Homework #1
Due W 1/24

You must turn in your code as well as output files. Please generate a report that contains the code and output in a single readable format.

1. (KLT 2.2)

   Calculate by hand, you will use Matlab later.

   Solution

   These problems are easily solved by solving for $y(n)$ in recursive form assuming $y(n) = 0$ for $n < 0$.

   (a) $y(n) = x(n) + 0.75y(n-1)$
   In recursive form the solution is

   $$y(n) = \left(\frac{3}{4}\right)^n$$

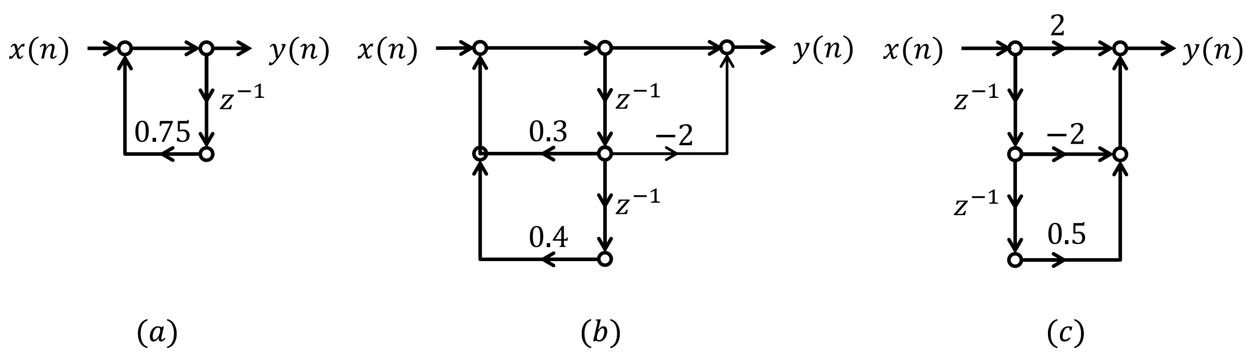
   which results in solution for $n = 0, 1, 2, 3, 4$, with input $x[n] = \delta[n]$ of
   $h(n) = [1, 0.75, 0.5625, 0.4219, 0.3164]$.

   (b) $y(n) = 0.3y(n-1) + 0.4y(n-2) + x(n) - 2x(n-1)$
   $h(n) = [1.0000, -1.7000, -0.1100, -0.7130, -0.2579]$.

   (c) $y(n) = 2x(n) - 2x(n-1) + 0.5x(n-2)$
   $h(n) = [2.0, -2.0, 0.5, 0, 0]$.

2. (KLT 2.3)

   Solution



3. (KLT 2.5)

   Solution

   The transfer function is found by taking the z-transform of both sides of the difference equation and solving for $H(z) = Y(z)/X(z)$.

   (a)

   $$H(z) = \frac{1}{1 - 0.75z^{-1}}.$$

(b)

$$H(z) = \frac{1 - 2z^{-1}}{1 - 0.3z^{-1} + 0.4z^{-2}}.$$

(c)

$$H(Z) = 2 - 2z^{-1} + 0.5z^{-2}.$$

4. (KLT 2.6)

Solution

The easiest way to find the poles and zeros is to convert $H(z)$ into polynomials of z and not z^{-1}. This is because the z^{-1} terms contribute both a pole and a zero. This can be verified in Matlab using `tf2zpk.m`. For stability, assume causal system and check if poles are within the unit circle.

(a)

$$H(z) = \frac{z}{z - 0.75}$$

Zeros: @ $z = 0$, Poles: @ $z = 3/4$

(b)

$$H(z) = \frac{z^2 - 2z}{z^2 - 0.3z + 0.4} = \frac{z(z-2)}{(z-0.8)(z+0.5)}$$

Zeros: @ $z = 0, 2$, Poles: @ $z = 0.8, -0.5$

(c)

$$H(z) = \frac{2z^2 - 2z + 0.5}{z^2} = 2\frac{z^2 - z + 0.25}{z^2} = 2\frac{(1-0.5)(1-0.5)}{z^2}$$

Zeros: 2 @ $z = 0.5$, Poles: 2 @ $z = 0$

5. (KLT 2.16)

Solution

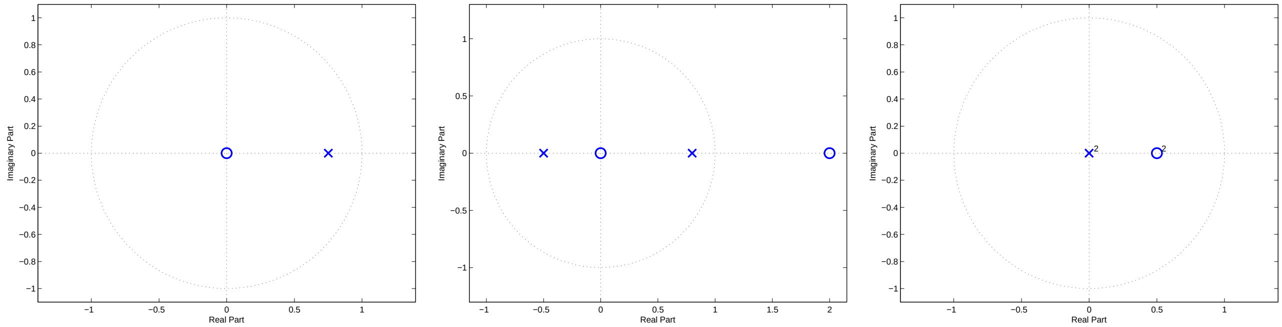
First define the digital filter coefficients

```
%a
aa = [1 -0.75];
ba = 1;

%b
ab = [1 -0.3 -0.4];
bb = [1 -2];

%c
ac = 1;
bc = [2 -2 0.5];
```

Then plot the poles and zeros using `zplane.m`



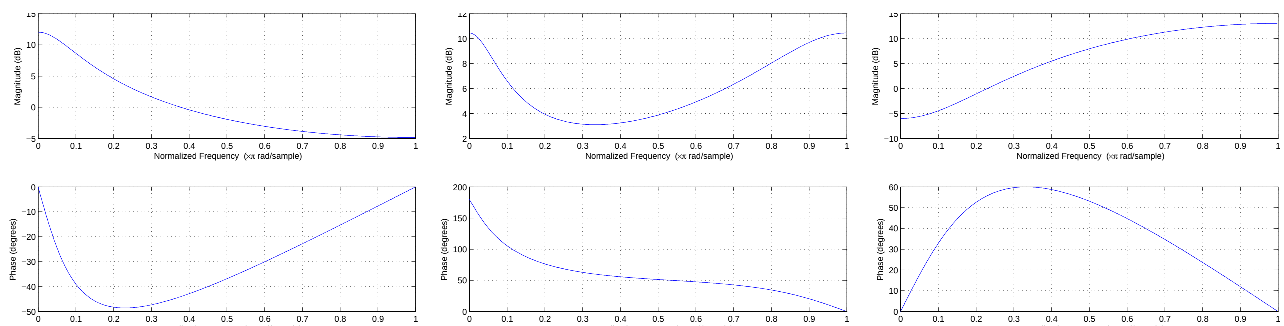
(a) (b) (c)

```
%a
h=figure;
[hz,hp,ht]=zplane(ba, aa);
%b
h=figure;
[hz,hp,ht]=zplane(bb, ab);
%c
h=figure;
[hz,hp,ht]=zplane(bc, ac);
```

6. (KLT 2.17)

Solution

Plot magnitude and phase using `freqz.m`

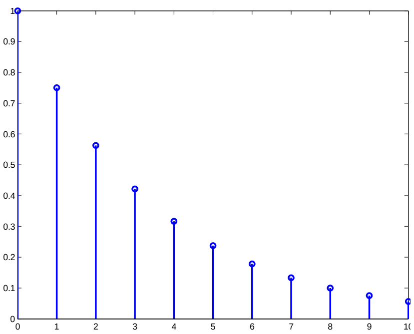


(a) (b) (c)

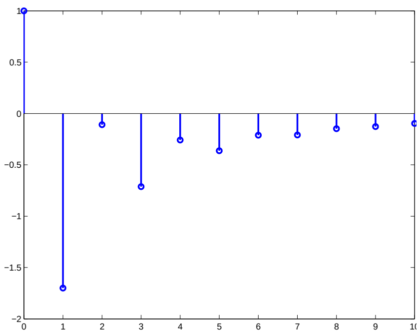
7. (KLT 2.21)

Solution

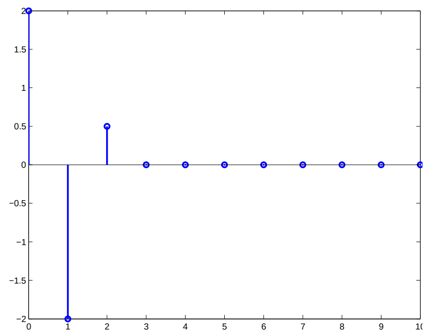
Use `filter.m` to find the impulse response.



(a)



(b)



(c)

8. (KLT 2.8)

Solution

The sampling period is

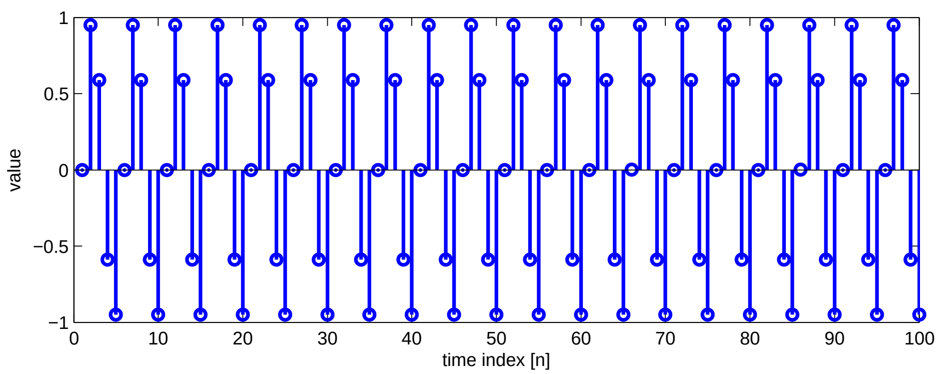
$$T = \frac{1}{f_s} = \frac{1}{10000} = 0.1\,\text{ms}.$$

The digital frequencies are

$$\omega = 2\pi f T = \frac{2\pi * 2000}{10000} = 0.4\pi \qquad\qquad F = \frac{\omega}{\pi} = 0.4$$

The total number of periods in 100 samples can be found by dividing the time in 100 samples by the period time of the sinusoid.

$$p_{100} = \frac{T * 100}{1/f_c} = \frac{10e^{-3}}{f_c} = 10^{-3} \times 2e^3 = 20.$$



9. (KLT 2.9)

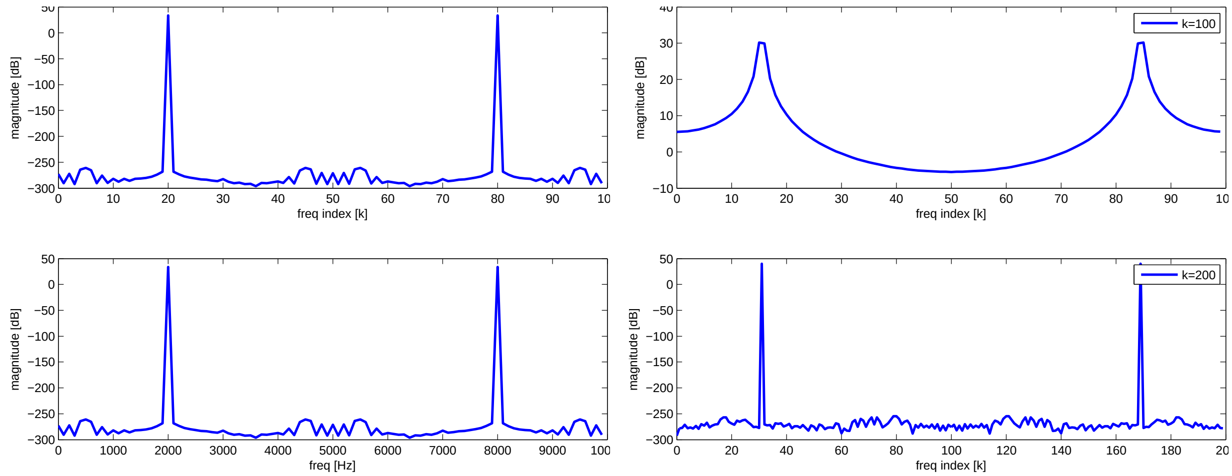
Solution

The frequency resolution is

$$\Delta_\omega = \frac{2\pi}{N} = 0.628\,\text{rad}\,\text{s}^{-1} \qquad\qquad \Delta_f = \frac{f_s}{N} = 100\,\text{Hz}$$

Since each bin is for 0.1 kHz, the peak comes at bin $k = 20$.

If the sine wave frequency is 1550 Hz, the peak will be between bins $k = 15$ and $k = 16$ instead of one distinct bin. This can be solved by increasing the size of N to have smaller bin resolution. E.g. $N' = 2N$.



(a) (b)

Figure 1: (a) 100 point DFT of 2000 Hz analog sine wave. (b) Top: 1550 Hz sine wave with $N = 100$. Bottom: 1550 Hz sine wave with $N' = 2N = 200$

10. (KLT 2.19)

Solution

The calculated SNR = 4.02 dB

```
N = 1024;

%a
A=1;
f=1000;
fs = 8000;

n=0:N-1;
w = 2*pi*f/fs;
x = A*sin(w*n);
%generate noise
v = sqrt(0.2)*randn(1,N);

xx = x+v;

%calculate SNR
Px = sum(x.^2) / length(x);
Pv = sum(v.^2) / length(v);
SNR = 10*log10(Px/Pv)
```

```
%mag spectrum
X = fft(x,N);
XX = fft(xx, N);
figure
K = length(X);

h=figure;
subplot(2,1,1)
plot(0:K-1, 20*log10(abs(XX)), 'linewidth', 2);
xlabel('freq index [k]');
ylabel('magnitude [dB]');
title('With Noise');
subplot(2,1,2)
plot(0:K-1, 20*log10(abs(X)), 'linewidth', 2);
xlabel('freq index [k]');
ylabel('magnitude [dB]');
title('Without Noise');
```