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EE482: Digital Signal Processing Applications

Spring 2014
TTh 14:30-15:45 CBC C222

Lecture 06
IIR Design 2
14/03/06

http://www.ee.unlv.edu/~b1morris/ee482/

Outline

- Review IIR Design
- Implementation Considerations
- Stability
- Coefficient Quantization
- Roundoff Effects
- Cascade Pairing and Ordering

IIR Design

- Reuse well studied analog filter design techniques (books and tables for design)
- Need to map between analog design and a digital design
  - Mapping between s-plane and z-plane

IIR Filter Design

- IIR transfer function

$$H(z) = \frac{\sum_{l=0}^{L-1} b_l z^{-l}}{1 + \sum_{l=0}^{M} a_l z^{-l}}$$

- Need to find coefficients a_l, b_l
  - Impulse invariance – sample impulse response
    - Have to deal with aliasing
  - Bilinear transform
    - Match magnitude response
    - “Warp” frequencies to prevent aliasing

Bilinear Transform Design

- Convert digital filter into an "equivalent" analog filter
  - Use bilinear "warping"
- Design analog filter using IIR design techniques
- Map analog filter into digital
  - Use bilinear transform

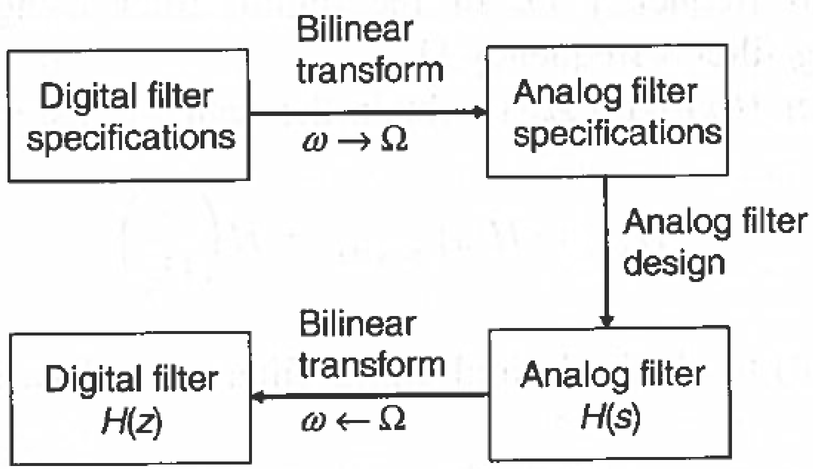
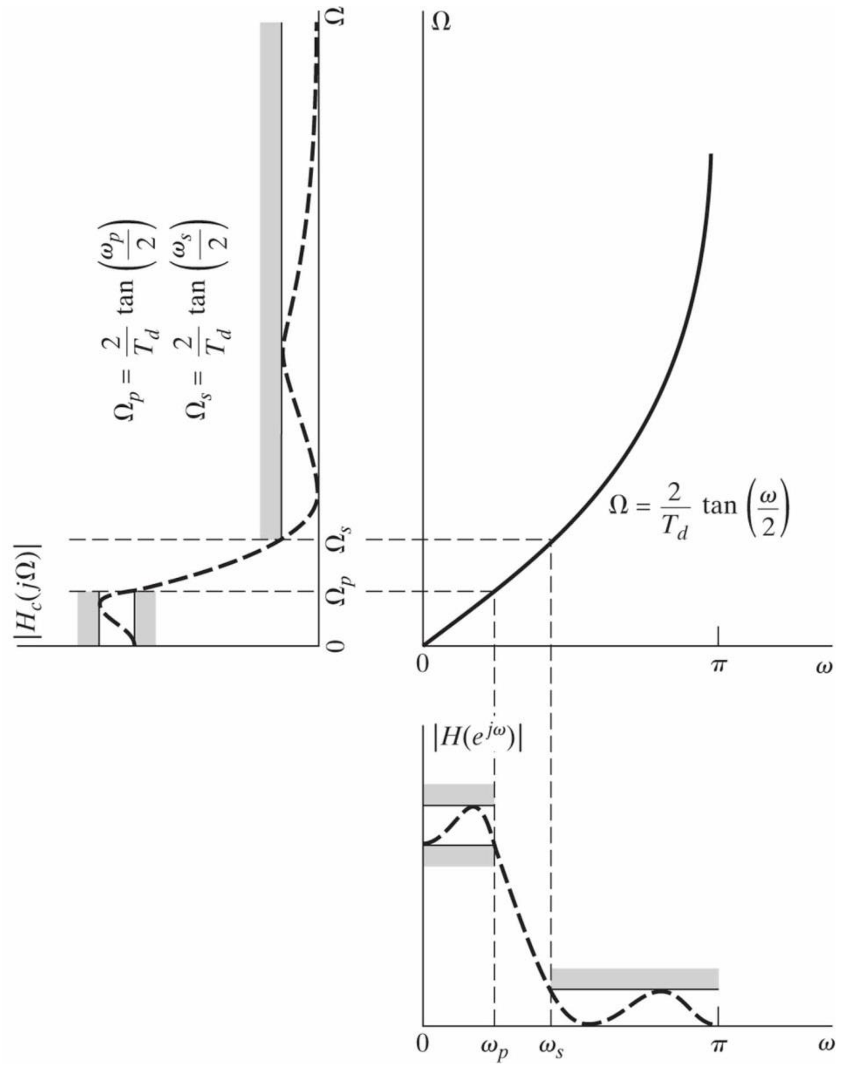


Figure 4.5 Digital IIR filter design using the bilinear transform

Bilinear Design Steps

1. Convert digital filter into an "equivalent" analog filter
   - Pre-warp using
     - $\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)$
2. Design analog filter using IIR design techniques
   - Butterworth, Chebyshev, Elliptical
3. Map analog filter into digital
   - $H(z) = H(s)\big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$

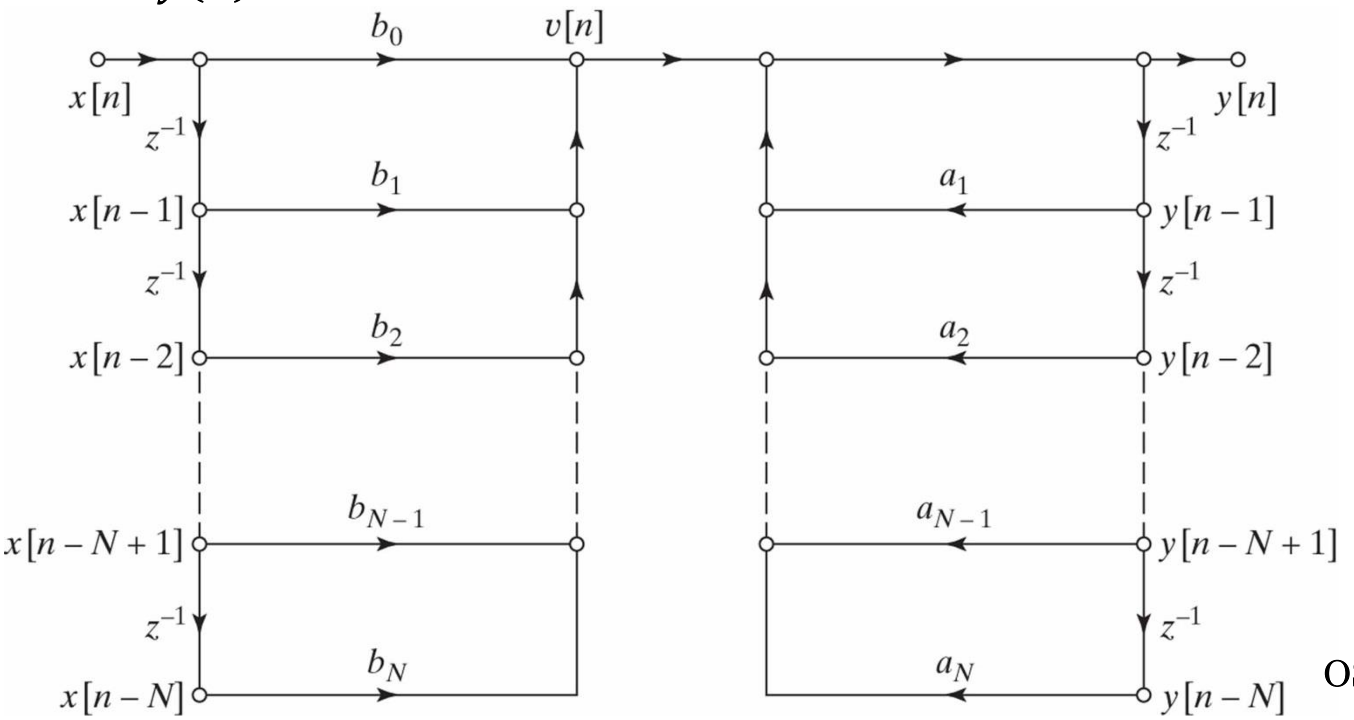


OS 3e

Direct Form I

- Straight-forward implementation of diff. eq.
  - b_l - feed forward coefficients
    - From $x(n)$ terms
  - a_l - feedback coefficients
    - From $y(n)$ terms
- Requires $(L + M)$ coefficients and delays

OS 3e



OS 3e

Direct Form II

- Notice that we can decompose the transfer function
  - $H(z) = H_1(z)H_2(z)$
    - Section to implement zeros section to implement poles
- Can switch order of operations
  - $H(z) = H_2(z)H_1(p)$
  - This allows sharing of delays and saving in memory

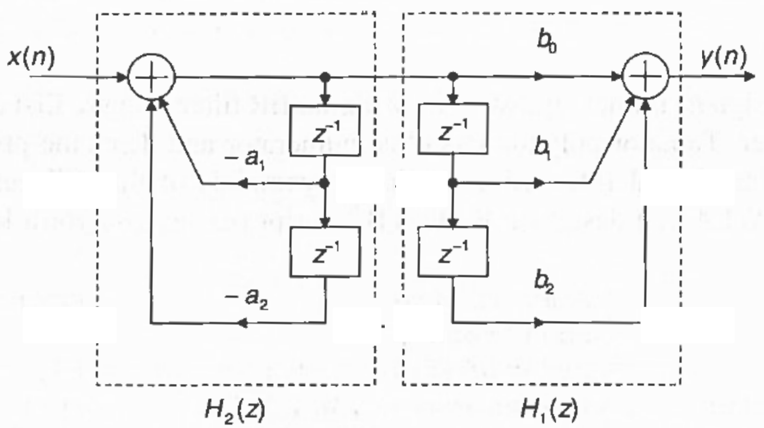
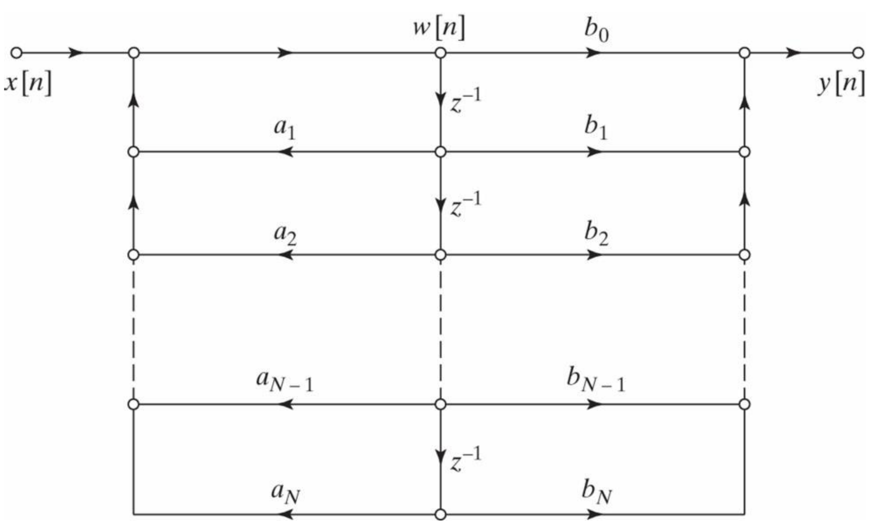


Figure 4.7 Direct-form I realization of second-order IIR filter



OS 3e

Cascade (Factored) Form

- Factor transfer function and decompose into smaller sub-systems
  - $H(z) = H_1(z)H_2(z) \ldots H_K(z)$

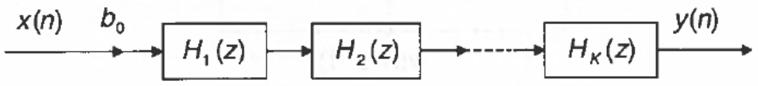
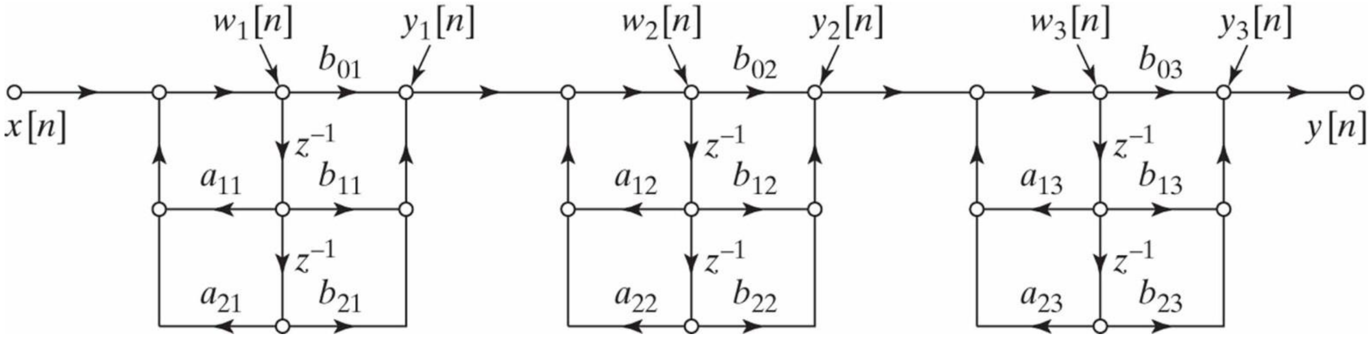


Figure 4.10 Cascade realization of digital filter

- Make each subsystem second order
  - Complex conjugate roots have real coefficients
  - Limit the order of subsystem (numerical effects)
    - Effects limited to single subsystem stage
    - Change in a single coefficient affects all poles in DF

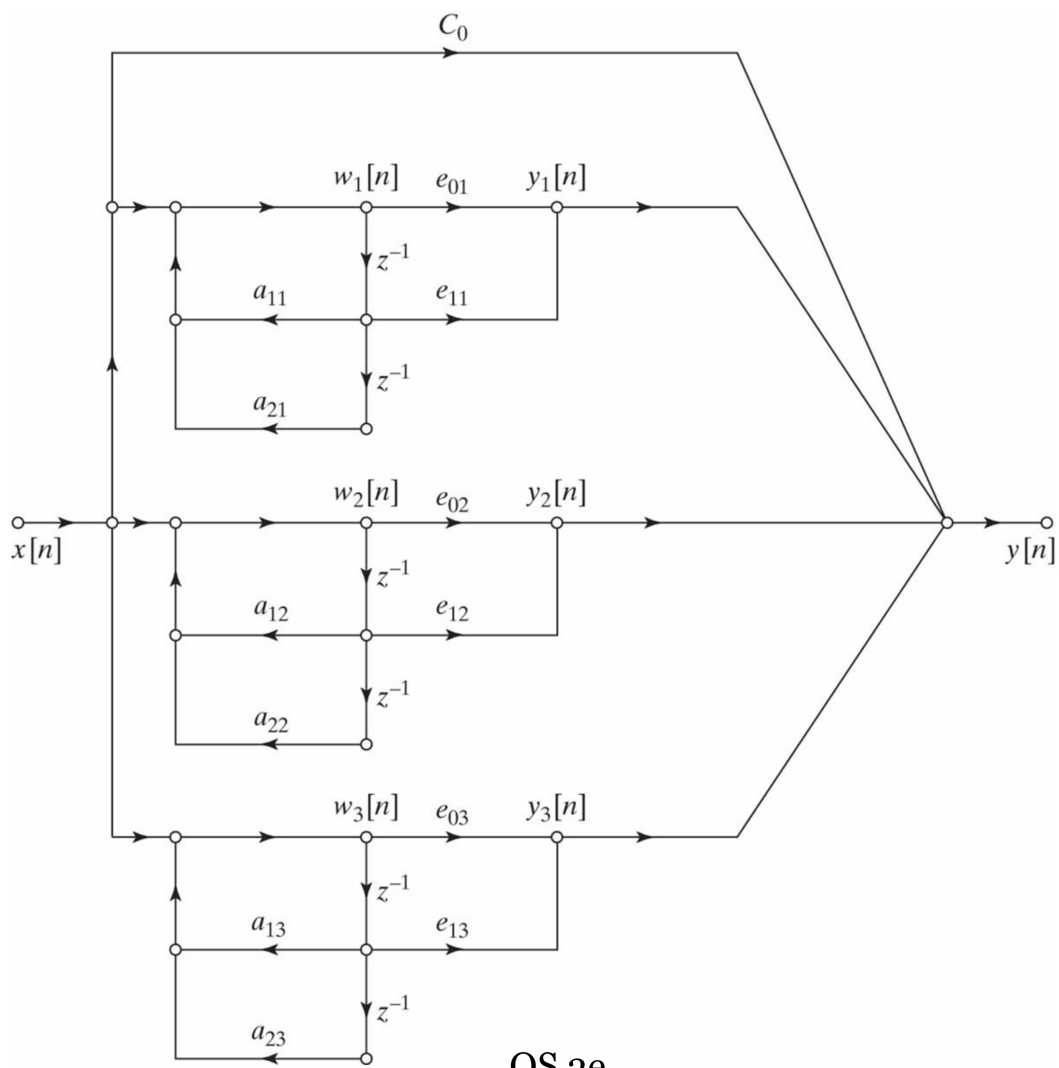


OS 3e

- Preferred over DF because of numerical stability

Parallel (Partial Fraction) Form

- Decompose transfer function using a partial fraction expansion
  - $H(z) = H_1(z) + H_2(z) + \ \ldots + H_K(z)$
    - $H_k(z) = \frac{b_{0k}+b_{1k}z^{-1}}{1+a_{1k}z^{-1}+a_{2k}z^{-2}}$
- Be sure to remember that PFE requires numerator order less than denominator
  - Use polynomial long division



OS 3e

Matlab Filter Design

- Realization tools:
- Finding polynomial roots
  - `roots.m`
  - `tf2zp.m`
- Cascade form
  - $H(z) = G \prod_{k=1}^{K} \frac{b_{0k}+b_{1k}z^{-1}+b_{2k}z^{-2}}{1+a_{1k}z^{-1}+a_{2k}z^{-2}}$
  - `zp2sos.m`
- Parallel form
  - `Residuez.m`

- Filter design tools:
- Order estimation tool
  - `butterord.m`
- Coefficient tool
  - `butter.m`
- Frequency transforms
  - `lp2hp.m, lp2bp.m, lp2bs.m`

- Useful exploration tool
  - `fvtool.m`
- Useful design tool
  - `fdatool.m`
- Useful processing tool
  - `sptool.m`

Stability

- (Causal) IIR filters are stable if all poles are within the unit circle
  - $|p_m| < 1$
  - We will not consider marginally stable (single pole on unit circle)
- Consider poles of 2nd order filter (used in cascade and parallel forms)
  - $A(z) = 1 + a_1 z^{-1} + a_2 z^{-2}$
- Factor
  - $A(z) = (1 - p_1 z^{-1})(1 - p_2 z^{-1})$
  - $A(z) = 1 - (p_1 + p_2) z^{-1} + p_1 p_2 z^{-2}$
- Because poles must be inside the unit circle
  - $|a_2| = |p_1 p_2| < 1$
  - $|a_1| < 1 + a_2$

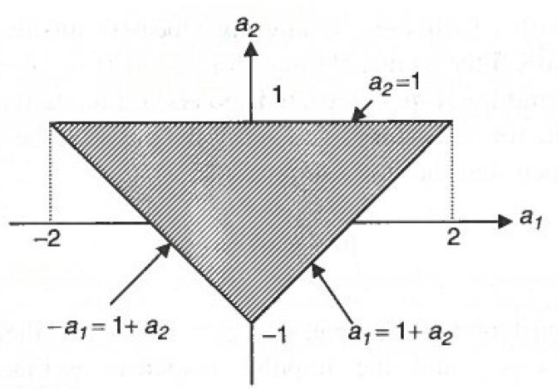


Figure 4.15 Region of coefficient values for the stable second-order IIR filters

Coefficient Quantization

- Using fixed word lengths results in a quantized approximation of a filter
  - $H'(z) = \frac{\sum_{k=0}^{L-1} b'_k z^{-k}}{1+\sum_{k=1}^{M} a'_k z^{-k}}$
- This can cause a mismatch from desired system $H(z)$
- Poles that are close to the unit circle may move outside and cause instability
  - This is exacerbated with higher order systems

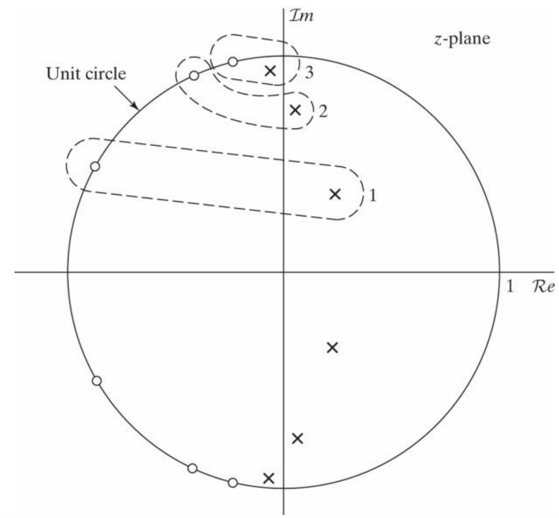
Rounding Effects

- Using B bit architecture, products require $2B$ bits
  - Must be rounded into smaller B bit container
- This results in noise error terms
  - Can be simply modeled as additive term
- The order of cascade sections influences power of noise at output
  - How should sections be paired and ordered?
- Need to optimize SQNR
  - Trade-off with probability of arithmetic overflow
  - Need to use scaling factors to prevent overflow
  - Optimality when signal level is maximized without overflow

Cascade Ordering and Pairing

- Good results are obtained using simple rules
- Cascade ordering and pairing algorithm

1. Pair pole closest to unit circle with zero that is closest in z-plane
   - Minimize the chance of overflow
2. Apply 1 repeatedly until all poles and zeros are paired
3. Resulting 2nd -order sections can be ordered in two alternative ways
   - Increasing closeness to unit circle
   - Decreasing closeness to unit circle



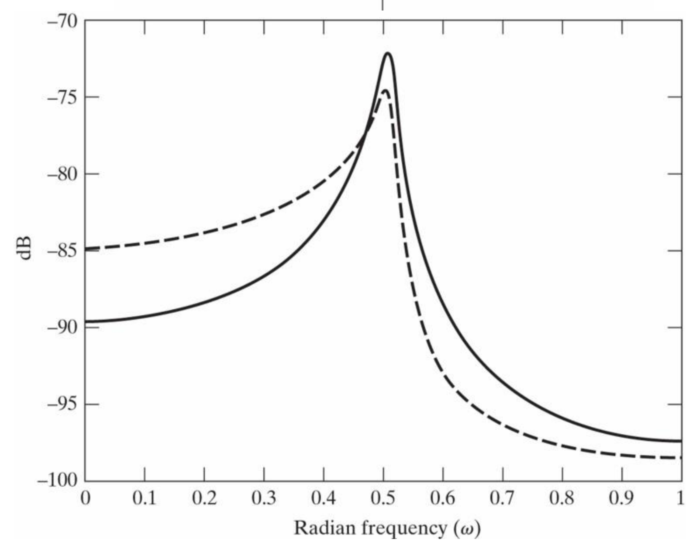


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Figure 6.67 Output noise power spectrum for 123 ordering (solid line) and 321 ordering (dashed line) of 2^{nd}-order sections.

Recursive Resonator

- Filter with frequency response dominated at a single peak
  - Use complex-conjugate pole pair inside unit circle
- $H(z) = \dfrac{A}{(1-r_p e^{j\omega_0} z^{-1})(1-r_p e^{-j\omega_0} z^{-1})}$
- $H(z) = \dfrac{A}{1-2r_p\cos(\omega_0)z^{-1}+r_p^2 z^{-2}}$
  - A – normalization constant for unity gain at ω_0
  - $0 < r_p < 1$
- Close to unit circle
  - Bandwidth $\cong 2(1-r_p)$
  - Closer to $r_p = 1$, more peaked

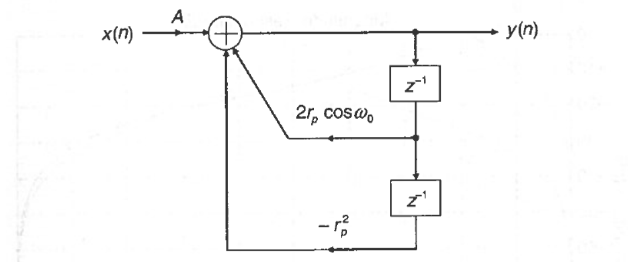
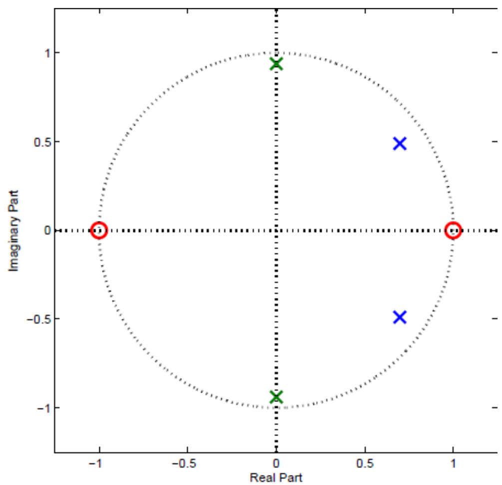
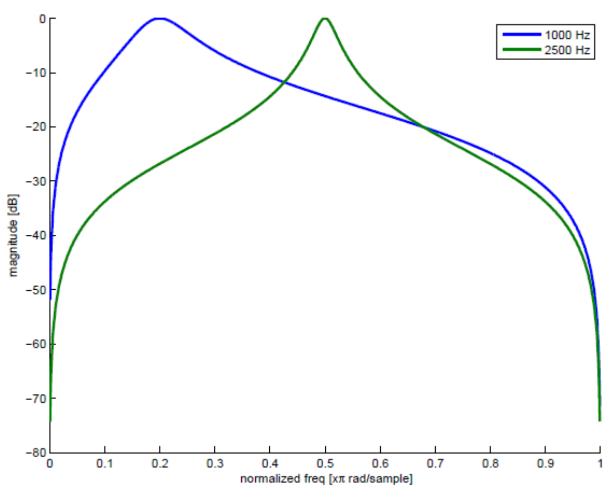


Figure 4.17 Signal-flow diagram of the second-order resonator filter





Parametric Equalizer

- Add nearby zeros to the resonator
  - At same angle as poles ω_0
  - Similar radius
- Pole and zero counter balance one another
- $r_z < r_p$
  - Pole dominates because it is closer to unit circle
  - Generates peak at $\omega = \omega_0$
    - Provides boost to freq
- $r_z > r_p$
  - Zero dominates pole
  - Generates dip at $\omega = \omega_0$
    - Cuts freq
- Bandwidth still determined by r_p

- Ex 4.18
  - Create equalizer by changing gain at given frequency