Homework #8
Due Tu. 12/06

Note:
OW  Oppenheim and Wilsky
SSS  Schaum’s Signals and Systems
SPR  Schaum’s Probability, Random Variables, and Random Processes

Be sure to show all your work for credit.

1. (SPR 5.84)
Consider a random process \( X(t) \) defined by

\[
X(t) = Y \cos(\omega t + \Theta)
\]

where \( Y \) and \( \Theta \) are independent r.v.’s and are uniformly distributed over \((-A, A)\) and \((-\pi, \pi)\) respectively.

(a) Find the mean of \( X(t) \).
(b) Find the autocorrelation function \( R_X(t, s) \) of \( X(t) \).

2. (SPR 5.85)
Suppose that a random process \( X(t) \) is wide-sense stationary with autocorrelation

\[
R_X(t, t + \tau) = e^{-|\tau|/2}.
\]

(a) Find the second moment of the r.v. \( X(5) \).
(b) Find the second moment of the r.v. \( X(5) - X(3) \).

3. (SPR 5.87)
Consider the random processes

\[
X(t) = A_0 \cos(\omega_0 t + \Theta) \quad Y(t) = A_1 \cos(\omega_1 t + \Phi)
\]

where \( A_0, A_1, \omega_0, \omega_1 \) are constants and r.v.’s \( \Theta \) and \( \Phi \) are independent and uniformly distributed over \((-\pi, \pi)\).

(a) Find the cross-correlation function \( R_{XY}(t, t + \tau) \) of \( X(t) \) and \( Y(t) \).
(b) Repeat (a) if \( \Theta = \Phi \).

4. (SPR 6.53)
A random process \( Y(t) \) is defined by

\[
Y(t) = AX(t) \cos(\omega_c t + \Theta)
\]

where \( A \) and \( \omega_c \) are constants, \( \Theta \) is a uniform r.v. over \((-\pi, \pi)\), and \( X(t) \) is a zero-mean WSS random process with the autocorrelation function \( R_X(\tau) \) and the power spectral density \( S_X(\omega) \). Furthermore, \( X(t) \) and \( \Theta \) are independent. Show that \( Y(t) \) is WSS, and find the power spectral density of \( Y(t) \).
5. (SPR 6.61)

The input $X(t)$ to the RC filter below is a white noise specified by $S_W(\omega) = \sigma^2$. Find the mean-square value of $Y(t)$.

![RC filter diagram](image)

Fig. 6-7  RC filter.

6. (SPR 6.65)

Suppose that the input to the discrete-time filter shown below is a discrete-time white noise with average power $\sigma^2$. Find the power spectral density of $Y[n]$.

![Discrete-time filter diagram](image)

Fig. 6-9