EE361: SIGNALS AND SYSTEMS II

CH3: FOURIER SERIES HIGHLIGHTS

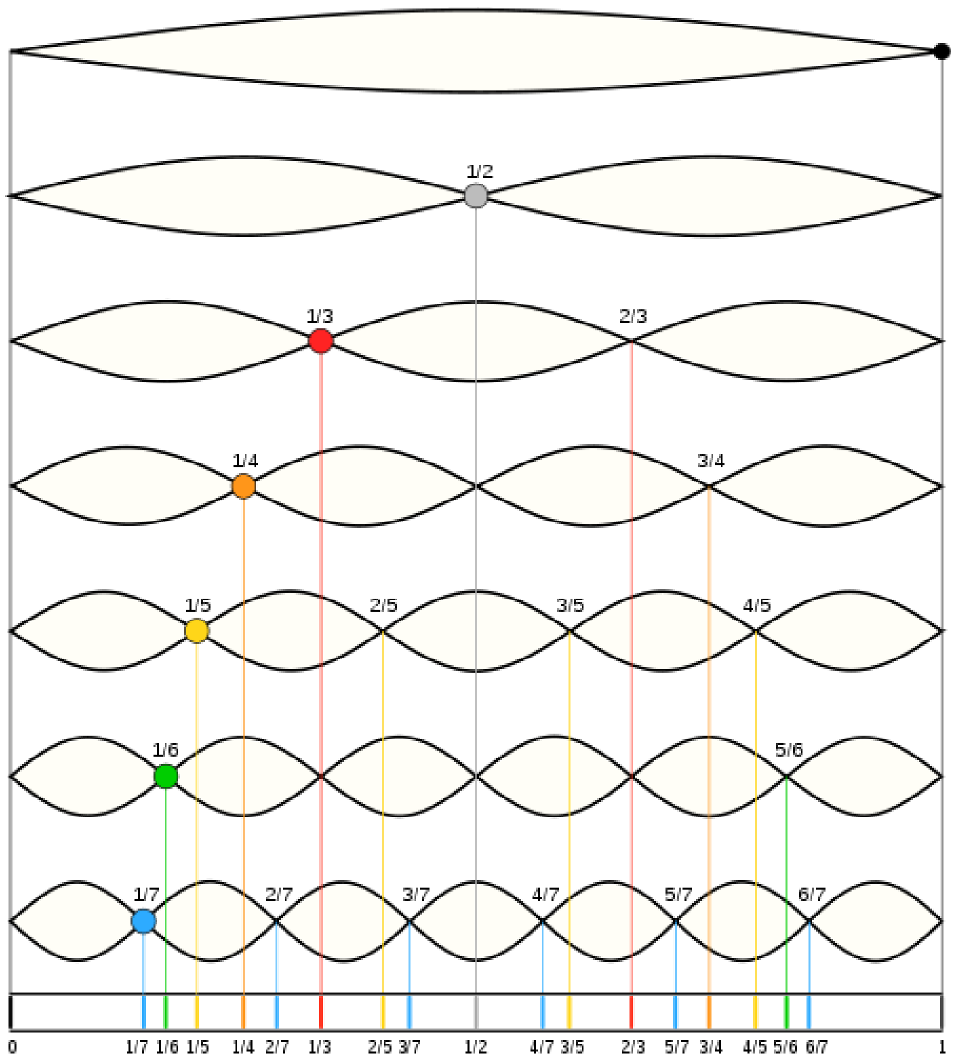
FOURIER SERIES OVERVIEW AND MOTIVATION

BIG IDEA: TRANSFORM ANALYSIS

- Make use of properties of LTI system to simplify analysis
- Represent signals as a linear combination of basic signals with two properties
  - Simple response: easy to characterize LTI system response to basic signal
  - Representation power: the set of basic signals can be use to construct a broad/useful class of signals

NORMAL MODES OF VIBRATING STRING

- When plucking a string, length is divided into integer divisions or harmonics
  - Frequency of each harmonic is an integer multiple of a "fundamental frequency"
  - Also known as the normal modes
- Any string deflection could be built out of a linear combination of "modes"



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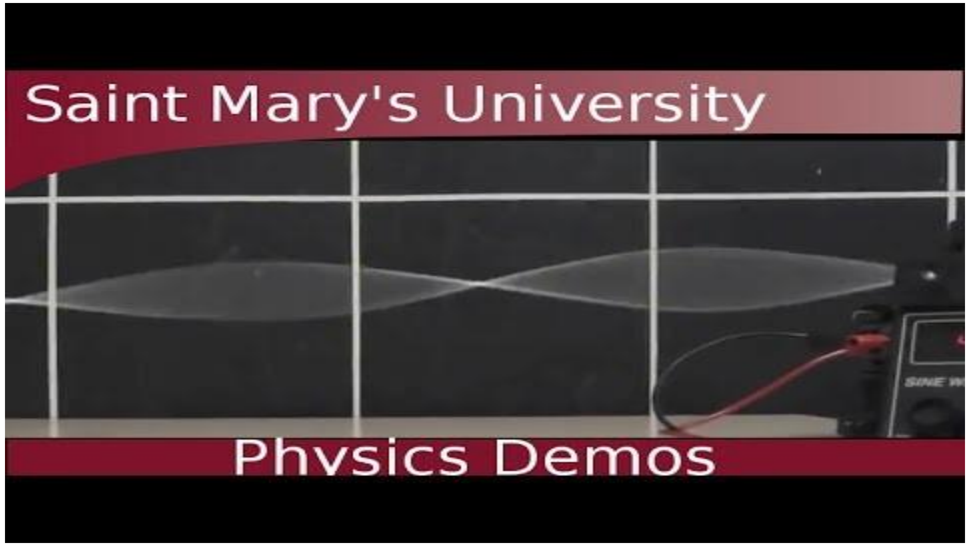


Caution: turn your sound down

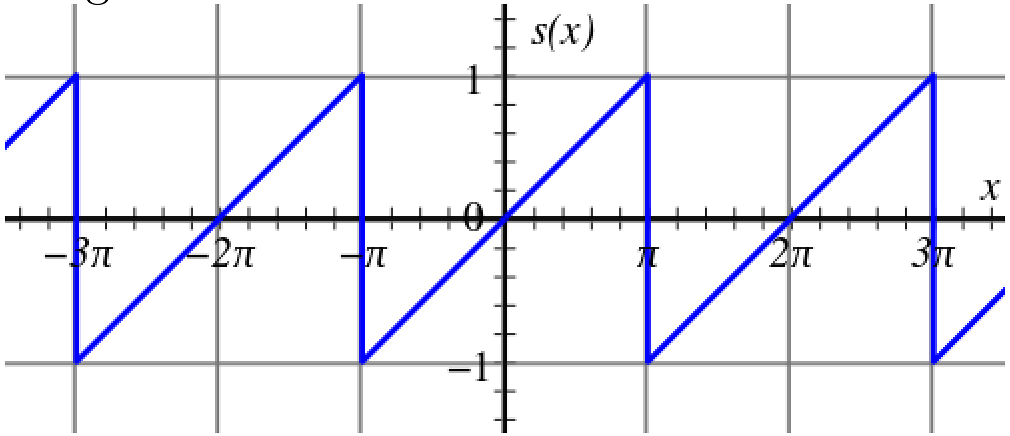
https://youtu.be/BSlw5SgUirg

FOURIER SERIES 1 SLIDE OVERVIEW

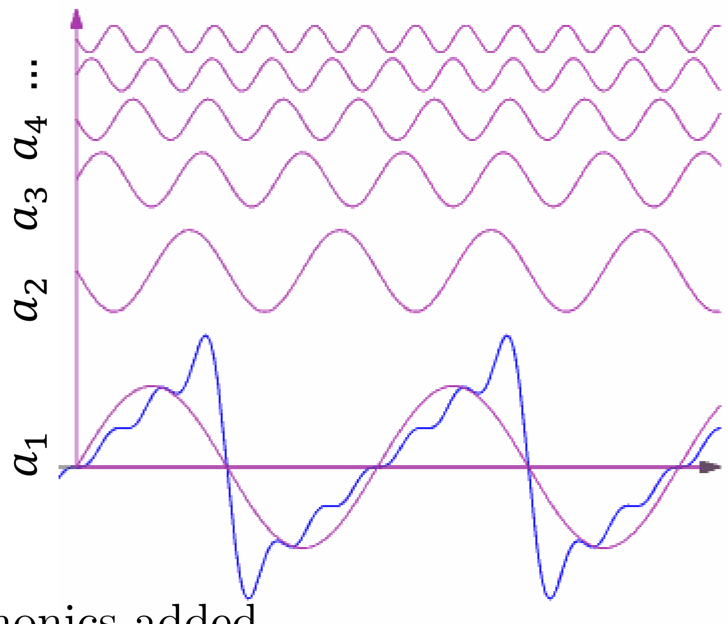
- Fourier argued that periodic signals (like the single period from a plucked string) were actually useful
  - Represent complex periodic signals
- Examples of basic periodic signals
  - Sinusoid: $x(t) = cos\omega_0 t$
  - Complex exponential: $x(t) = e^{j\omega_0 t}$
  - Fundamental frequency: ω_0
  - Fundamental period: $T = \frac{2\pi}{\omega_0}$
- Harmonically related period signals form family
  - Integer multiple of fundamental frequency
  - $\phi_k(t) = e^{jk\omega_0 t}$ for $k = 0, \pm 1, \pm 2, \ldots$
- Fourier Series is a way to represent a periodic signal as a linear combination of harmonics
  - $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$
  - a_k coefficient gives the contribution of a harmonic (periodic signal of k times frequency)

SAWTOOTH EXAMPLE

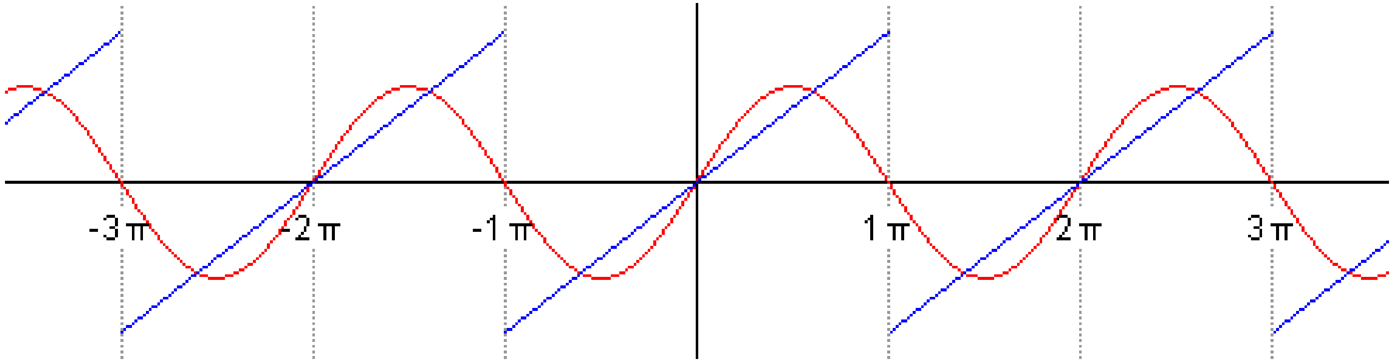
signal



Harmonics: height given by coefficient



Animation showing approximation as more harmonics added



SQUARE WAVE EXAMPLE

- Better approximation of square wave with more coefficients

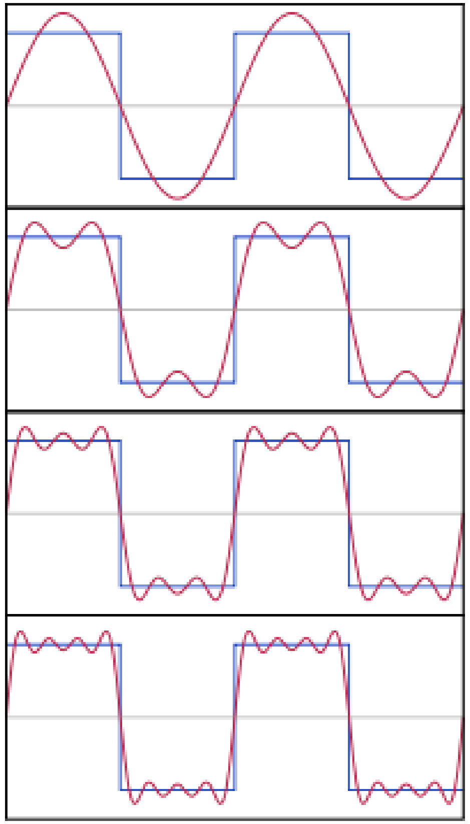
$\#a_k$ coefficients

1

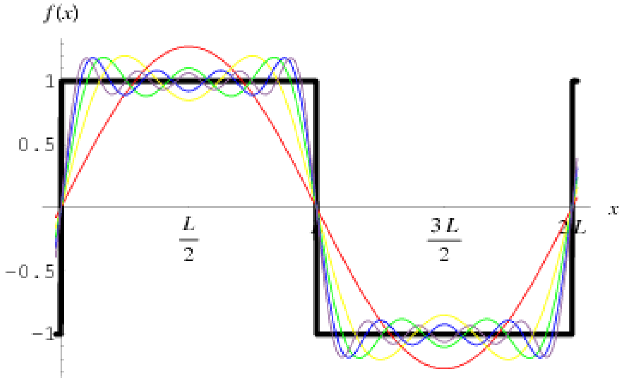
2

3

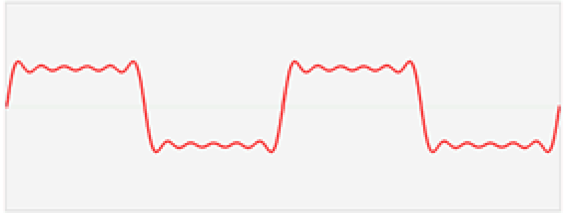
4



- Aligned approximations

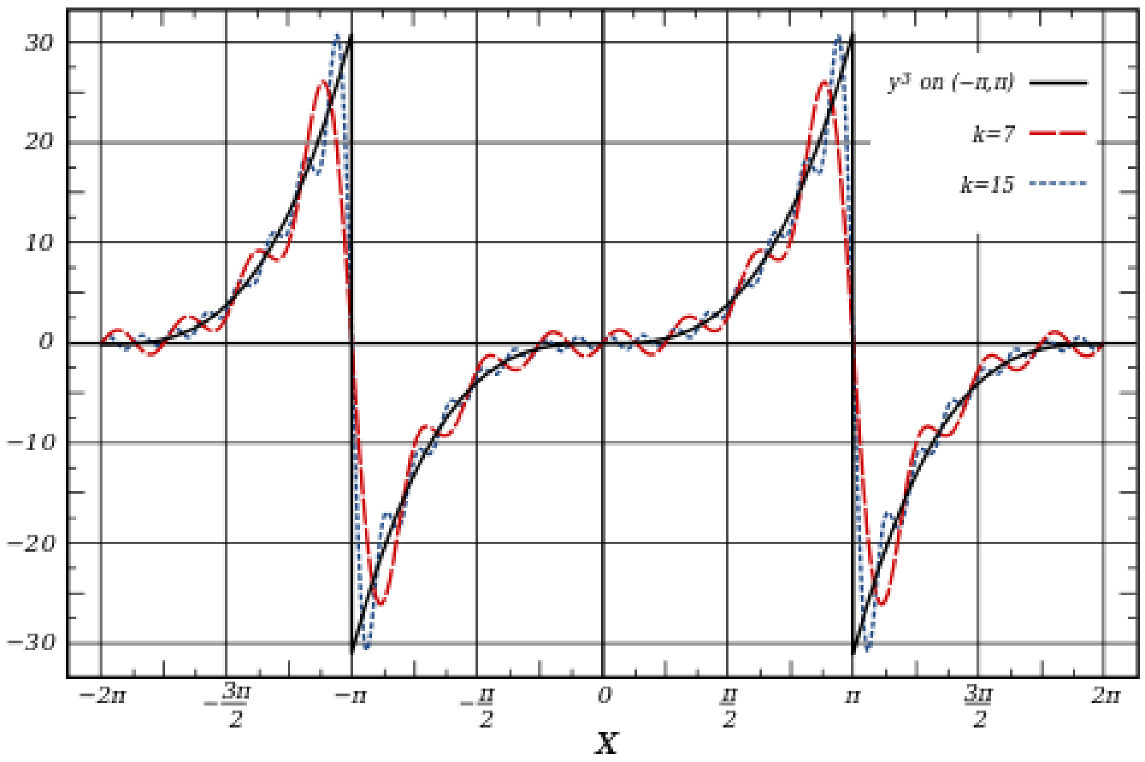


- [Animation of FS]



Note: $S(f) \sim a_k$

ARBITRARY EXAMPLES



- Interactive examples [flash (dated)][html]

RESPONSE OF LTI SYSTEMS TO COMPLEX EXPONENTIALS

CHAPTER 3.2

TRANSFORM ANALYSIS OBJECTIVE

- Need family of signals $\{x_k(t)\}$ that have 1) simple response and 2) represent a broad (useful) class of signals

1. Family of signals Simple response – every signal in family pass through LTI system with scale change

$$x_k(t) \longrightarrow \lambda_k x_k(t)$$

2. "Any" signal can be represented as a linear combination of signals in the family

$$x(t) = \sum_{k=-\infty}^{\infty} a_k x_k(t)$$

- Results in an output generated by input $x(t)$

$$x(t) \longrightarrow \sum_{k=-\infty}^{\infty} a_k \lambda_k x_k(t)$$

IMPULSE AS BASIC SIGNAL

- Previously (Ch2), we used shifted and scaled deltas
  - $\{\delta(t-t_0)\} \Longrightarrow x(t) = \int x(\tau)\delta(t-\tau)d\tau \longrightarrow y(t) = \int x(\tau)h(t-\tau)d\tau$
- Thanks to Jean Baptiste Joseph Fourier in the early 1800s we got Fourier analysis
  - Consider signal family of complex exponentials
    - $x(t) = e^{st}$ or $x[n] = z^n$, $s, z \in \mathbb{C}$

COMPLEX EXPONENTIAL AS EIGENSIGNAL

- Using the convolution
  - $e^{st} \longrightarrow H(s)e^{st}$
  - $z^n \longrightarrow H(z)z^n$

$$
\begin{aligned}
y(t) &= x(t) * h(t) \\
&= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau \\
&= \int_{-\infty}^{\infty} h(\tau)e^{s(t-\tau)}d\tau \\
&= e^{st} \underbrace{\int_{-\infty}^{\infty} h(\tau)e^{-s\tau}d\tau}_{H(s)} \\
&= \underbrace{H(s)}_{\text{eigenvalue}} \cdot \underbrace{e^{st}}_{\text{eigenfunction}}
\end{aligned}
$$

- Notice the eigenvalue $H(s)$ depends on the value of $h(t)$ and s
  - Transfer function of LTI system
  - Laplace transform of impulse response

TRANSFORM OBJECTIVE

- Simple response
  - $x(t) = e^{st} \longrightarrow y(t) = H(s)x(t)$
- Useful representation?
  - $x(t) = \sum a_k e^{s_k t} \longrightarrow y(t) = \sum a_k H(s_k) e^{s_k t}$
    - Input linear combination of complex exponentials leads to output linear combination of complex exponentials
  - Fourier suggested limiting to subclass of period complex exponentials $e^{jk\omega_0 t}, k \in \mathbb{Z}, \omega_0 \in \mathbb{R}$
  - $x(t) = \sum a_k e^{jk\omega_0 t} \longrightarrow y(t) = \sum a_k H(jk\omega_0) e^{s_k t}$
    - Periodic input leads to periodic output.
    - $H(j\omega) = H(s)|_{s=j\omega}$ is the frequency response of the system