

Homework #7
Due Tu. 11/15

Note:

OW Oppenheim and Wilsky
 SSS Schaum's Signals and Systems
 SPR Schaum's Probability, Random Variables, and Random Processes

Be sure to show all your work for credit.

1. (SPR 3.55)

Consider an experiment of tossing a fair coin three times. Let (X, Y) be a bivariate r.v., where X denotes the number of heads in the first two tosses and Y the number of heads on the third toss.

- (a) Find the range of X .
- (b) Find the range of Y .
- (c) Find the range of (X, Y) .
- (d) Find (i) $P(X \leq 2, Y \leq 1)$; (ii) $P(X \leq 1, Y \leq 1)$; and (iii) $P(X \leq 0, Y \leq 0)$.

2. (SPR 3.57)

Let the joint pmf of (X, Y) be given by

$$p_{XY}(x_i, y_j) = \begin{cases} k(x_i + y_j) & x_i = 1, 2, 3; y_j = 1, 2 \\ 0 & \text{else} \end{cases}$$

where k is a constant.

- (a) Find the value of k .
- (b) Find the marginal pmf's of X and Y .

3. (SPR 3.58)

The joint pdf of (X, Y) is given by

$$f_{XY}(x, y) = \begin{cases} ke^{-(x+2y)} & x > 0, y > 0 \\ 0 & \text{else} \end{cases}$$

where k is a constant.

- (a) Find the value of k .
- (b) Find $P(X > 1, Y < 1)$, $P(X < Y)$, and $P(X \leq 2)$.

4. (SPR 3.63)

The joint pdf of (X, Y) is given by

$$f_{XY}(x, y) = \begin{cases} e^{-(x+y)} & x > 0, y > 0 \\ 0 & \text{else} \end{cases}$$

where k is a constant.

- (a) Are X and Y independent?
- (b) Find the conditional pdf's of X and Y .

5. (SPR 3.66)

Consider a bivariate r.v. (X, Y) with joint pdf

$$f_{XY}(x, y) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/(2\sigma^2)} \quad -\infty < x, y < \infty.$$

Find $P[(X, Y)|x^2 + y^2 \leq a^2]$.

6. (SPR 4.87)

Let $Y = 2X + 3$. Find the pdf of Y if X is a uniform r.v. over $(-1, 2)$.

7. (SPR 4.92)

Let X denote the number of heads obtained when three independent tossings of a fair coin are made. Let $Y = X^2$. Find $E[Y]$.