1. (OS 5.21)
   
   Solution

   (a) This is a highpass filter with cutoff frequency = $\pi/4$.
   
   $$y[n] = x[n] - x[n] * h_{lp}[n] = x[n] * (\delta[n] - h_{lp}[n])$$
   
   $$h[n] = \delta[n] - h_{lp}[n] \rightarrow H(e^{j\omega}) = 1 - H_{lp}(e^{j\omega})$$

   ![Highpass Filter Diagram]

   (b) This is a highpass filter with cutoff frequency = $3\pi/4$.
   
   The signal $x[n]$ is modulated by $(-1)^n = e^{j\pi n}$ which shifts the lp filter by $\pi$.

   ![Highpass Filter Diagram]

   (c) This is a lowpass filter with cutoff frequency = $\pi/2$.
   
   This system is a downsampled version of the lp filter. This causes compression in time
   by a factor of 2 which results in a expansion in frequency by a factor of 2. In addition,
   the gain of the filter is halved.

   ![Lowpass Filter Diagram]

   (d) This is a bandstop filter with cutoff frequencies $\pi/8 < \omega < 7\pi/8$.
   
   This is an upsampling version of the lp filter. This causes an expansion in time by a
   factor of 2 which results in a compression in frequency by a factor of 2. This time, the
   gain is unaffected.
(e) This is a lowpass filter with $\omega_c = \pi/2$.

The expander and filter can be interchanged using the relationships in Section 4.7.1. Notice this results in upsampling followed by downsampling for a unity equivalence. This results in a filter with double the bandwidth ($H_{lp}(z^{1/2}) = H_{lp}(e^{j\omega/2})$).

2. (OS 5.24)

Use Matlab to plot the magnitude and phase response of $H(z)$, $H_{\text{min}}(z)$, and $H_{\text{ap}}(z)$. Verify the magnitude response of your decomposition is valid.

**Solution**

Notice the pole/zero plot specifies a system function

$$H(z) = \frac{(z - 4)}{(z - 3)(z - 1/2)}$$

$$= \frac{z^{-1}(1 - 4z^{-1})}{(1 - 3z^{-1})(1 - 1/2z^{-1})}$$

To find the minimum phase system $H_{\text{min}}(z)$ the poles and zeros outside of the unit circle must be moved inside to conjugate reciprocal locations.

$$H_{\text{min}}(z) = K \frac{(1 - 1/4z^{-1})}{(1 - 1/3z^{-1})(1 - 1/2z^{-1})}$$

Notice that there is a zero at the origin because the order of the denominator is greater than the numerator. The factor $K$ accounts for the unknown scale in $H(z)$.

The all-pass system $H_{\text{ap}}(z)$ is selected to move the poles and zeros from the min-phase system back to the original locations.

$$H_{\text{ap}}(z) = z^{-1} \left( \frac{z^{-1} - 3}{1 - 3z^{-1}} \right) \left( \frac{z^{-1} - 1/4}{1 - 1/4z^{-1}} \right)$$

This is a unique decomposition up to a scale factor since there are no additional all-pass terms that could be included that will be canceled by the min-phase system. (Extra poles and zeros would appear outside of the unit circle).
The following Figure shows plots of $H(z), H_{\text{min}}(z)$, and $H_{\text{ap}}(z)$. Notice that $H_{\text{min}}$ is offset from $H(z)$ by a constant factor $K = 4/3$.

3. (OS 5.27)

Solution
Do not let this problem trick you with the group delay statement. This is an eigen-signal problem not the short-time FT example with mixed cosines. We know that a sinusoid into a LTI system will result in a sinusoid out (see Example 2.15)

$$x[n] = \cos(\omega_0 n + \phi) \rightarrow y[n] = |H(e^{j\omega_0})| \cos (\omega_0 n + \phi + \angle H(e^{j\omega})).$$

Therefore the solution is (a) $y[n] = \cos(0.3\pi n + \theta)$.

4. (OS 5.34)

Solution
This problem can be approached the same way as described at the end of Section 5.1 with Figures 5.4–5.6.

Notice the highest frequency component is destroyed due to the zeroing of the magnitude response. The lowest frequency component is delayed by 40 samples and the mid frequency component is delayed by 80 samples. The two remaining pulses are also scaled by approximately the same factor of 1.7. With these constraints, it is easy to determine $y_2[n]$ is the correct output.
5. (OS 5.36)

Use Matlab to plot the magnitude and phase response of \( H(z) \), \( H_{min}(z) \), and \( H_{ap}(z) \). Verify the magnitude response of your decomposition is valid.

Solution

\[
H(z) = \frac{1 + 4z^{-2}}{1 - \frac{1}{2}z^{-1} - \frac{3}{8}z^{-2}} \\
= \frac{(1 - 2jz^{-1})(1 + 2jz^{-1})}{(1 - \frac{3}{4}z^{-1})(1 + \frac{1}{2}z^{-1})}
\]

Select the AP section from zeros outside the unit circle as

\[
H_{ap}(z) = \frac{(1 - 2jz^{-1})(1 + 2jz^{-1})}{(1 - \frac{1}{2}jz^{-1})(1 + \frac{1}{2}jz^{-1})}.
\]

Then \( H_{min}(z) \) can be found by solving

\[
H_{min}(z) = \frac{H(z)}{H_{ap}(z)} = \frac{(1 - \frac{1}{2}jz^{-1})(1 + \frac{1}{2}jz^{-1})}{(1 - \frac{3}{4}z^{-1})(1 + \frac{1}{2}z^{-1})}
\]

The following Figure shows plots of \( H(z) \), \( H_{min}(z) \), and \( H_{ap}(z) \). Notice that \( H_{min} \) is offset from \( H(z) \) by a constant factor \( K = 4 \) which is the magnitude of the all-pass system.
6. (OS 5.45)

Solution

(a) Systems (B), (C), (D), (E)
IIR systems are those that have poles in general position, meaning poles in places other than just the origin or infinity.

(b) Systems (A), (F)
FIR systems have only zeros, except for poles at the origin.

(c) Systems (A), (B), (C), (E), (F)
Stable systems have ROC that contains the unit circle. Since these are causal systems, the ROC extends outward from the outermost pole which means causal and stable systems must have poles inside the unit circle.

(d) System (E)
A stable and causal system is minimum phase if its inverse system is also stable and causal. This requirement implies that all the poles and zeros must lie within the unit circle.

(e) System (A), (F)
Generalized linear-phase systems must be FIR systems. These systems have zeros that occur in reciprocal conjugate pairs or at \( z = \pm 1 \).

(f) System (C)
Constant magnitude response is the definition of an all pass system. All pass systems have poles and zeros in conjugate reciprocal pairs.

(g) System (E)
Stable and causal inverse implies minimum phase.

(h) System (F)
The shortest impulse response must arise from an FIR system. The number of samples in the impulse response is related to the number of zeros in the pole/zero plot. System (A) has 12 nonzeros samples while System (E) has only 7.

(i) Systems (A), (F)
Systems (B) and (D) cannot be lowpass since they have zeros at \( \omega = 0 (z = 1) \). System (C) is allpass therefore has constant magnitude response. System (E) has magnitude response that peaks near the pole locations. Since one is near \( \omega = \pi \) it is not lowpass.

(j) System (E)
Minimum group delay is another property of minimum phase systems. This system has minimum group delay among all systems with the same magnitude response.

7. (OS 5.55)

Solution

(A) A zero-phase system has all poles and zeros in conjugate reciprocal pairs. A generalized linear phase system is a zero phase system with additional poles or zeros at \( z = 0, \infty, 1 \) or \( -1 \).

(B) The inverse system will be stable if the ROC contains the unit circle.
(a) B
   The poles are not in conjugate reciprocal pairs so it is not generalized linear phase. $H_i(z)$ has a pole at $z = 0$ so the ROC is $0 < |z| < \infty$ and includes the unit circle.

(b) B
   The poles/zeros are not in conjugate reciprocal pairs so it is not generalized linear phase. $H_i(z)$ has poles inside the unit circle and the ROC is $|z| >$ outer-most pole.

(c) A, B
   The zeros occur in conjugate reciprocal pairs so it is generalized linear phase. Since the inverse system has poles both inside and outside the unit circle, a stable inverse does exist when the ROC is $1/4 < |z| < 4$.

(d) A
   The zeros occur in conjugate reciprocal pairs so it is generalized linear phase. Since the inverse system has poles on the unit circle, it cannot be stable.