Homework #8
Due Tu. 12/06

Note:

OW	Oppenheim and Wilsky
SSS	Schaum's Signals and Systems
SPR	Schaum's Probability, Random Variables, and Random Processes

Be sure to show all your work for credit.

1. (SPR 5.84)

   Consider a random process $X(t)$ defined by

   $$X(t) = Y\cos(\omega t + \Theta)$$

   where Y and Θ are independent r.v.'s and are uniformly distributed over $(-A, A)$ and $(-\pi, \pi)$ respectively.

   (a) Find the mean of $X(t)$.

   (b) Find the autocorrelation function $R_X(t, s)$ of $X(t)$.

2. (SPR 5.85)

   Suppose that a random process $X(t)$ is wide-sense stationary with autocorrelation

   $$R_X(t, t+\tau) = e^{-|\tau|/2}.$$

   (a) Find the second moment of the r.v. $X(5)$.

   (b) Find the second moment of the r.v. $X(5) - X(3)$.

3. (SPR 5.87)

   Consider the random processes

   $$X(t) = A_0\cos(\omega_0 t + \Theta) \qquad\qquad Y(t) = A_1\cos(\omega_1 t + \Phi)$$

   where $A_0, A_1, \omega_0, \omega_1$ are constants and r.v.'s Θ and Φ are independent and uniformly distributed over $(-\pi, \pi)$.

   (a) Find the cross-correlation function $R_{XY}(t, t+\tau)$ of $X(t)$ and $Y(t)$.

   (b) Repeat (a) if $\Theta = \Phi$.

4. (SPR 6.53)

   A random process $Y(t)$ is defined by

   $$Y(t) = AX(t)\cos(\omega_c t + \Theta)$$

   where A and ω_c are constants, Θ is a uniform r.v. over $(-\pi, \pi)$, and $X(t)$ is a zero-mean WSS random process with the autocorrelation function $R_X(\tau)$ and the power spectral density $S_X(\omega)$. Furthermore, $X(t)$ and Θ are independent. Show that $Y(t)$ is WSS, and find the power spectral density of $Y(t)$.

5. (SPR 6.61)

   The input $X(t)$ to the RC filter below is a white noise specified by $S_W(\omega) = \sigma^2$. Find the mean-square value of $Y(t)$.

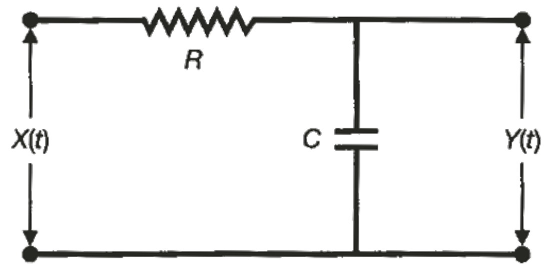

Fig. 6-7 *RC* filter.

6. (SPR 6.65)

   Suppose that the input to the discrete-time filter shown below is a discrete-time white noise with average power σ^2. Find the power spectral density of $Y[n]$.

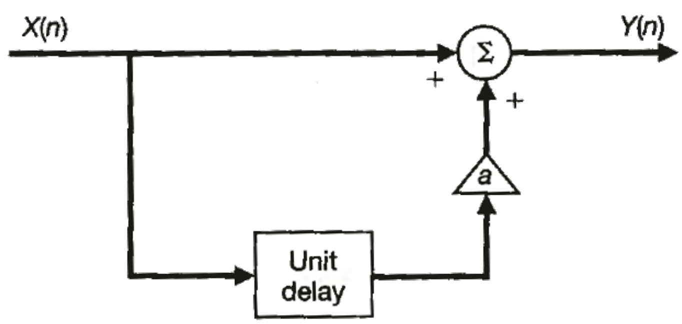

Fig. 6-9