Homework #7
Due Th. 11/16

Note:
      OW   Oppenheim and Wilsky
      SSS   Schaum's Signals and Systems
      SPR   Schaum's Probability, Random Variables, and Random Processes

Be sure to show all your work for credit.

1. (SPR 3.55)

   Consider an experiment of tossing a fair coin three times. Let (X, Y) be a bivariate r.v.,
   where X denotes the number of heads in the first two tosses and Y the number of heads on
   the third toss.

   (a) Find the range of X.

   (b) Find the range of Y.

   (c) Find the range of (X, Y).

   (d) Find (i) $P(X \leq 2, Y \leq 1)$; (ii) $P(X \leq 1, Y \leq 1)$; and (iii) $P(X \leq 0, Y \leq 0)$.

2. (SPR 3.57)

   Let the joint pmf of (X, Y) be giving by

   $$p_{XY}(x_i, y_j) = \begin{cases} k(x_i + y_j) & x_i = 1, 2, 3; y_j = 1, 2 \\ 0 & \text{else} \end{cases}$$

   where k is a constant.

   (a) Find the value of k.

   (b) Find the marginal pmf's of X and Y.

3. (SPR 3.58)

   The joint pdf of (X, Y) is given by

   $$f_{XY}(x, y) = \begin{cases} ke^{-(x+2y)} & x > 0, y > 0 \\ 0 & \text{else} \end{cases}$$

   where k is a constant.

   (a) Find the value of k.

   (b) Find $P(X > 1, Y < 1), P(X < Y)$, and $P(X \leq 2)$.

4. (SPR 3.59)

   Let (X, Y) be a bivariate RV, where $X \sim U[0, 1]$ and Y is an exponential RV with parameter
   5, and X and Y are independent.

   (a) Find the joint pdf of (X, Y).

   (b) Find $P(Y \leq X)$.

5. (SPR 3.63)

   The joint pdf of (X, Y) is given by

   $$f_{XY}(x, y) = \begin{cases} e^{-(x+y)} & x > 0, y > 0 \\ 0 & \text{else} \end{cases}$$

   where k is a constant.

   (a) Are X and Y independent?

   (b) Find the conditional pdf's of X and Y.

6. (SPR 3.65)

   Consider the bivariate RV (X, Y) of Prob 3.14 (below)

   $$p_{XY}(x_i, y_j) = \begin{cases} k(2x_i + y_j) & x_i = 1, 2; y_j = 1, 2 \\ 0 & \text{otherwise} \end{cases}.$$

   (a) Find the mean and variance of X.

   (b) Find the mean and variance of Y.

   (c) Find the covariance of X and Y.

   (d) Find the correlation coefficient of X and Y.

7. (SPR 3.66)

   Consider a bivariate r.v. (X, Y) with joint pdf

   $$f_{XY}(x, y) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/(2\sigma^2)} \qquad -\infty < x, y < \infty.$$

   Find $P[(X, Y)|x^2 + y^2 \le a^2]$.

8. (SPR 3.68)

   The joint pdf of a bivariate RV (X, Y) is given by

   $$f_{XY}(x, y) = \frac{1}{\sqrt{3}\pi} \exp\left[-\frac{2}{3}(x^2 - xy + y^2)\right].$$

   (a) Find the means and variances of X and Y.

   (b) Find the correlation coefficient of X and Y.