EE482/682: DSP APPLICATIONS

CH3 FIR FILTER DESIGN

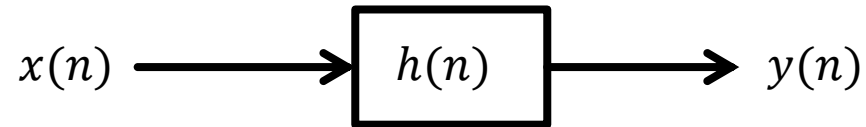# INTRODUCTION TO FIR FILTERS

CHAPTER 3.1

WHY FIR FILTERS?

- Always stable
  - Finite length
- Linear phase property (delay) is guaranteed
  - Even/odd symmetry
- Finite precision errors are less severe
  - No feedback
- FIR filtering is efficient for implementation

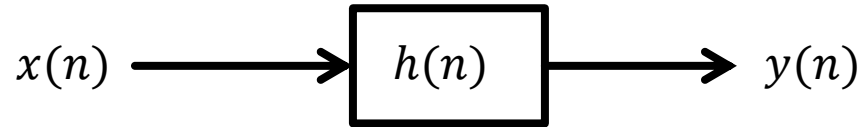- Modern filter design is FIR design

TRANSIENT RESPONSE

- Remember the LTI system

$$x(n) \longrightarrow \boxed{h(n)} \longrightarrow y(n)$$

- Transient response
  - Rising-time – how fast output can change (changing rate)
  - Settling-time – how long to settle to stable value
 - Overshoot – if output goes over the desired value

STEADY STATE RESPONSE

$$x(n) \longrightarrow \boxed{h(n)} \longrightarrow y(n)$$

- $Y(\omega) = H(\omega)X(\omega)$
- Magnitude response
  - $|Y(\omega)| = |H(\omega)||X(\omega)|$
- Phase response
  - $\Phi_Y(\omega) = \Phi_H(\omega) + \Phi_X(\omega)$

- Linear phase filters
  - $\Phi_H(\omega) = -\alpha\omega,$ or $\pi - \alpha\omega$
  - All frequencies delayed by same amount
  - Simple phase relationship indicates a time shift by α
    - $y(n) = x(n - \alpha)$
- Group delay
  - $T_d(\omega) = -\dfrac{d\Phi_H(\omega)}{d\omega}$
  - Constant group delay for linear phase → no phase distortion

FILTER TYPES

- Ideal filters

(a) Lowpass filter.

(b) Highpass filter.

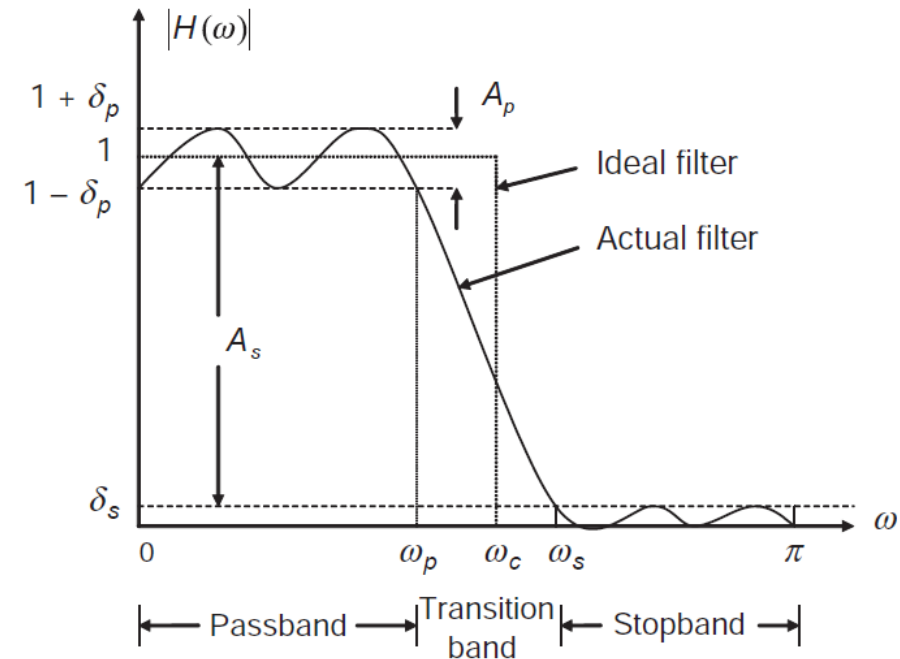(c) Bandpass filter.
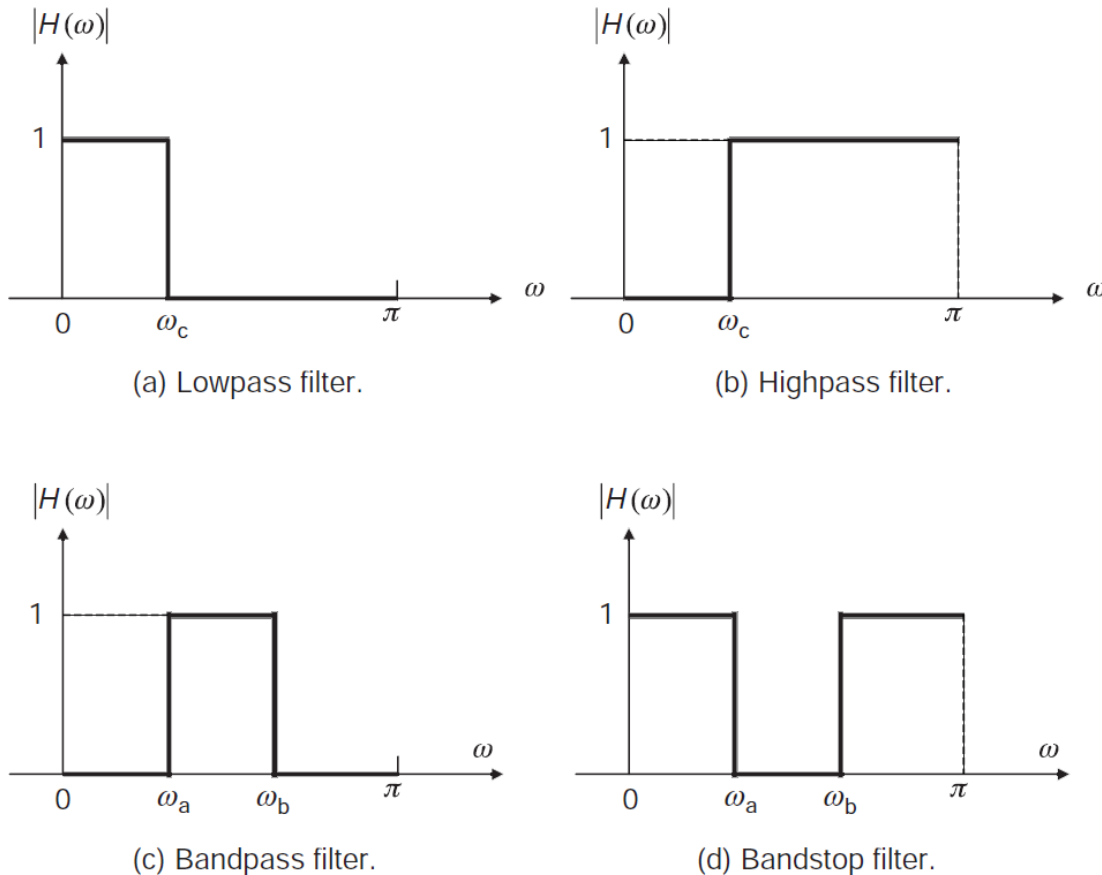
(d) Bandstop filter.

Figure 3.2 Magnitude responses of four different ideal filters

- Defined in terms of magnitude response

- Note: only $[0, \pi]$ given because with real filter coefficients $H(\omega)$ is even symmetric across $\omega = 0$

- Remember this is 2π periodic

- Bandstop with a narrow band is called a notch filter

- Allpass filter has $|H(\omega)| = 1$, $\forall \omega$

FILTER SPECIFICATIONS

- Defined by magnitude response
- Must give a tolerance scheme
  - Cannot practically make ideal filters with sharp transitions

- ω_p- passband edge frequency
- ω_s- stopband edge frequency
- δ_p - passband ripple

  - $A_p = 20 \log_{10}\left(\frac{1+\delta_p}{1-\delta_p}\right) dB$

- δ_s - stopband attenuation
  - $A_s = -20 \log_{10} \delta_s \ dB$

Figure 3.4 Magnitude response and performance measurement of lowpass filter

- $1 - \delta_p \leq |H(\omega)| \leq 1 + \delta_p \quad 0 \leq \omega \leq \omega_p$
- $|H(\omega)| \leq \delta_s \qquad \omega_s \leq \omega \leq \pi$

LINEAR PHASE FIR FILTERS

- Systems have symmetry which can be exploited

- Even

  - $b_l = b_{L-1-l}, \quad l = 0, 1, \ldots, L-1$

- Odd

  - $b_l = -b_{L-1-l}, \quad l = 0, 1, \ldots, L-1$
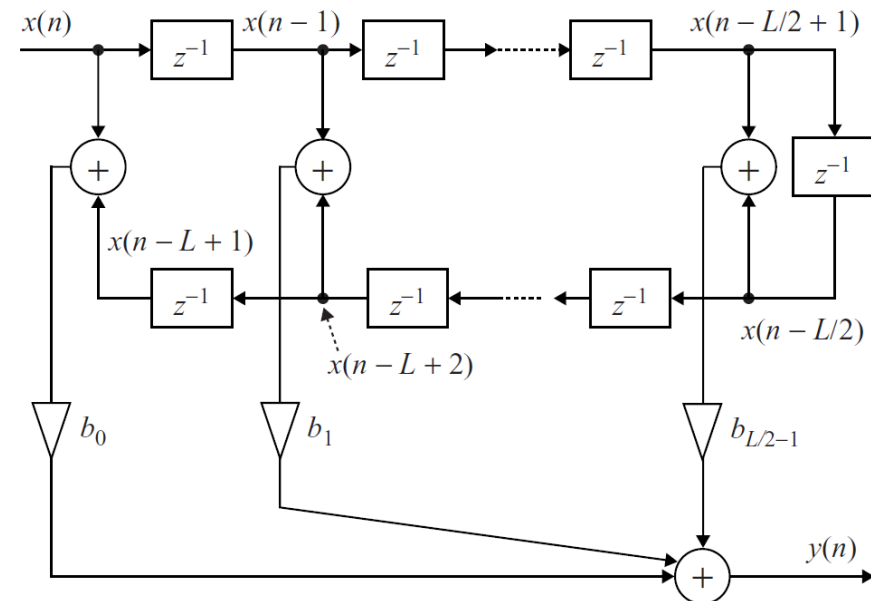
- Group delay is constant

  - $T_d(\omega) = M = \begin{cases} L/2 & L \text{ even} \\ \dfrac{L-1}{2} & L \text{ odd} \end{cases}$

Figure 3.5 Signal-flow diagram of symmetric FIR filter; L is an even number

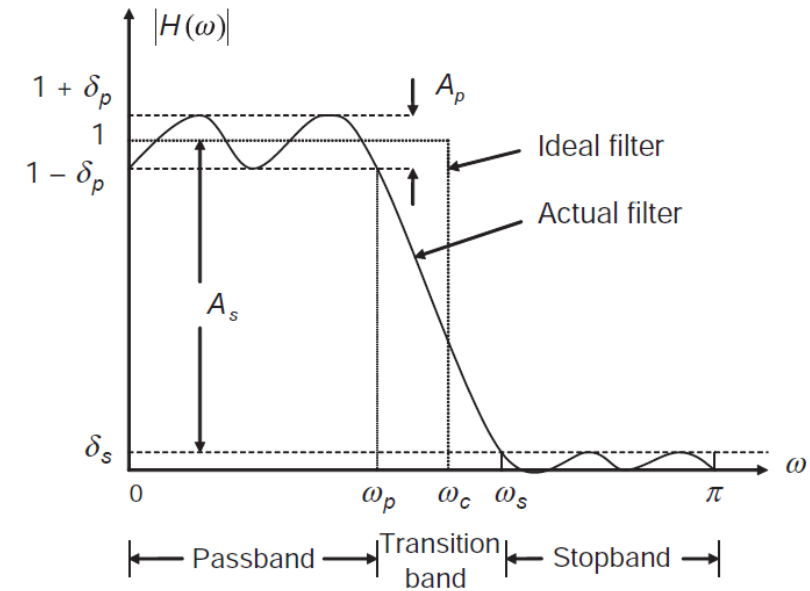- Less multiplications are required because coefficients are shared

DESIGN OF FIR FILTERS

CHAPTER 3.2

FIR FILTER DESIGN

- Determine filter coefficients to meet specifications

  - a_k, b_k coeffiecents

Figure 3.4 Magnitude response and performance measurement of lowpass filter

FOURIER SERIES (WINDOWING) METHOD

- Find a desired impulse response from desired frequency response
  - $H_d(\omega) = \sum_{n=-\infty}^{\infty} h_d(n)e^{-j\omega n}$
  - $h_d(n) = \int_{-\pi}^{\pi} H_d(\omega)e^{j\omega n}\, d\omega$
- Notice the impulse response is in general infinite
  - Can make this finite only taking some of the samples (truncate)
    - $h(n) = \begin{cases} h_d(n) & -M \leq n \leq M \\ 0 & else \end{cases}$
  - This can be made causal by shifting to the right by M samples
    - $b_l = h(l - M), \quad l = 0, \dots, 2M$
- Notice that $h(n)$ can be thought of as FS coefficients for $H_d(\omega)$
  - More coefficients, better approximation

EXAMPLES: LP FILTER WITH WINDOWING

- Example 3.5

- $H_d(\omega) = \begin{cases} 1 & |\omega| \leq \omega_c \\ 0 & else \end{cases}$

- Use FT equation or in a Table of common pairs

  - $h_d(n) = \sin\frac{\omega_c n}{\pi n} = \frac{\omega_c}{\pi} sinc\left(\frac{\omega_c n}{\pi}\right)$

- Window the impulse response and shift to make causal

  - $b_l = \begin{cases} \frac{\omega_c}{\pi} sinc\left(\frac{\omega_c(l-M)}{\pi}\right) & 0 \leq l \leq L-1 \\ 0 & else \end{cases}$

- Example 3.7

- Design a LP filter with $\omega_c = 0.4\pi$ with $L = 61$.

  - $M = \frac{L-1}{2} = 30$

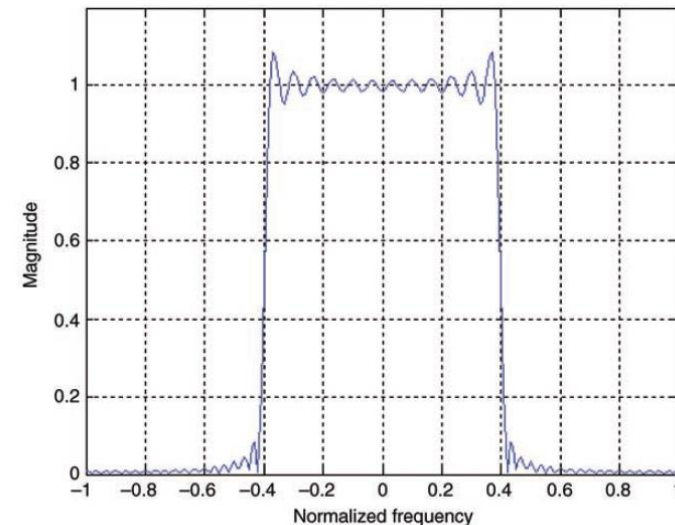 - $b_l = 0.4 sinc(0.4(l-30)), \quad l = 0, 1, \dots, 60$

Figure 3.10 Magnitude response of lowpass filter designed by Fourier series method

WINDOWING APPROXIMATION ACCURACY

- Notice the rippling effect known as Gibbs phenomenon
- Windowing is equivalent to multiplication in time domain

  - $h(n) = h_d(n)w(n)$

  - Rectangular window

    - $w(n) = \begin{cases} 1 & -M \leq n \leq M \\ 0 & else \end{cases}$

- Multiplication in time is convolution in frequency domain

  - $H(\omega) = \dfrac{1}{2\pi} H_d(\omega) * W(\omega)$

    - $W(\omega) = \dfrac{\sin\left(\frac{(2M+1)}{2}\right)\omega}{\sin\frac{\omega}{2}}$
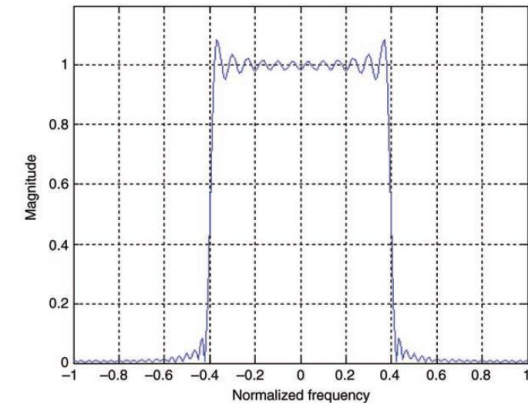

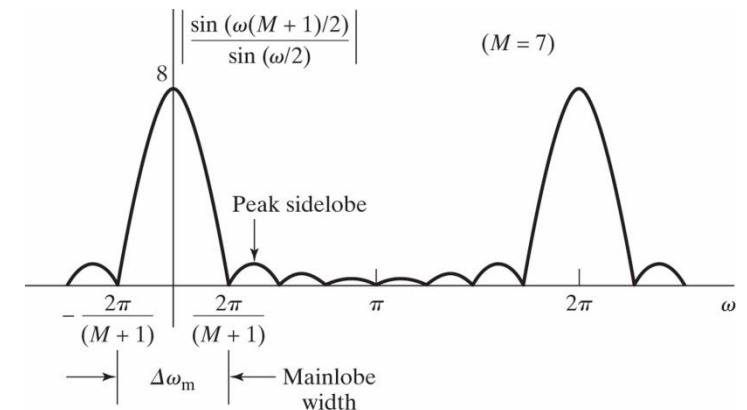Figure 3.10 Magnitude response of lowpass filter designed by Fourier series method

Figure 7.28   Magnitude of the Fourier transform of a rectangular window ($M = 7$).

WINDOWING IN FREQUENCY DOMAIN
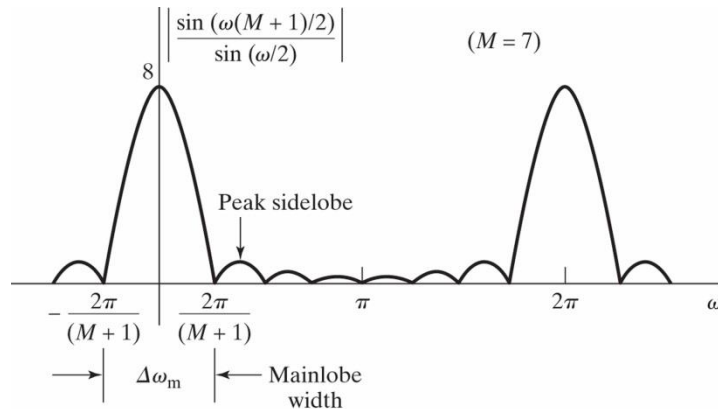
- $H(\omega) = \dfrac{1}{2\pi} H_d(\omega) * W(\omega)$

Figure 7.28 Magnitude of the Fourier transform of a rectangular window ($M = 7$).
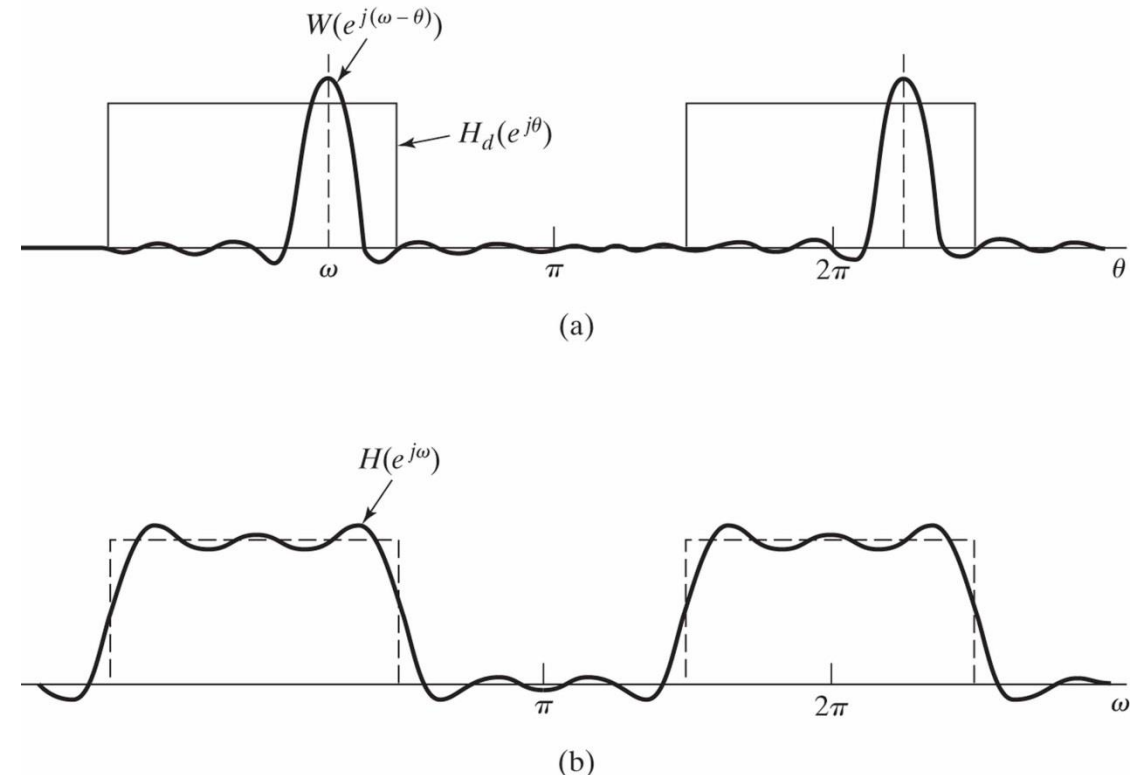

Figure 7.27 (a) Convolution process implied by truncation of the ideal impulse response. (b) Typical approximation resulting from windowing the ideal impulse response.

- Ideal frequency response is smoothed by window DTFT

RECTANGULAR WINDOW

- $W(\omega) = \dfrac{\sin\left(\frac{(2M+1)}{2}\right)\omega}{\sin\frac{\omega}{2}}$

- This window spectrum has ripples which causes ripples in $H(\omega)$ at sharp transitions
  - Can't make perfectly sharp edges

- Mainlobe – centered at $\omega = 0$
  - Care about width

- Sidelobes – all other ripples
  - Care about height

- Gibbs phenomenon can be managed by smoothing the window edges
  - Results in lower sidelobe height and increased mainlobe width
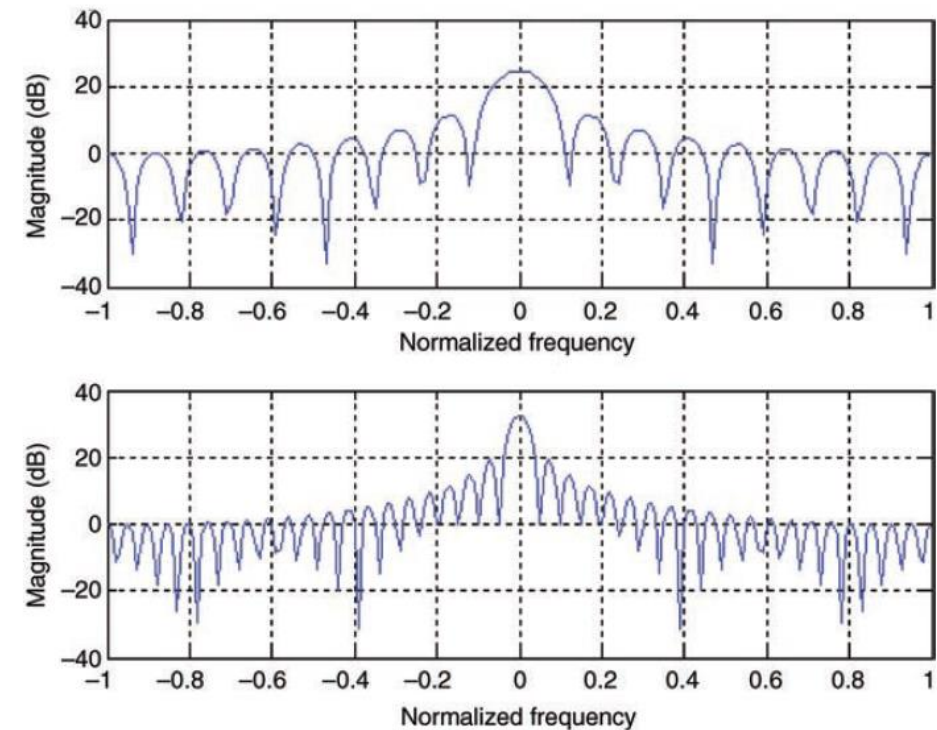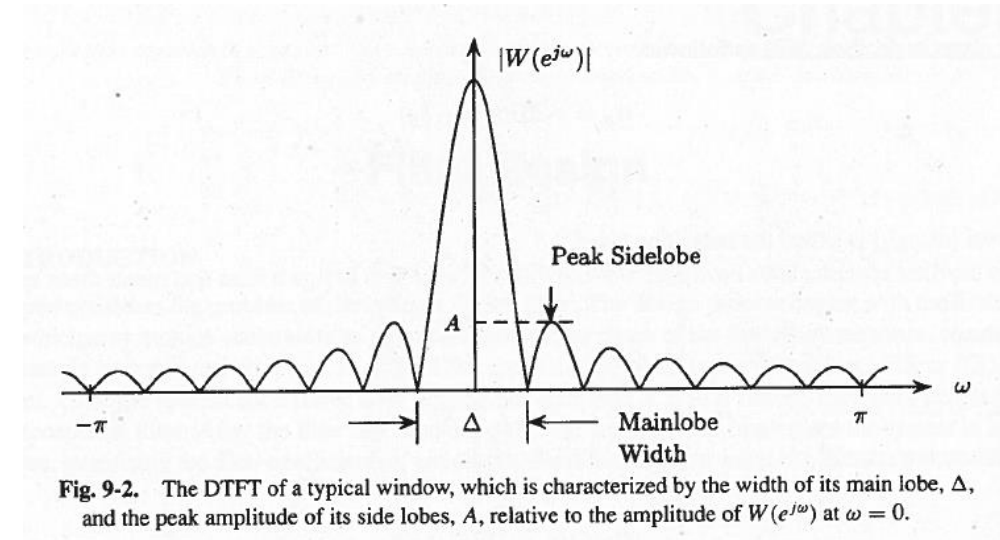 - Larger transition width at discontinuity but less ringing

Figure 3.11 Magnitude responses of the rectangular windows for $M = 8$ (top) and $M = 20$ (bottom).

WINDOWING DESIGN CONSIDERATIONS

- $H(\omega) = \frac{1}{2\pi} H_d(\omega) * W(\omega)$
  - Ideal frequency response is smoothed by window DTFT
- The quality of the FIR approximation is dependent on two factors
  - The width of the main lobe
  - The peak side-lobe amplitude
- Want narrow main-lobe width with small side lobe amplitude
  - More impulse-like
  - Cannot optimize both at the same time

  - $N\Delta_f = c$
    - N – length of filter
    - See Shaum's DSP notes

Fig. 9-2. The DTFT of a typical window, which is characterized by the width of its main lobe, Δ, and the peak amplitude of its side lobes, A, relative to the amplitude of $W(e^{j\omega})$ at $\omega = 0$.

- Increasing length of window the decreases the width of the mainlobe
  - Decreases width of the transition band
- Peak sidelobe amplitude is practically independent of length only depends on shape of window
 - Decrease in sidelobe amplitude results in greater mainlobe width

WINDOW FUNCTIONS

- Many windows have been designed to trade off mainlobe width and sidelobe height

  - All have smooth transitions at edge of window

Table 9-1   Some Common Windows

|	
Rectangular	$w(n) = \begin{cases} 1 & 0 \leq n \leq N \\ 0 & \text{else} \end{cases}$
Hanning[1]	$w(n) = \begin{cases} 0.5 - 0.5\cos\left(\dfrac{2\pi n}{N}\right) & 0 \leq n \leq N \\ 0 & \text{else} \end{cases}$
Hamming	$w(n) = \begin{cases} 0.54 - 0.46\cos\left(\dfrac{2\pi n}{N}\right) & 0 \leq n \leq N \\ 0 & \text{else} \end{cases}$
Blackman	$w(n) = \begin{cases} 0.42 - 0.5\cos\left(\dfrac{2\pi n}{N}\right) + 0.08\cos\left(\dfrac{4\pi n}{N}\right) & 0 \leq n \leq N \\ 0 & \text{else} \end{cases}$

[1]In the literature, this window is also called a Hann window or a von Hann window.
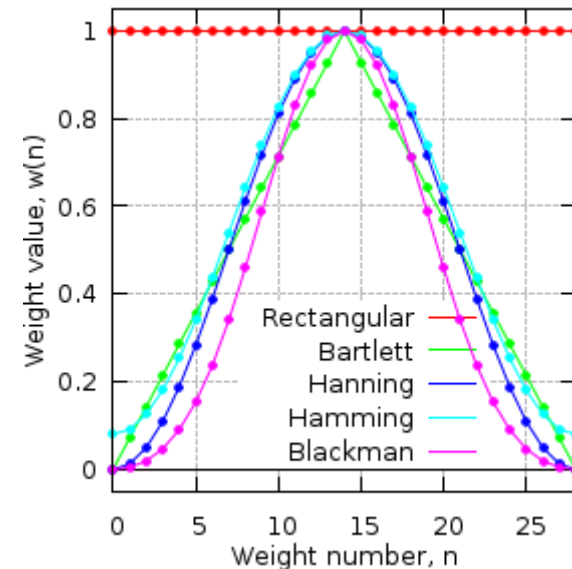
WINDOW PERFORMANCE

Table 9-2 The Peak Side-Lobe Amplitude of Some Common Windows and the Approximate Transition Width and Stopband Attenuation of an Nth-Order Low-Pass Filter Designed Using the Given Window.

| Window	Side-Lobe Amplitude (dB)	Transition Width (Δf)	Stopband Attenuation (dB)
Rectangular	−13	0.9/N	−21
Hanning	−31	3.1/N	−44
Hamming	−41	3.3/N	−53
Blackman	−57	5.5/N	−74

http://www.labbookpages.co.uk/audio/firWindowing.html
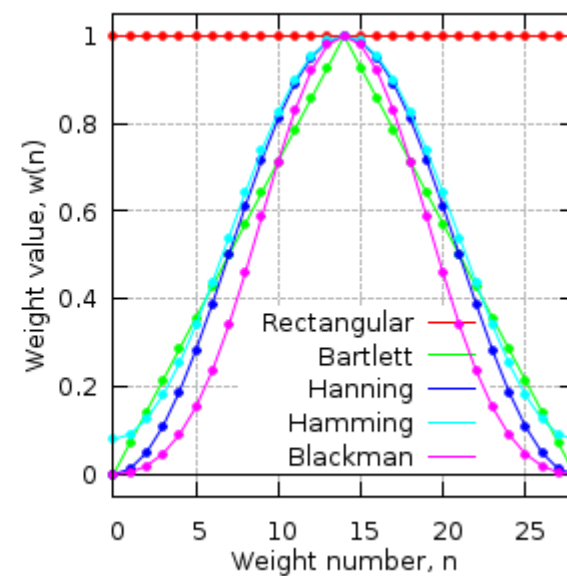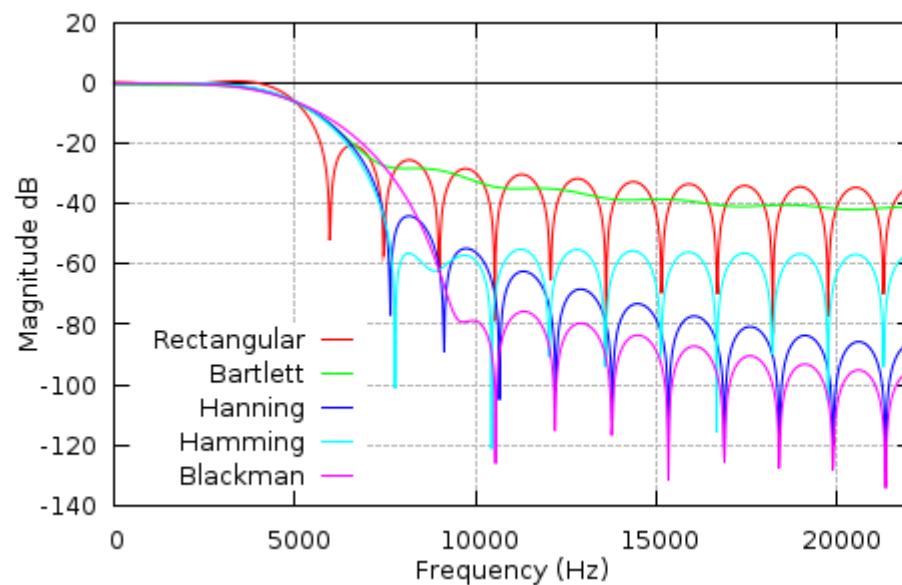
FIR DESIGN STEPS

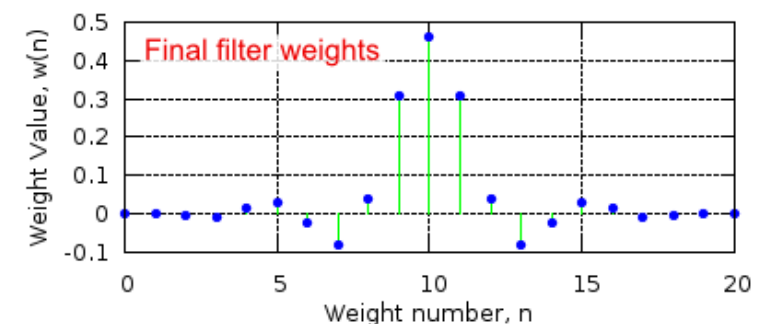- Select window type to satisfy stopband attenuation requirements

- Determine window size L based on transition width (N in Schaum's)

- Calculate window values/coefficients

- Calculate impulse response of desired filter
  - Truncate to fixed length L
  - Shift to make causal

- Calculate final filter coefficients as product of window and desired response
  - $b_l = h_d[l - M]w[l]$
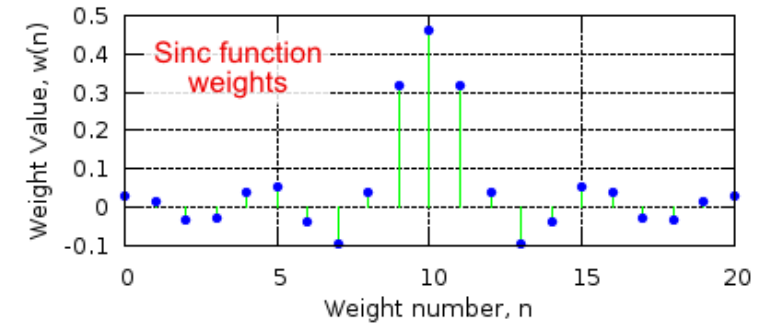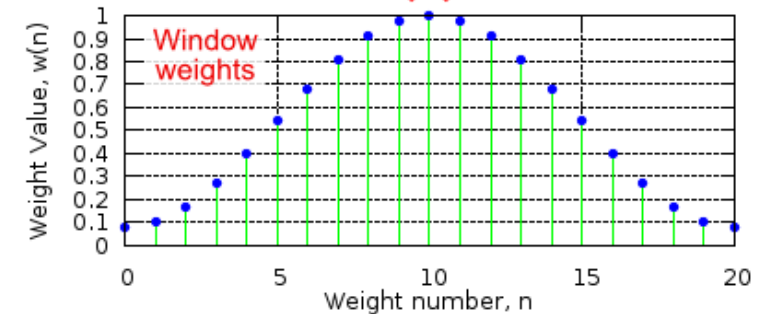
APPLICATIONS: INTERPOLATION AND DECIMATION FILTERS

CHAPTER 3.4

UPSAMPLING/INTERPOLATION

- Increase the sampling rate of a signal by factor L

- Accomplished by inserting zeros into a sequence and then lowpass filtering

- Zero insertion is upsampling

  - $x_u(n) = \begin{cases} x\left(\dfrac{n}{L}\right) & n = 0, \pm L, \pm 2L, \ldots \\ 0 & else \end{cases}$

    - Resulting signal has more samples but gaps between values

- LP filtering is interpolation to fill gaps

  - LP filter using gain L and cutoff $= \pi/L$

  - Gain of L to "spread" sample energy to neighbor zeros
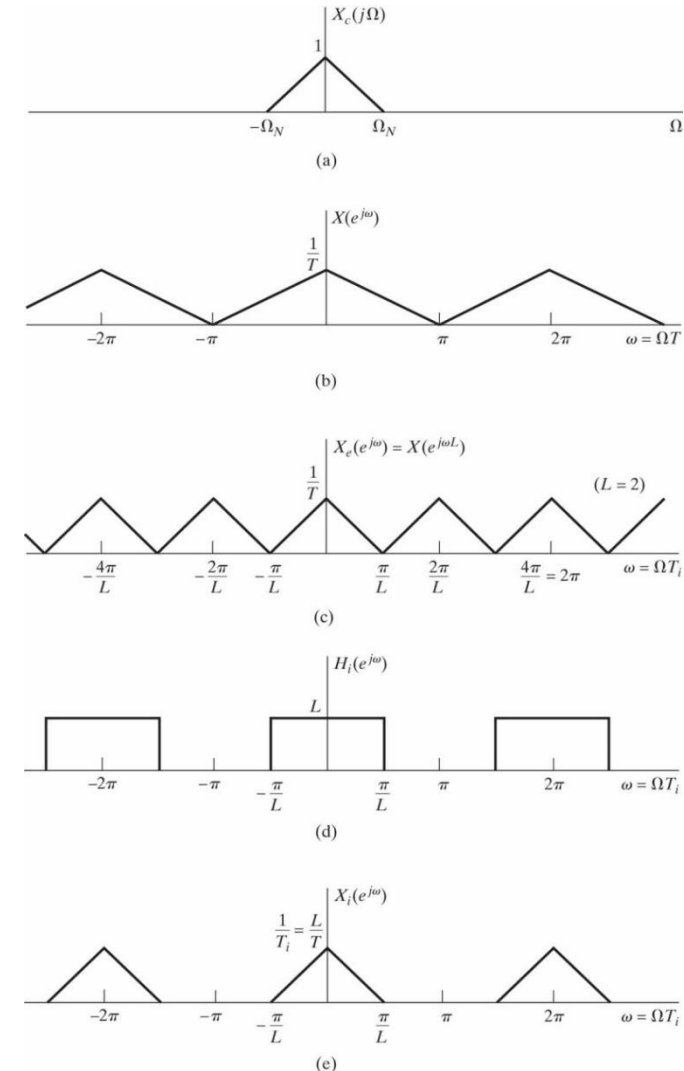

Figure 4.24   Frequency-domain illustration of interpolation.

DOWNSAMPLING/DECIMATION

- Reduce the sampling rate of a signal by factor M

- Accomplished by dropping samples

  - $x_d(n) = x(nM)$

- Remember bandwidth is controlled by sampling rate

  - Both sampling rate and bandwidth decrease by factor M

  - This may result in aliasing of the signal

- Avoid aliasing by pre-filtering signal with LP filter with cutoff $= \pi/M$ before decimation
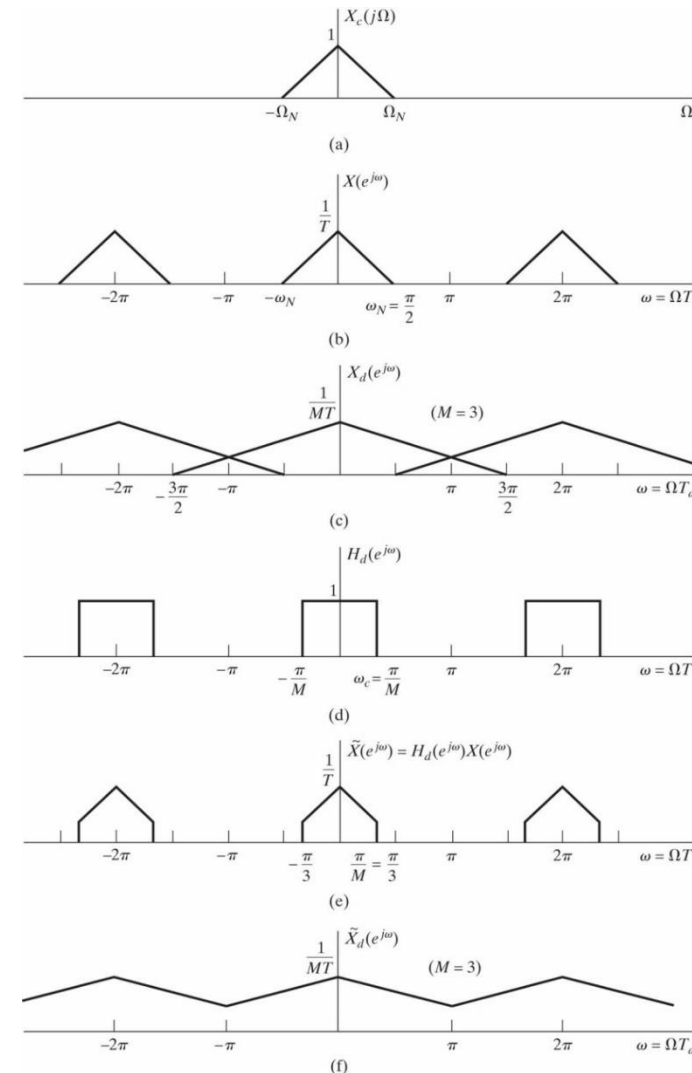

Figure 4.21   (a)–(c) Downsampling with aliasing. (d)–(f) Downsampling with prefiltering to avoid aliasing.

ARBITRARY SAMPLE RATE CONVERSION

- Conversion to arbitrary sample rate is possible
  - $R = U/D$
    - Must find appropriate upsample factor U and downsample factor D
- First perform interpolation followed by decimation
  - Minimize reduction in signal bandwidth
    - No fear of aliasing in upsample
  - Downsampling first could result in loss of high frequency content
- Can combine interpolation LP filter with LP for decimation
  - Cuttoff should be minimum of either operation
- Use Matlab interp.m, decimate.m, and upfindn.m/resample.m