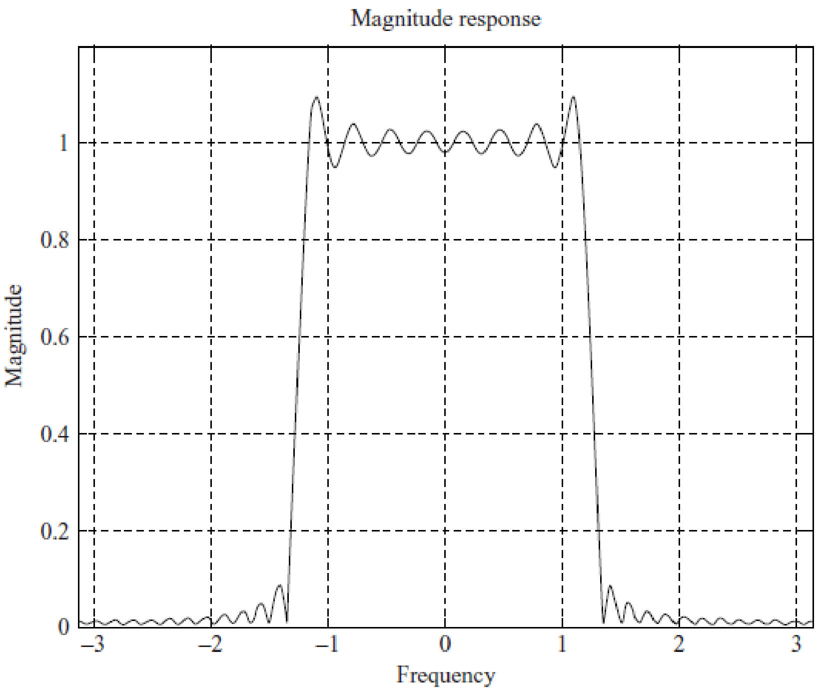
- Fourier series (windowing) method
  - Find a desired impulse response from desired frequency response
  - $H_d(\omega) = \sum_{n=-\infty}^{\infty} h_d(n) e^{-j\omega \mathrm{n}}$
  - $h_d(n) = \int_{-\pi}^{\pi} H_d(\omega) e^{j\omega n}\, d\omega$
- Notice the impulse response is in general infinite
  - Can make this finite only taking some of the samples (truncate)
  - $h(n) = \begin{cases} h_d(n) & -M \leq n \leq M \\ 0 & else \end{cases}$
  - This can be made causal by shifting to the right by M samples
  - $b_l = h(l - M), \quad l = 0, \dots, 2M$

- Notice that $h(n)$ can be thought of as FS coefficients for $H_d(\omega)$
  - More coefficients, better approximation

Examples

- Example 3.5
- Design a LP filter using windowing
- $H_d(\omega) = \begin{cases} 1 & |\omega| \le \omega_c \\ 0 & else \end{cases}$
- Use FT equation or in a Table of common pairs
- $h_d(n) = \sin\frac{\omega_c n}{\pi n} = \frac{\omega_c}{\pi} sinc\left(\frac{\omega_c n}{\pi}\right)$
- Window the impulse response and shift to make causal
- $b_l = \begin{cases} \frac{\omega_c}{\pi} sinc\left(\frac{\omega_c(l-M)}{\pi}\right) & 0 \le l \le L-1 \\ 0 & else \end{cases}$

- Example 3.7
- Design a LP filter with $\omega_c = 0.4\pi$ with $L = 61$.
  - $M = \frac{L-1}{2} = 30$
  - $b_l = 0.4 sinc(0.4(l-30))$
    - $l = 0, 1, \ldots, 60$



Windowing Approximation Accuracy

- Notice the rippling effect known as Gibbs phenomenon
- Windowing is equivalent to multiplication in time domain
  - $h(n) = h_d(n)w(n)$
  - Rectangular window
    - $w(n) = \begin{cases} 1 & -M \leq n \leq M \\ 0 & else \end{cases}$
- Multiplication in time is convolution in frequency domain
  - $H(\omega) = \frac{1}{2\pi} H_d(\omega) * W(\omega)$
    - $W(\omega) = \frac{\sin\left(\frac{(2M+1)}{2}\right)\omega}{\sin\frac{\omega}{2}}$

Windowing in Frequency Domain

- $H(\omega) = \frac{1}{2\pi} H_d(\omega) * W(\omega)$
  - Ideal frequency response is smoothed by window DTFT

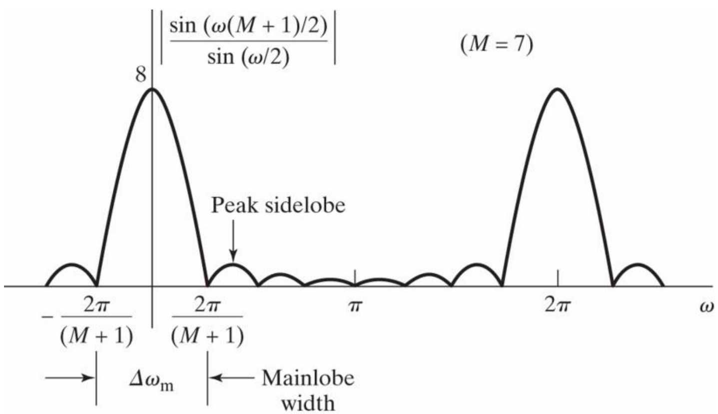


Figure 7.28 Magnitude of the Fourier transform of a rectangular window (M = 7).

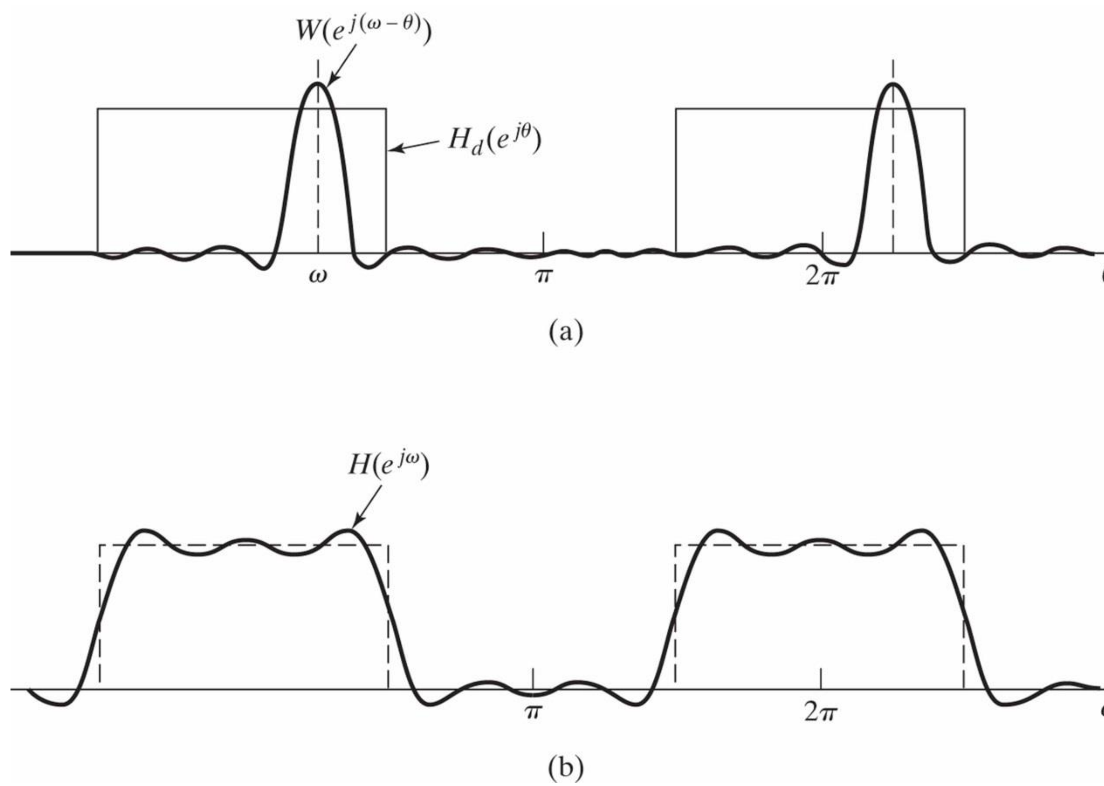
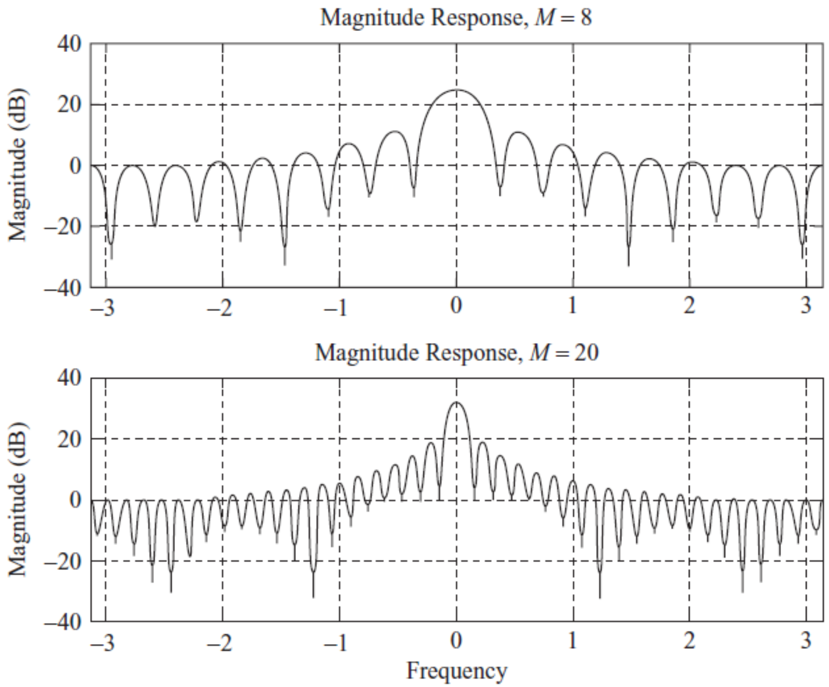


Figure 7.27 (a) Convolution process implied by truncation of the ideal impulse response. (b) Typical approximation resulting from windowing the ideal impulse response.

Rectangular Window

- $W(\omega) = \dfrac{\sin\left(\frac{(2M+1)}{2}\right)\omega}{\sin\frac{\omega}{2}}$
- This window spectrum has ripples which causes ripples in $H(\omega)$ at sharp transitions
    - Can't make perfectly sharp edges
- Mainlobe – centered at $\omega = 0$
    - Care about width
- Sidelobes – all other ripples
    - Care about height
- Gibbs phenomenon can be managed by smoothing the window edges
    - Results in lower sidelobe height and increased mainlobe width
    - Larger transition width at discontinuity but less ringing



Normalized frequency

Windowing Design Considerations

- $H(\omega) = \frac{1}{2\pi} H_d(\omega) * W(\omega)$
  - Ideal frequency response is smoothed by window DTFT
- The quality of the FIR approximation is dependent on two factors
  - The width of the main lobe
  - The peak side-lobe amplitude

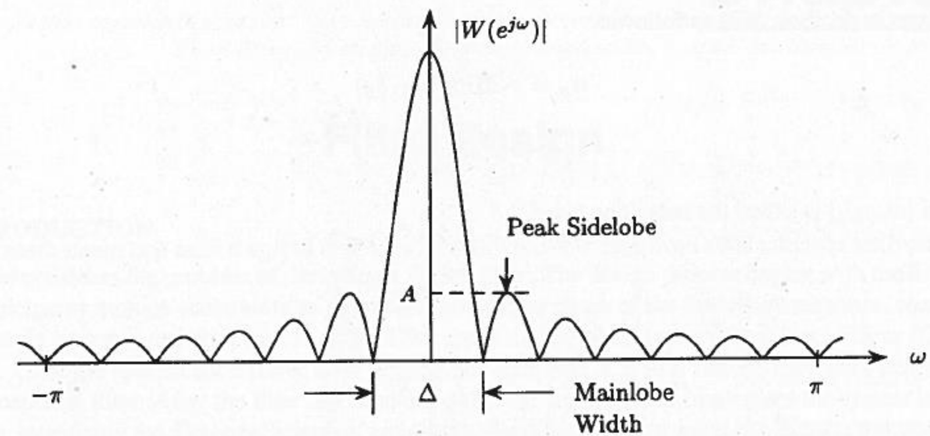


Fig. 9-2. The DTFT of a typical window, which is characterized by the width of its main lobe, Δ, and the peak amplitude of its side lobes, A, relative to the amplitude of $W(e^{j\omega})$ at $\omega = 0$.

- Want narrow main-lobe with small side lobe amplitude
  - More impulse-like
  - Cannot optimize both at the same time
  - $N\Delta_f = c$
    - N – length of filter
    - See Shaum's DSP notes
- Increasing length of window the decreases the width of the mainlobe
  - Decreases width of the transition band
- Peak sidelobe amplitude is practically independent of length only depends on shape of window
  - Decrease in sidelobe amplitude results in greater mainlobe width

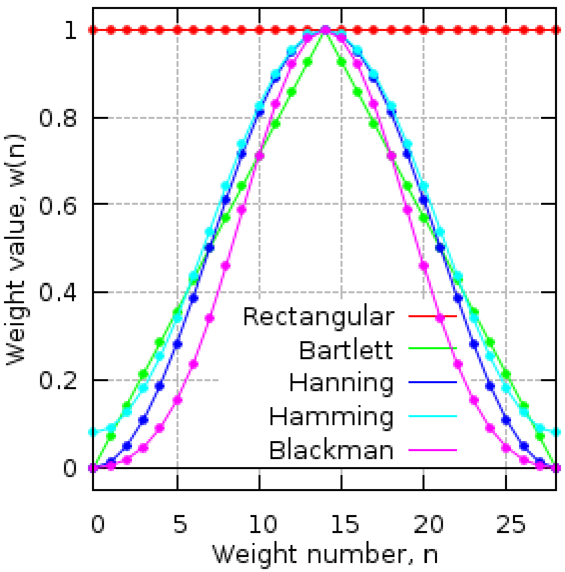
Window Functions

- Many windows have been designed to trade off mainlobe width and sidelobe height
  - All have smooth transitions at edge of window

Table 9-1 Some Common Windows

Rectangular	$w(n) = \begin{cases} 1 & 0 \le n \le N \\ 0 & \text{else} \end{cases}$
Hanning[1]	$w(n) = \begin{cases} 0.5 - 0.5\cos\left(\frac{2\pi n}{N}\right) & 0 \le n \le N \\ 0 & \text{else} \end{cases}$
Hamming	$w(n) = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{N}\right) & 0 \le n \le N \\ 0 & \text{else} \end{cases}$
Blackman	$w(n) = \begin{cases} 0.42 - 0.5\cos\left(\frac{2\pi n}{N}\right) + 0.08\cos\left(\frac{4\pi n}{N}\right) & 0 \le n \le N \\ 0 & \text{else} \end{cases}$

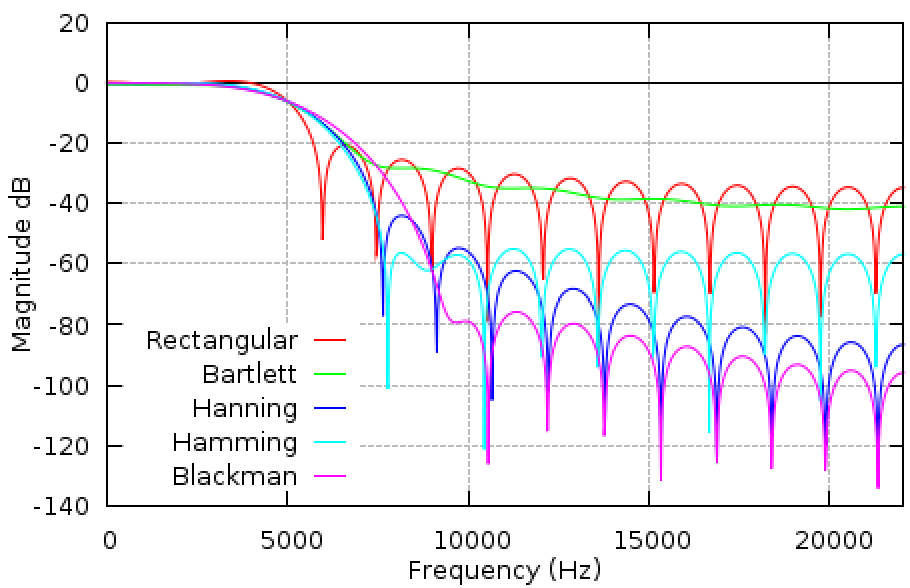
[1] In the literature, this window is also called a Hann window or a von Hann window.

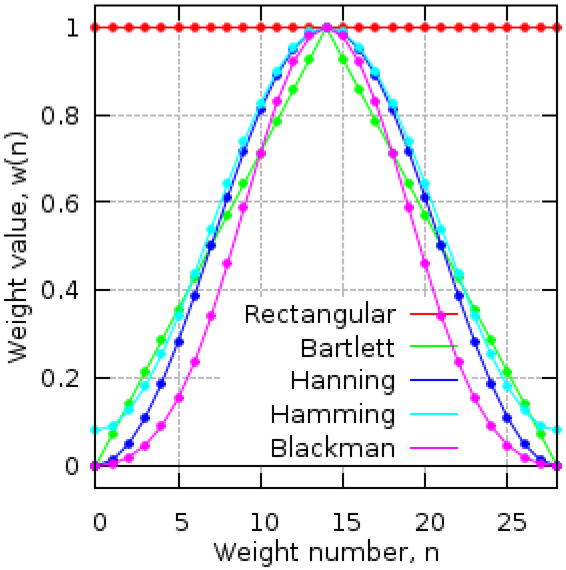


Window Performance

Table 9-2 The Peak Side-Lobe Amplitude of Some Common Windows and the Approximate Transition Width and Stopband Attenuation of an Nth-Order Low-Pass Filter Designed Using the Given Window.

Window	Side-Lobe Amplitude (dB)	Transition Width (Δf)	Stopband Attenuation (dB)
Rectangular	−13	$0.9/N$	−21
Hanning	−31	$3.1/N$	−44
Hamming	−41	$3.3/N$	−53
Blackman	−57	$5.5/N$	−74

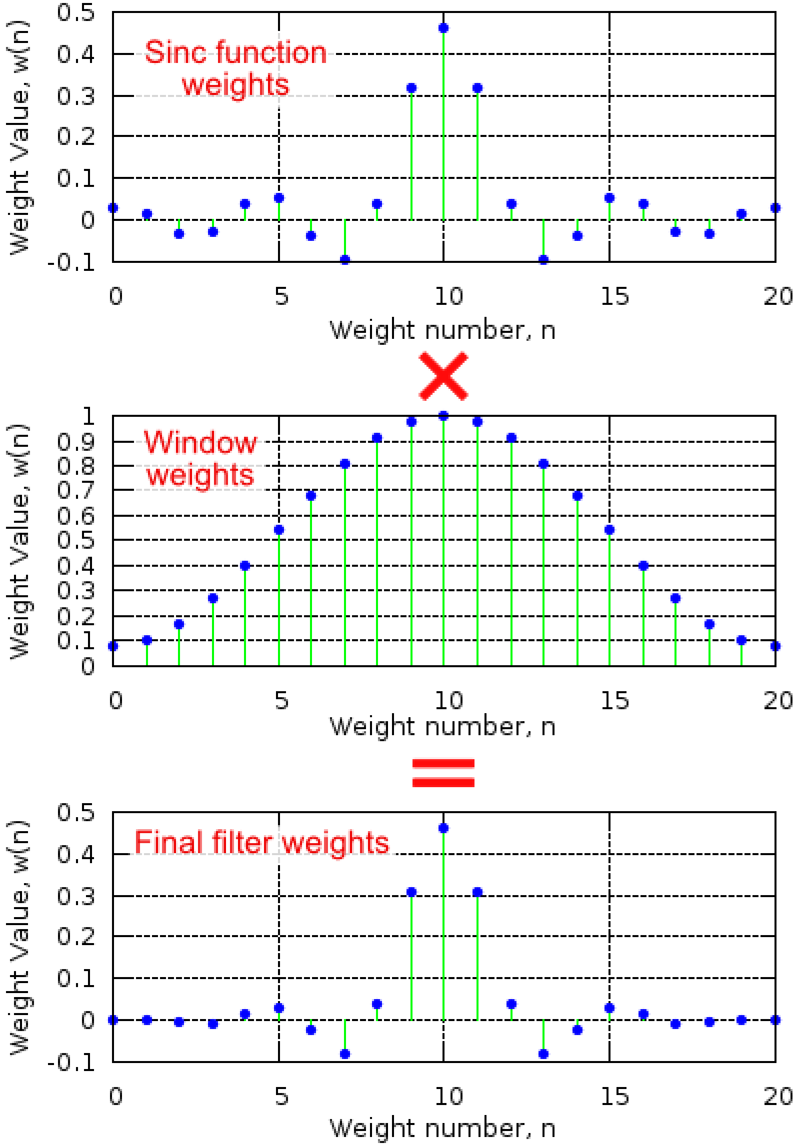




http://www.labbookpages.co.uk/audio/firWindowing.html

FIR Design Steps

1. Select window type to satisfy stopband attenuation requirements
2. Determine window size L based on transition width
3. Calculate window values
4. Calculate impulse response of desired filter
   - Truncate to fixed length L
   - Shift to make causal
5. Calculate final filter coefficients as product of window and desired response
   - $b_l = h_d[l - M]w[l]$



Upsampling/Interpolation

- Increase the sampling rate of a signal by factor L
- Accomplished by inserting zeros into a sequence and then lowpass filtering
    - Zero insertion is upsampling
    - LP filtering is interpolation
- $x_u(n) =$
$$\begin{cases} x\left(\frac{n}{L}\right) & n = 0, \pm L, \pm 2L, \ldots \\ 0 & else \end{cases}$$
    - Resulting signal has more samples but gaps between values
- LP filter using gain L and cutoff $= \pi/L$
    - Gain of L to "spread" sample energy to neighbor zeros

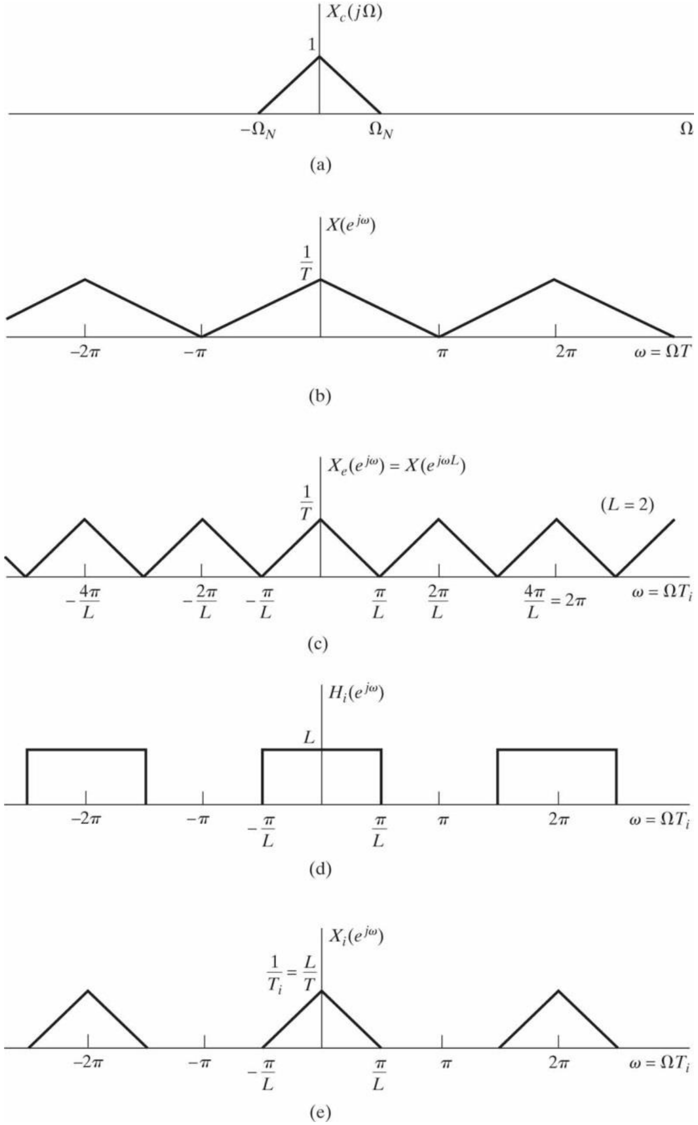


Figure 4.24 Frequency-domain illustration of interpolation.

Downsampling/Decimation

- Reduce the sampling rate of a signal by factor M
- Accomplished by dropping samples
- $x_d(n) = x(nM)$
- Remember bandwidth is controlled by sampling rate
    - Both sampling rate and bandwidth decrease by factor M
    - This may result in aliasing of the signal
- Avoid aliasing by pre-filtering signal with LP filter with cutoff $= \pi/M$ before decimation

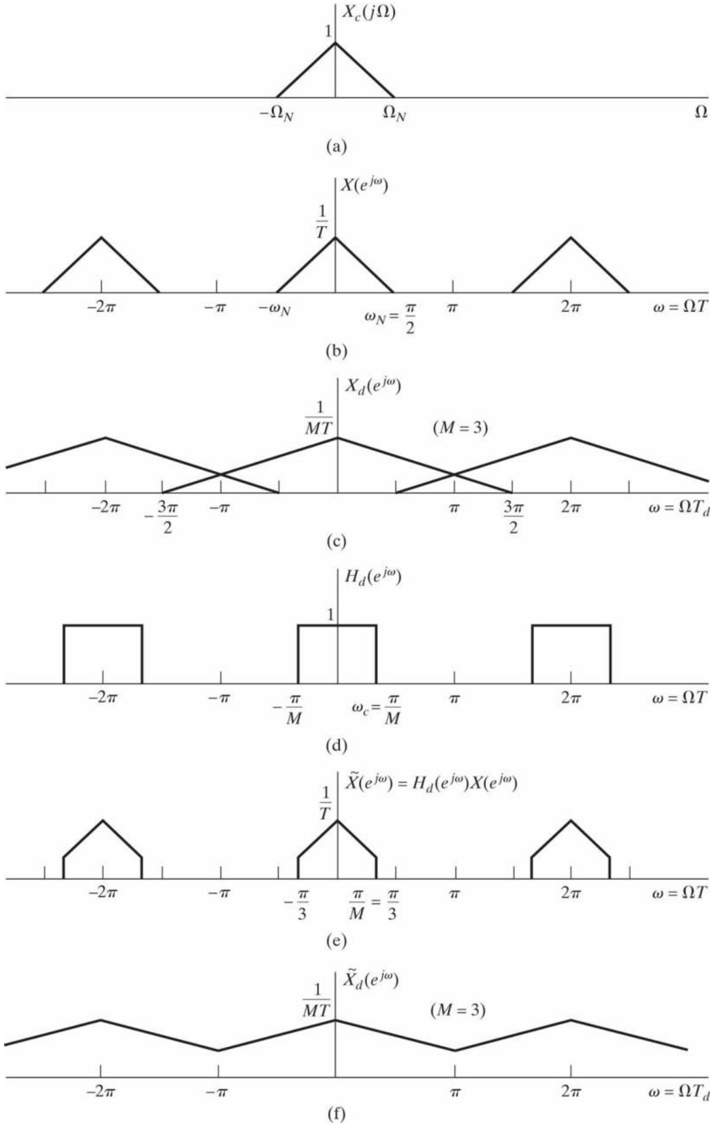


Figure 4.21 (a)–(c) Downsampling with aliasing. (d)–(f) Downsampling with prefiltering to avoid aliasing.

Arbitrary Sample Rate Conversion

- Conversion to arbitrary sample rate is possible
  - $R = U/D$
    - Must find appropriate upsample factor U and downsample factor D
- First perform interpolation followed by decimation
  - Minimize reduction in signal bandwidth
    - No fear of aliasing in upsample
  - Downsampling first could result in loss of high frequency content
- Can combine interpolation LP filter with LP for decimation
  - Cuttoff should be minimum of either operation
- Use Matlab `interp.m`, `decimate.m`, and `upfindn.m/resample.m`