Big Idea: Transform Analysis

- Make use of properties of LTI system to simplify analysis

- Represent signals as a linear combination of basic signals with two properties
  - Simple response: easy to characterize LTI system response to basic signal
  - Representation power: the set of basic signals can be used to construct a broad/useful class of signals
Normal Modes of Vibrating String

• Consider plucking a string
• Dividing the string length into integer divisions results in harmonics
  ▫ The frequency of each harmonic is an integer multiple of a “fundamental frequency”
  ▫ Also known as the normal modes
• It was realized that the vertical deflection at any point on the string at a given time was a linear combination of these normal modes
  ▫ Any string deflection could be built out of a linear combination of “modes”
Fourier Series

- Fourier argued that periodic signals (like the single period from a plucked string) were actually useful
  - Represent complex periodic signals
- Examples of basic periodic signals
  - Sinusoid: \( x(t) = \cos \omega_0 t \)
  - Complex exponential: \( x(t) = e^{j\omega_0 t} \)
  - Fundamental frequency: \( \omega_0 \)
  - Fundamental period: \( T = \frac{2\pi}{\omega_0} \)

- Harmonically related period signals form family
  - Integer multiple of fundamental frequency
    - \( \phi_k(t) = e^{jk\omega_0 t} \) for \( k = 0, \pm 1, \pm 2, \ldots \)

- Fourier Series is a way to represent a periodic signal as a linear combination of harmonics
  - \( x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \)
  - \( a_k \) coefficient gives the contribution of a harmonic (periodic signal of \( k \) times frequency)
Square Wave Example

- Better approximation of square wave with more coefficients
- Aligned approximations

![Animation of FS](image-url)
Sawtooth Example
Arbitrary Examples

- Interactive examples