Homework #5
Due Th. 10/26

Note:
OW Oppenheim and Wilsky
SSS Schaum’s Signals and Systems
SPR Schaum’s Probability, Random Variables, and Random Processes (2nd edition)

Be sure to show all your work for credit.

1. (SPR 1.85)
   A random experiment has sample space $S = \{a, b, c\}$. Suppose that $P(\{a, c\}) = 0.75$ and $P(\{b, c\}) = 0.6$. Find the probabilities of the elementary elements.

2. (SPR 1.86)
   Show that
   (a) $P(\overline{A} \cup \overline{B}) = 1 - P(A \cap B)$
   (b) $P(A \cap B) \geq 1 - P(\overline{A}) - P(\overline{B})$
   (c) $P(A \Delta B) = P(A \cup B) - P(A \cap B)$

3. (SPR 1.87)
   Let $A, B, \text{and } C$ be three events in $S$. If $P(A) = P(B) = \frac{1}{4}, P(C) = \frac{1}{3}, P(A \cap B) = \frac{1}{5}, P(A \cap C) = \frac{1}{6}$, and $P(B \cap C) = 0$, find $P(A \cup B \cup C)$.

4. (SPR 1.90)
   In an experiment consisting of 10 throws of a pair of fair dice, find the probability of the event that at least one double 6 occurs.

5. (SPR 1.94)
   An urn contains 8 white balls and 4 red balls. The experiment consisting of drawing 2 balls from the urn without replacement. Find the probability that both balls drawn are white.

6. (SPR 1.97)
   Let $A$ and $B$ be two independent events in $S$. It is known that $P(A \cap B) = 0.16$ and $P(A \cup B) = 0.64$. Find $P(A)$ and $P(B)$.

7. Let $A, B, \text{and } C$ be events. Find expressions for the following events:
   (a) Exactly one of the three events occurs.
   (b) Exactly two of the events occur.
   (c) One or more of the events occur.
   (d) Two or more of the events occur.
   (e) none of the events occur.

8. The number $U$ is selected at random from the unit interval. Let the events $A$ and $B$ be: $A = "U \text{ differs from } 1/2 \text{ by more than } 1/4"$ and $B = "1 - U \text{ is less than } 1/2"$. Find the events:
   (a) $A \cap B$
   (b) $\overline{A} \cap B$
   (c) $A \cup B$