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EE361: Signals and System II

Probability Distributions

http://www.ee.unlv.edu/~b1morris/ee361/

Big Idea: Probability Distribution

- Assign a probability to each of the possible outcomes of a random experiment

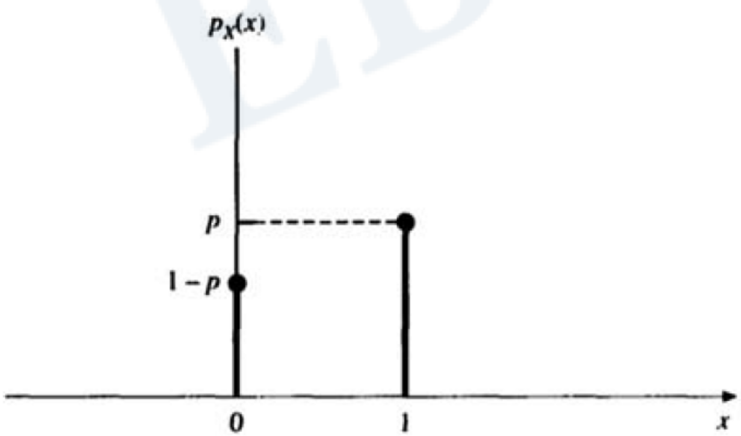
- Discrete
  - Probability mass function (pmf) – probability of each possible outcome
  - E.g. probability a roll of die will come up with a 3
- Continuous
  - Probability density function (pdf) – probability the outcome is within a range of values (interval)
  - E.g. probability that a 500 g package is between 490-510 g

Special Distributions

- Discrete
  - Bernoulli
  - Binomial
  - Geometric
  - Negative Binomial
  - Poisson
  - Uniform
- Continuous
  - Uniform
  - Exponential
  - Gamma
  - Normal

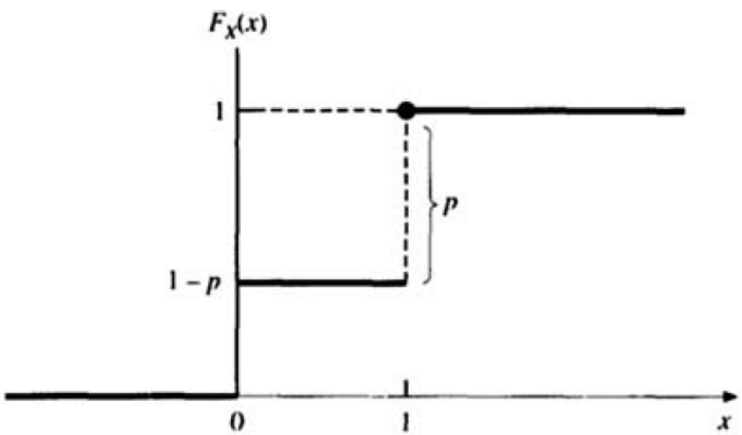
Bernoulli Distribution

- Binary RV with probability p of 1 ("success")
  - E.g. a coin flip with heads a "success" or "1" and tails a "failure" or "0"
- $p_X(k) = P(X = k) = p^k(1-p)^{1-k}$
  - $0 < p < 1$ is probability of success
  - $(1-p)$ is probability of failure



(a)

pmf

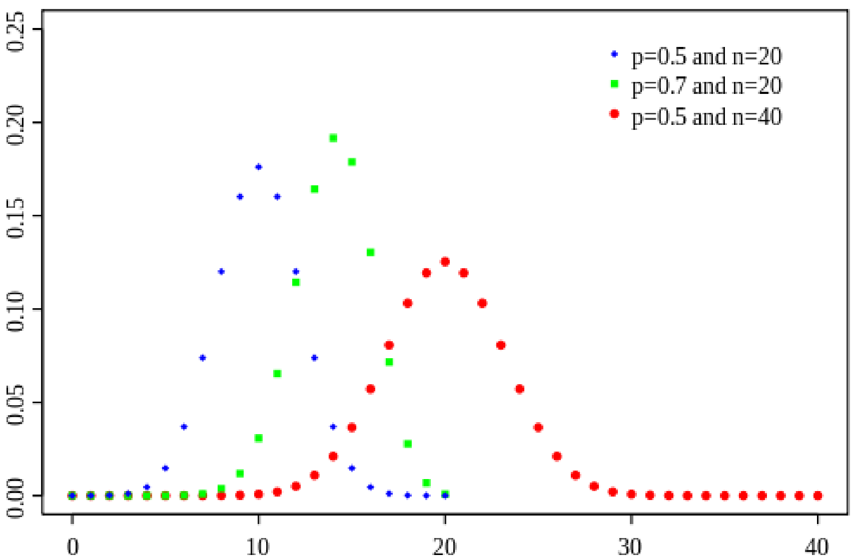


(b)

cdf

Binomial Distribution

- RV to count the number of successes with n independent Bernoulli trials
- $p_X(k) = P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$
  - $\binom{n}{k}$ - n choose k ways to get k successes



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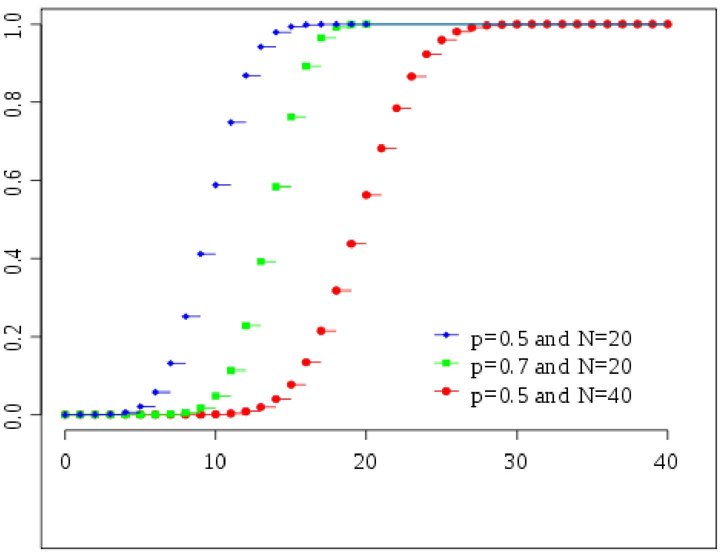

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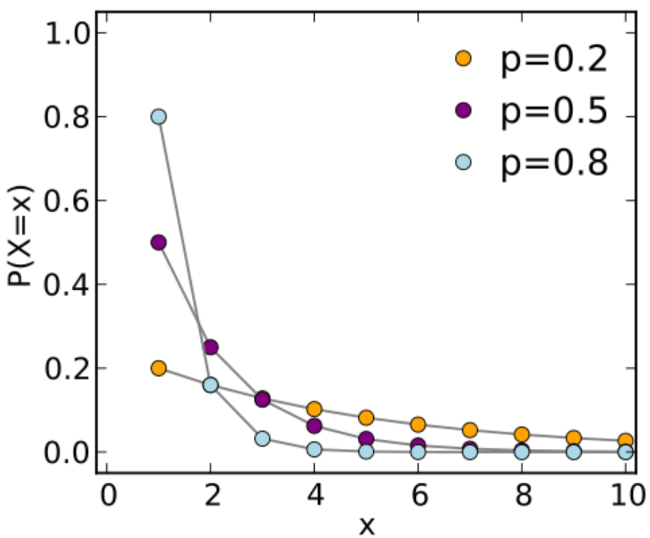
Geometric Distribution

- Sequence of Bernoulli trials observed until first success
- $p_X(x) = P(X = x) = (1 - p)^{x-1} p$



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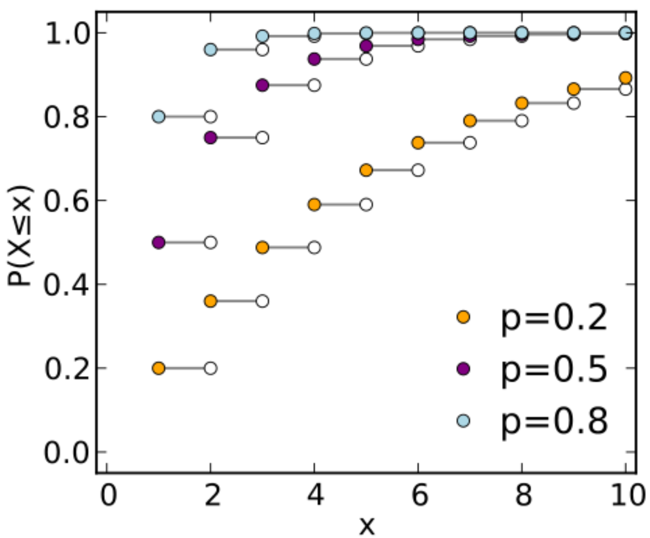

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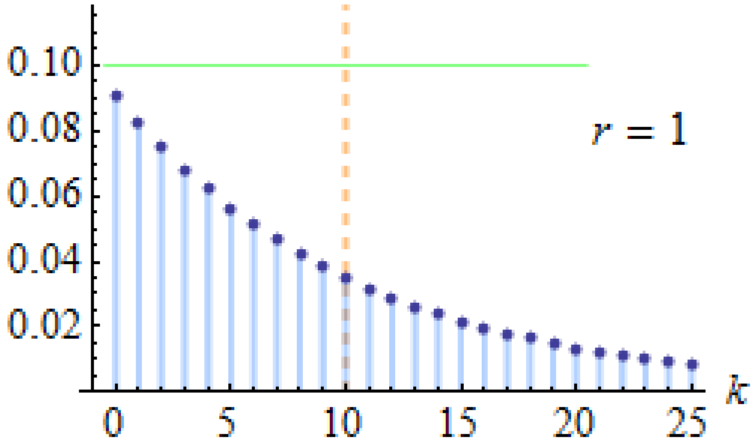
Negative Binomial Distribution

- Number of trials until kth success in sequence of Bernoulli trials
- $p_X(x) = P(X = x) = \binom{x-1}{k-1} p^k (1-p)^{x-k}$

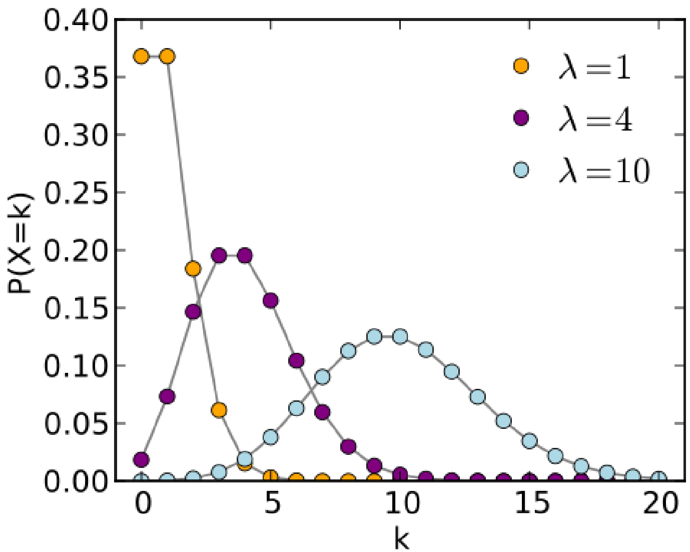


pmf

Note: parameter $r = x - k$

Poisson Distribution

- The number of events occurring in a fixed interval (time or space) given a known event average rate λ
- $p_X(k) = P(X = k) = e^{-\lambda}\frac{\lambda^k}{k!}$



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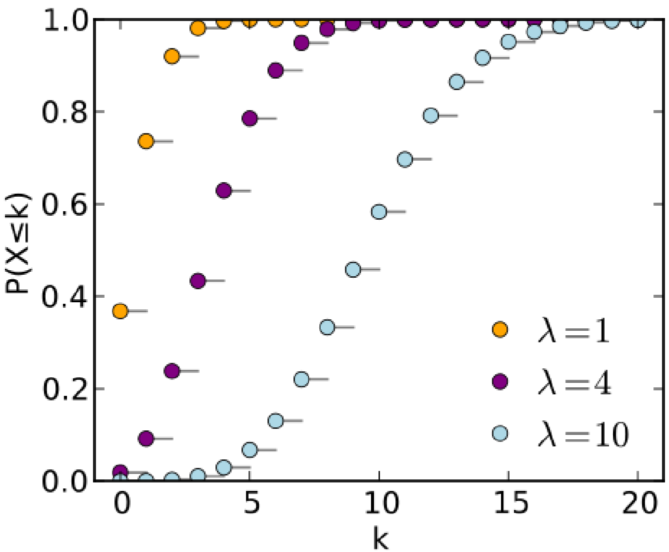

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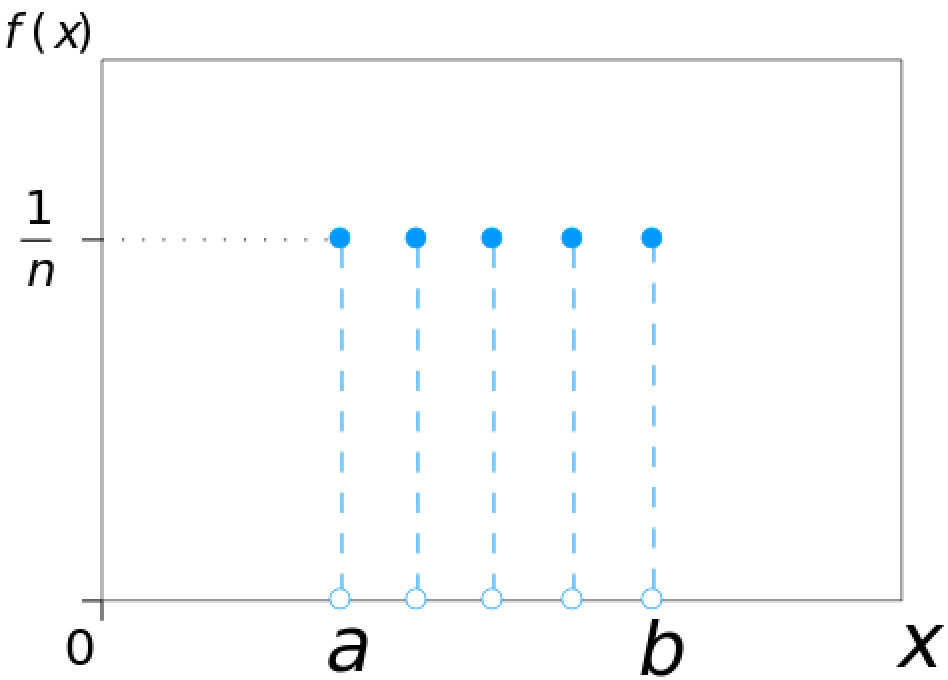
Discrete Uniform Distribution

- $p_X(x) = P(X = x) = \frac{1}{n}$



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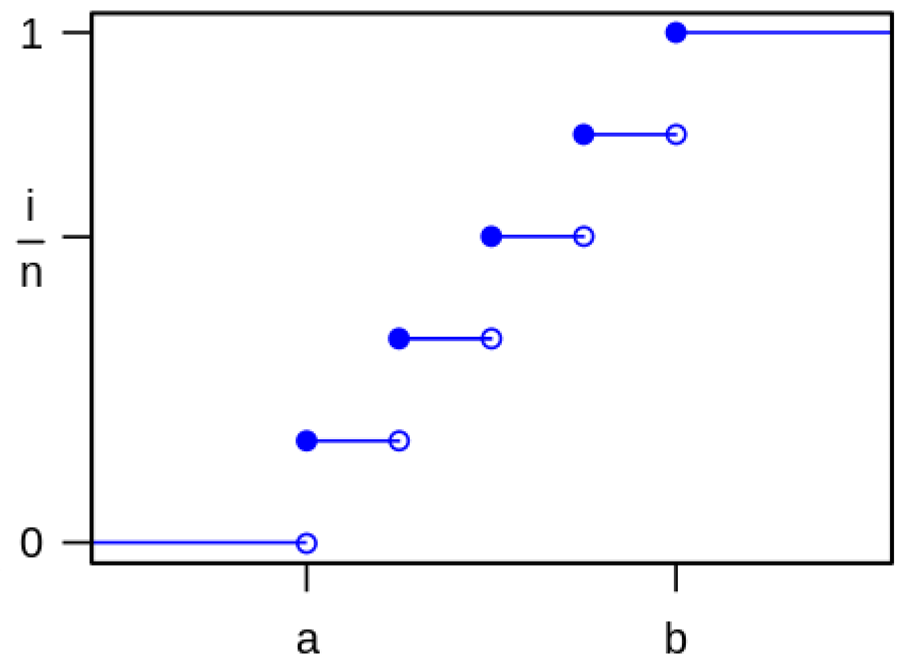

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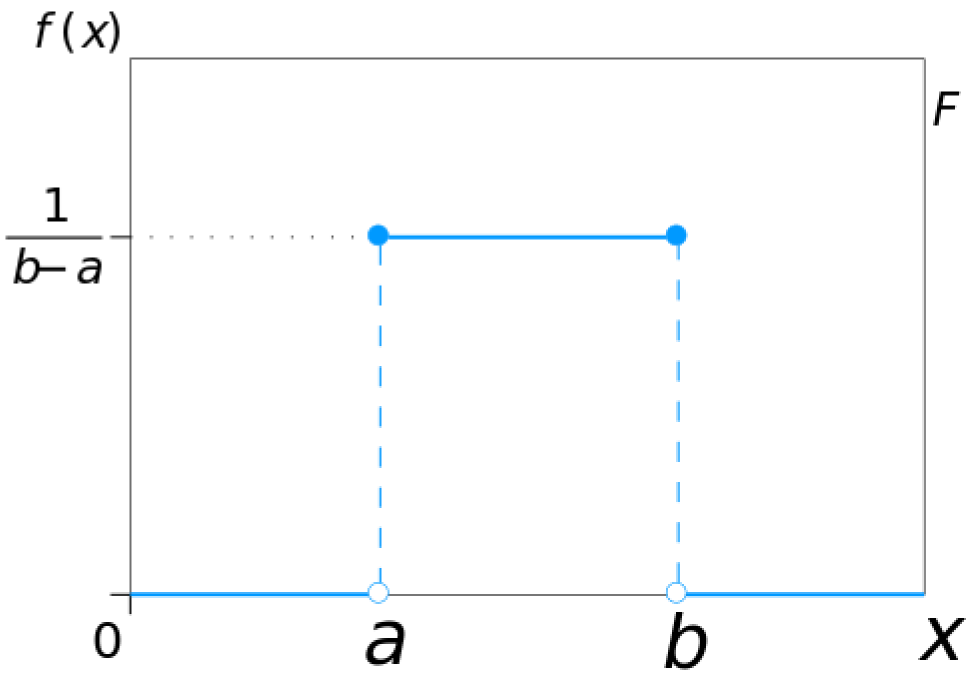
Continuous Uniform Distribution

- $f_X(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & else \end{cases}$



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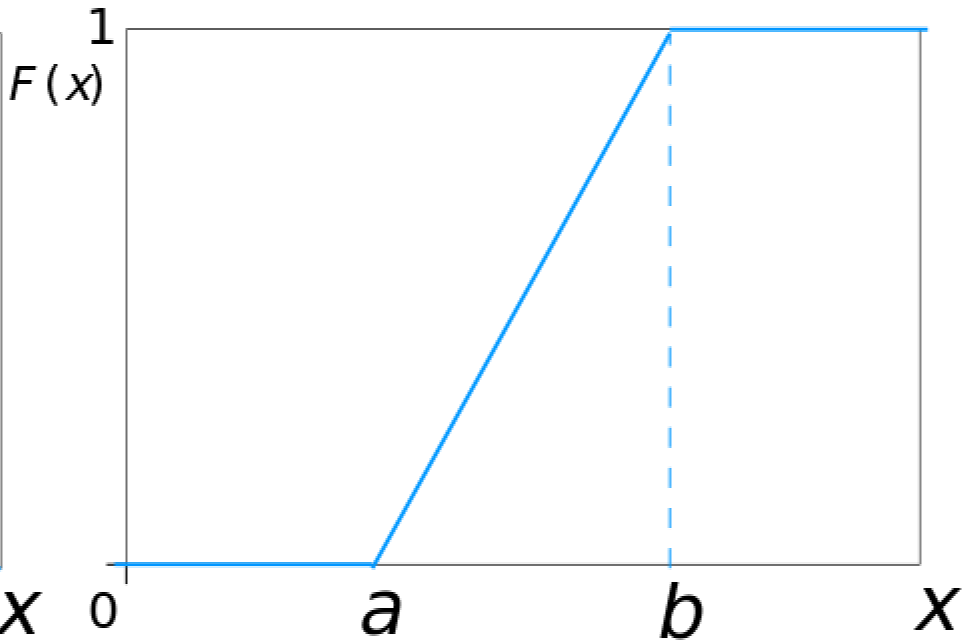

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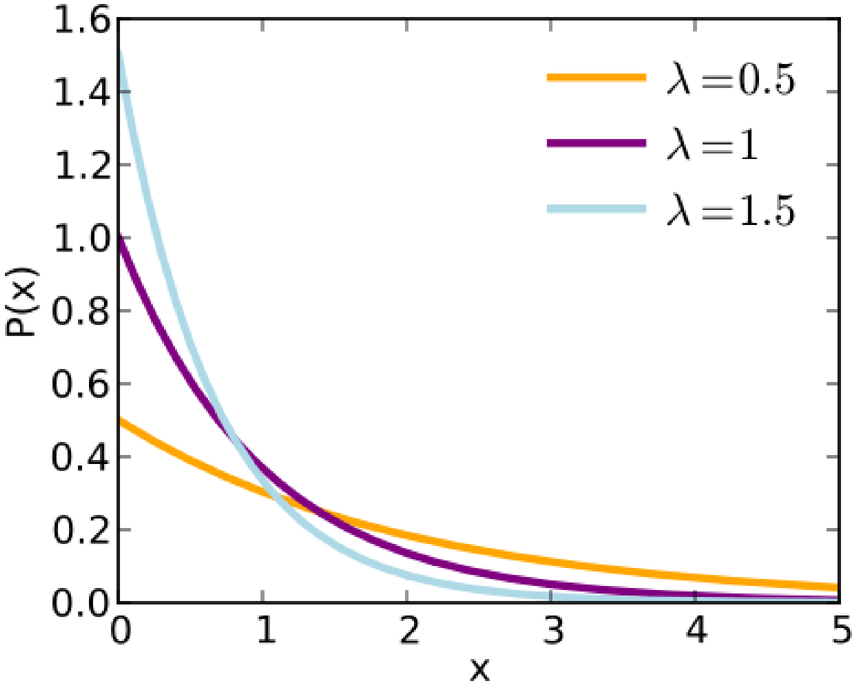
Exponential Distribution

- $f_X(x) = \lambda e^{-\lambda x} \quad x > 0$



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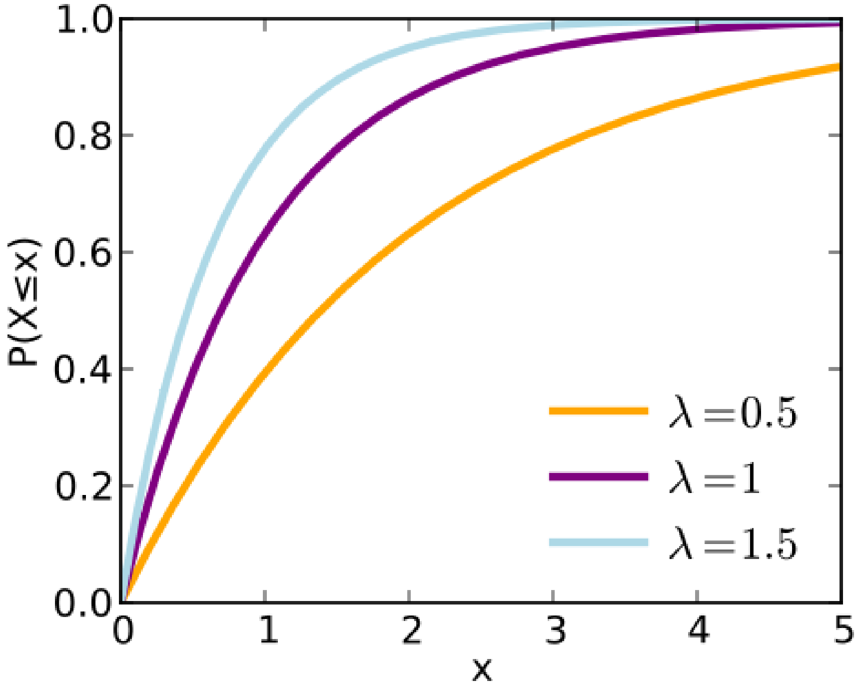

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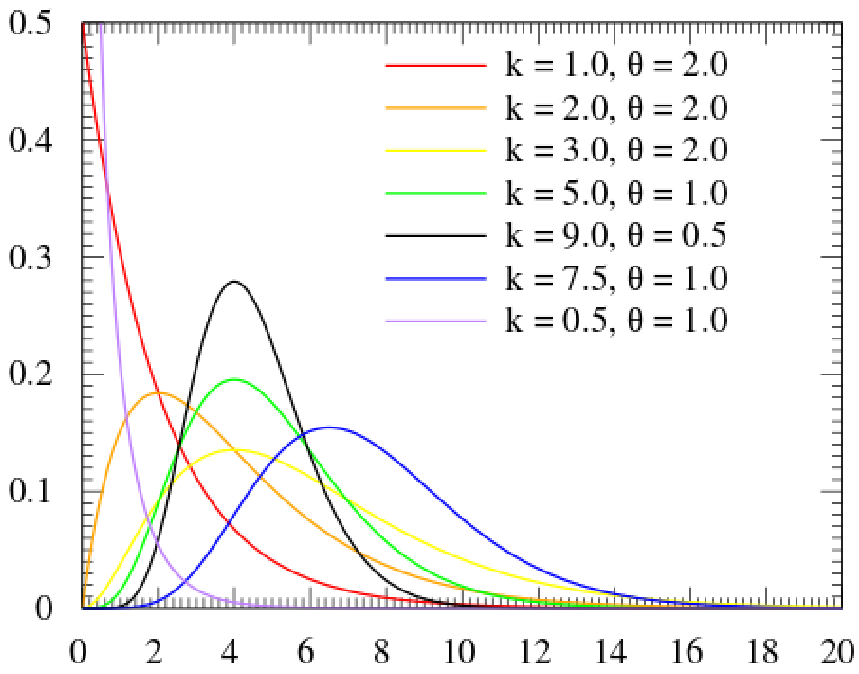
Gamma Distribution

- $f_X(x) = \dfrac{\lambda e^{-\lambda x}(\lambda x)^{\alpha-1}}{\Gamma(\alpha)} \quad x > 0$



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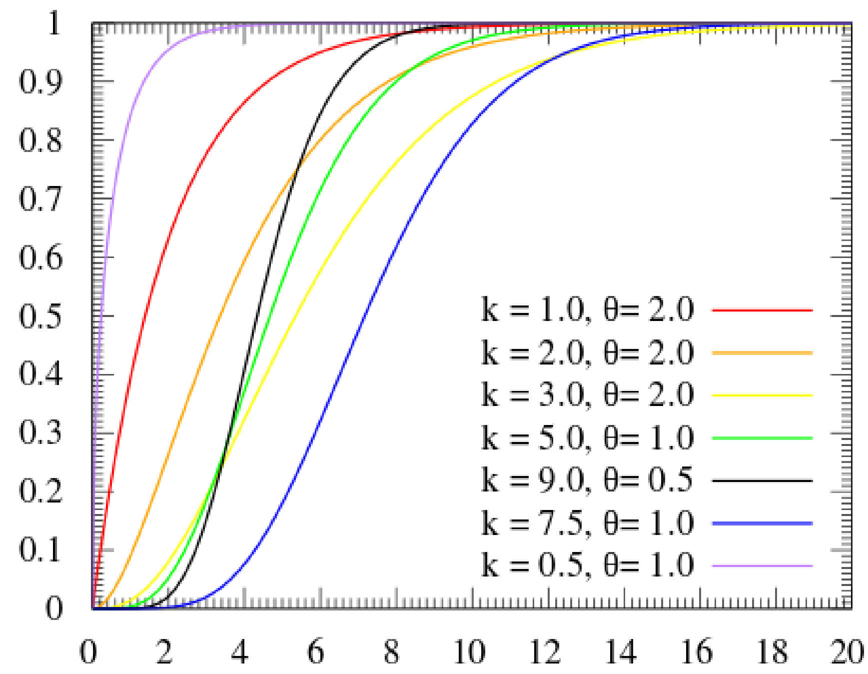

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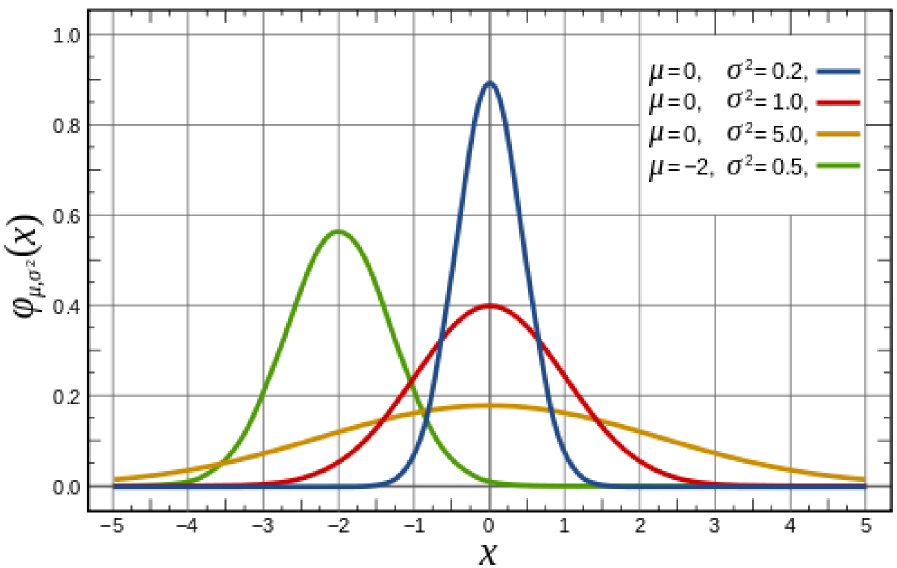
α – shape parameter

$\theta = \frac{1}{\lambda}$ rate parameter (inverse scale)

Normal (Gaussian) Distribution

- $f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$



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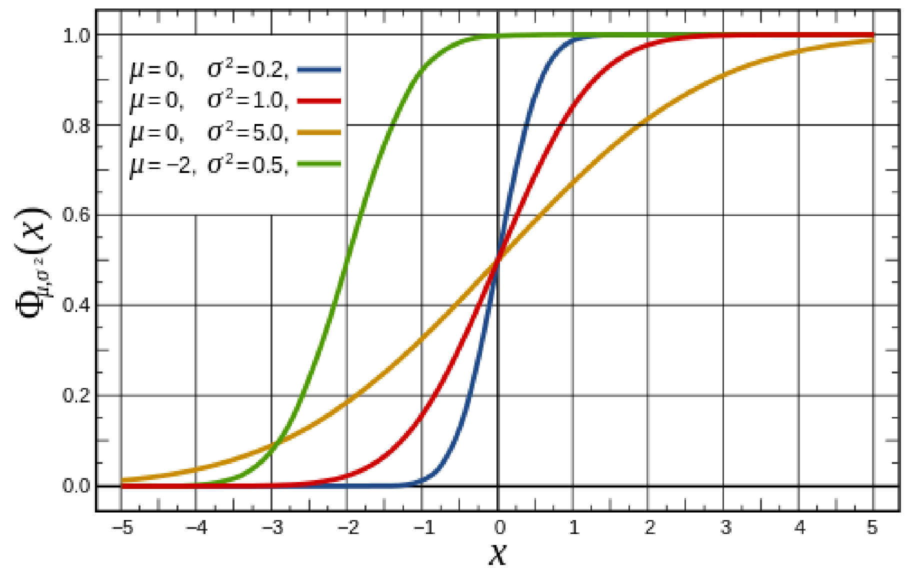

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