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EE361: Signals and System II

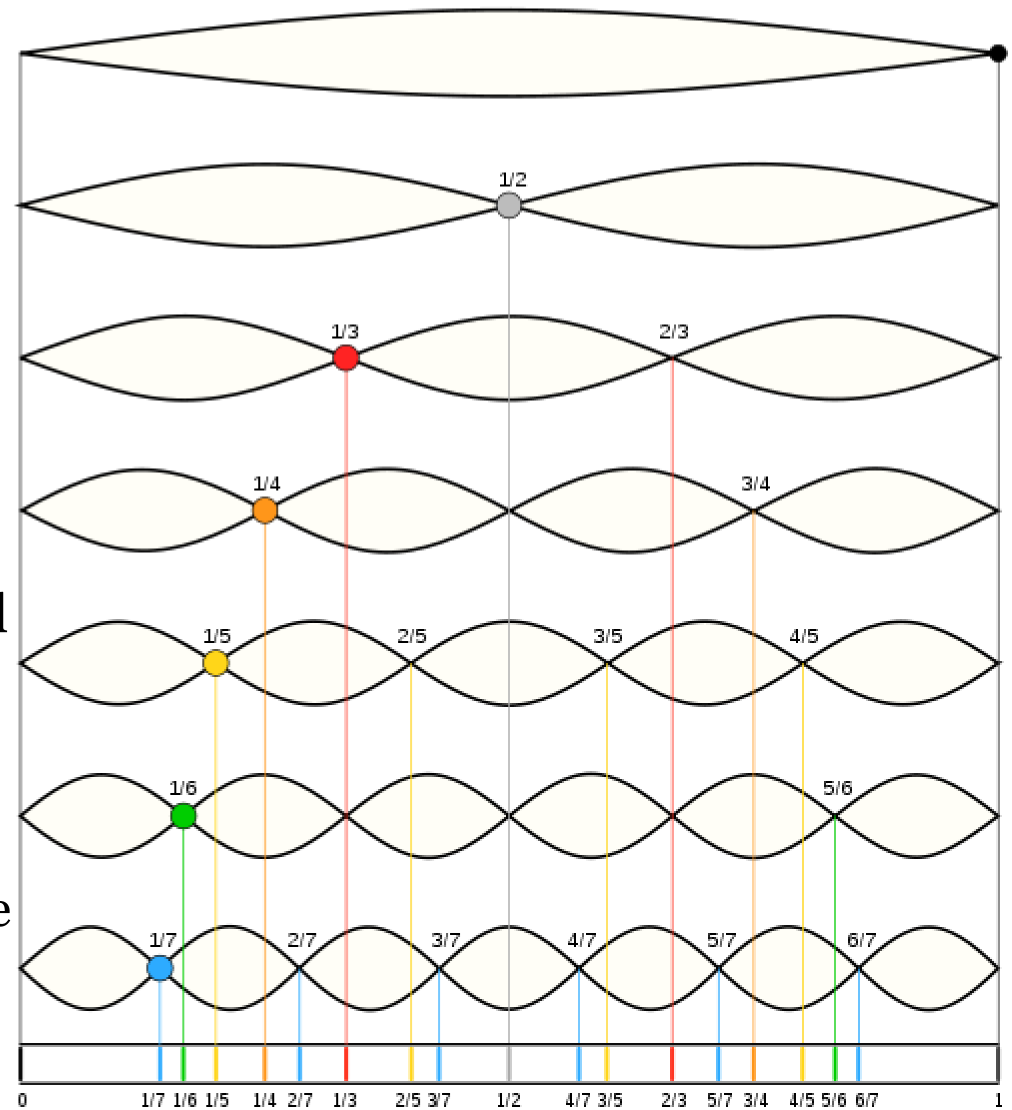
Fourier Series Highlights

Big Idea: Transform Analysis

- Make use of properties of LTI system to simplify analysis
- Represent signals as a linear combination of basic signals with two properties
  - Simple response: easy to characterize LTI system response to basic signal
  - Representation power: the set of basic signals can be use to construct a broad/useful class of signals

Normal Modes of Vibrating String

- Consider plucking a string
- Dividing the string length into integer divisions results in harmonics
  - The frequency of each harmonic is an integer multiple of a “fundamental frequency”
  - Also known as the normal modes
- It was realized that the vertical deflection at any point on the string at a given time was a linear combination of these normal modes
  - Any string deflection could be built out of a linear combination of “modes”

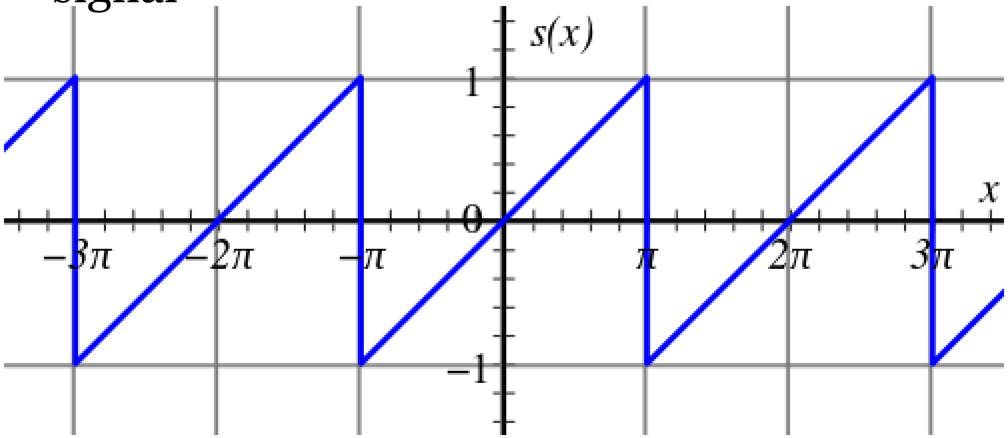


Fourier Series

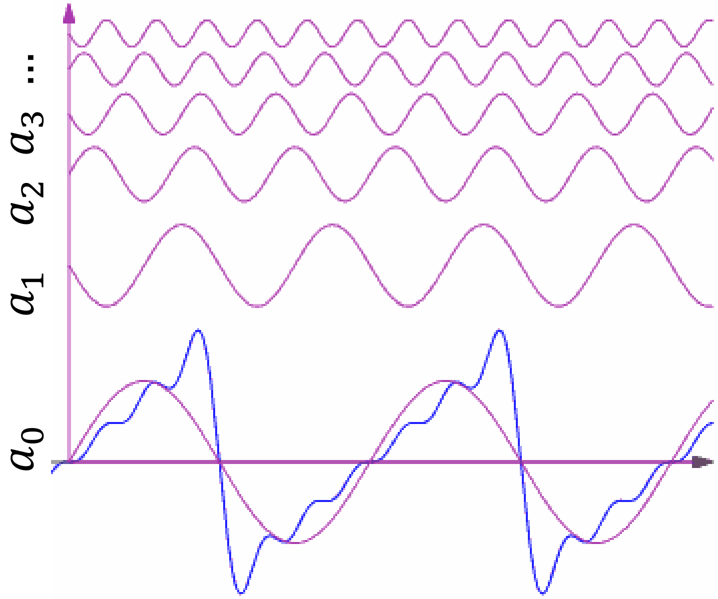
- Fourier argued that periodic signals (like the single period from a plucked string) were actually useful
  - Represent complex periodic signals
- Examples of basic periodic signals
  - Sinusoid: $x(t) = cos\omega_0 t$
  - Complex exponential: $x(t) = e^{j\omega_0 \mathrm{t}}$
  - Fundamental frequency: ω_0
  - Fundamental period: $T = \frac{2\pi}{\omega_0}$
- Harmonically related period signals form family
  - Integer multiple of fundamental frequency
  - $\phi_k(t) = e^{jk\omega_0 t}$ for $k = 0, \pm 1, \pm 2, \dots$
- Fourier Series is a way to represent a periodic signal as a linear combination of harmonics
  - $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$
  - a_k coefficient gives the contribution of a harmonic (periodic signal of k times frequency)

Sawtooth Example

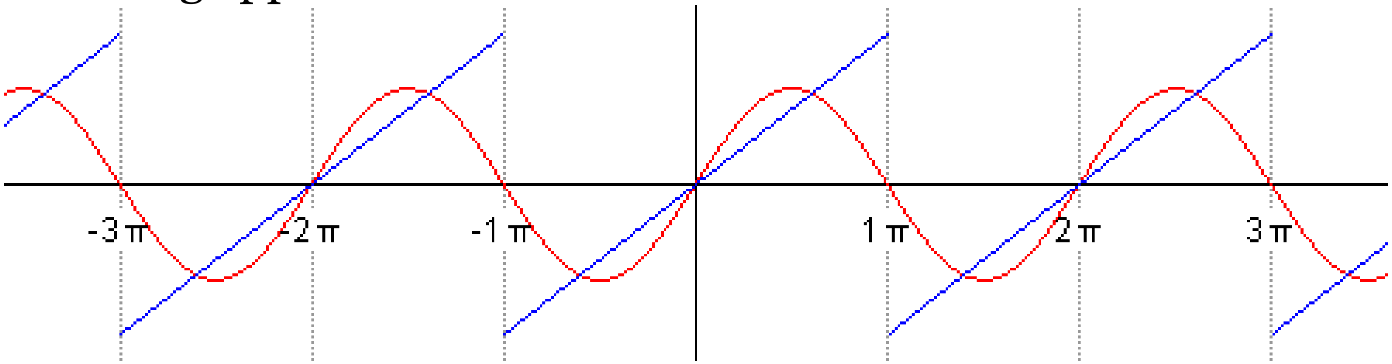
signal



Harmonics: height given by coefficient



Animation showing approximation as more harmonics added



Square Wave Example

- Better approximation of square wave with more coefficients

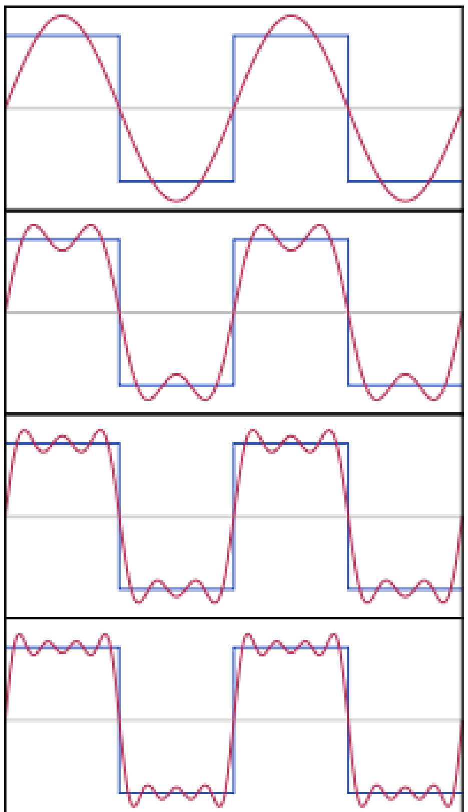
$\#\boldsymbol{a_k}$ coeff

1

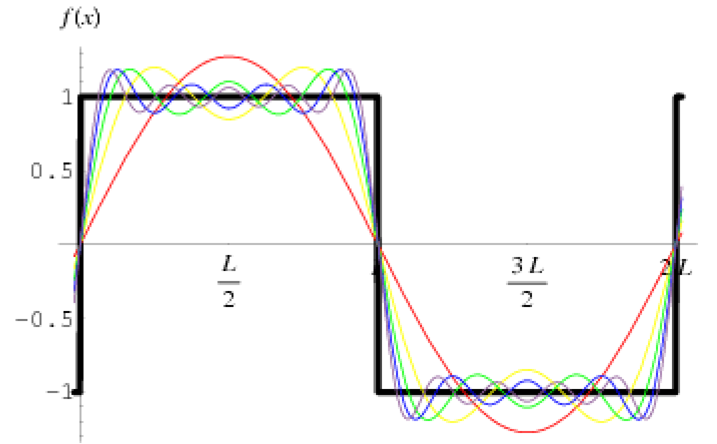
2

3

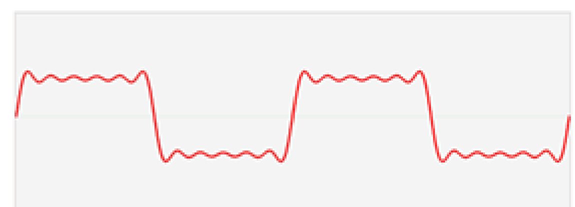
4



- Aligned approximations

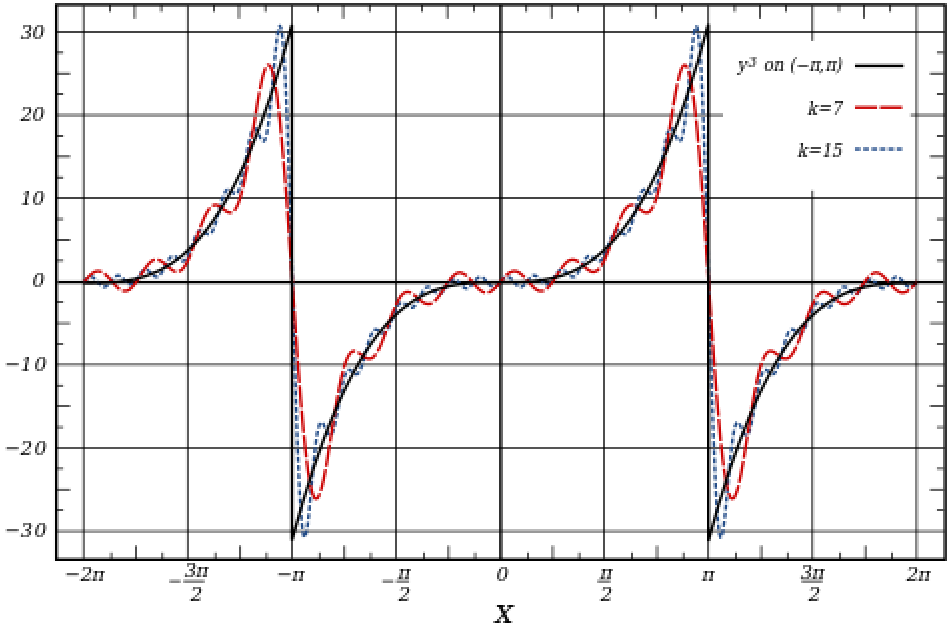


- Animation of FS



Note: $S(f) \sim a_k$

Arbitrary Examples



- Interactive examples