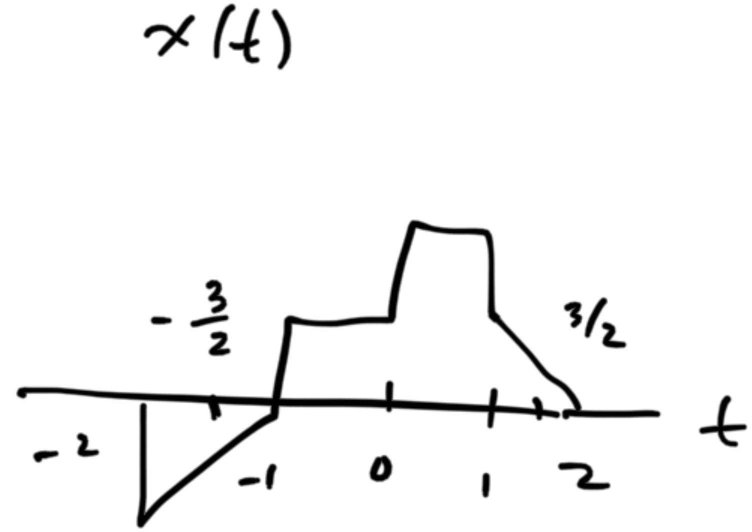
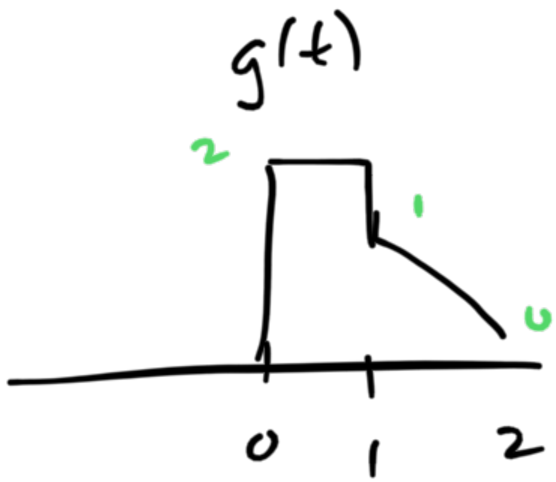
class 03

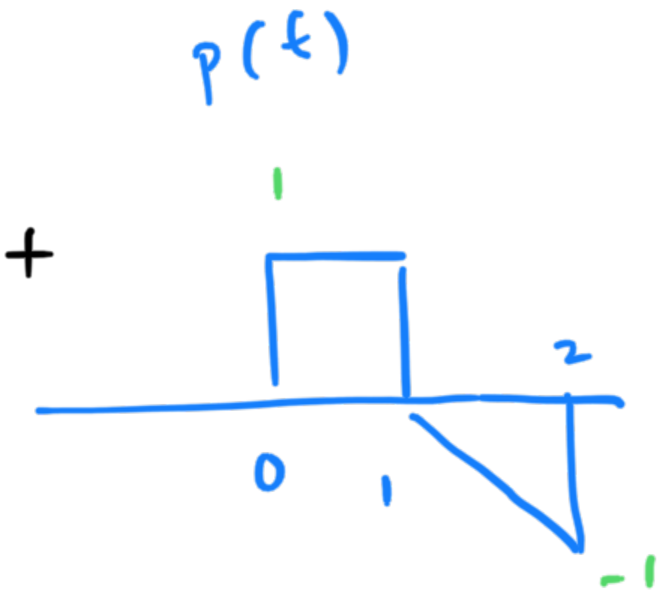
1.21

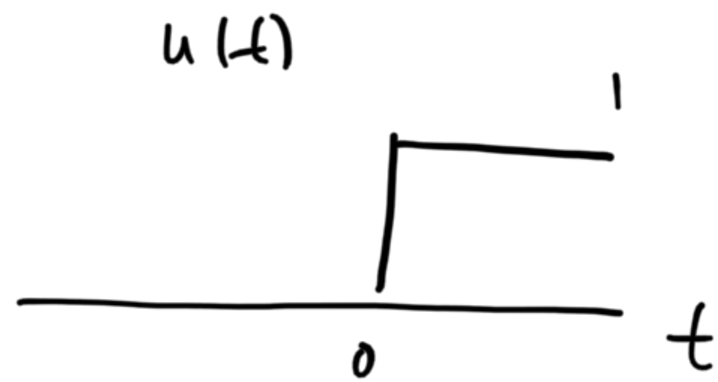
e) $\underbrace{x(t)u(t)}_{g(t)} + \underbrace{x(-t)u(t)}_{p(t)}$

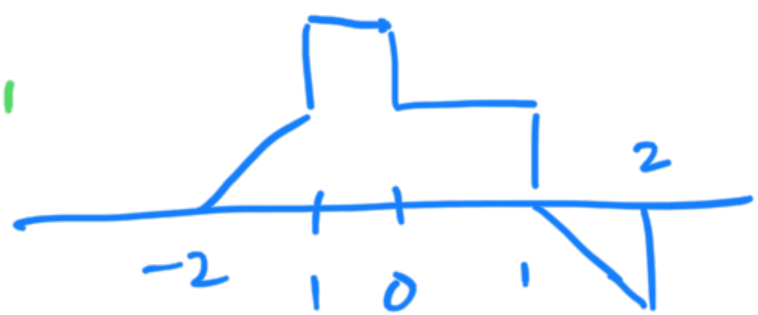


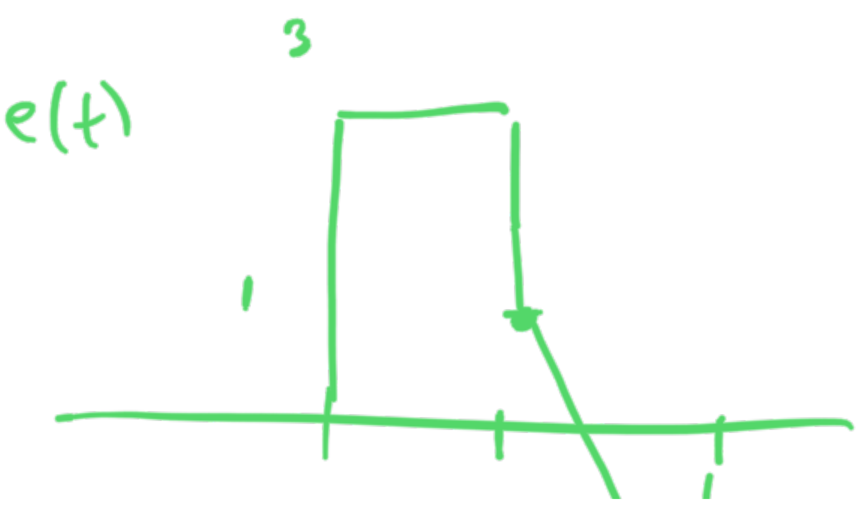


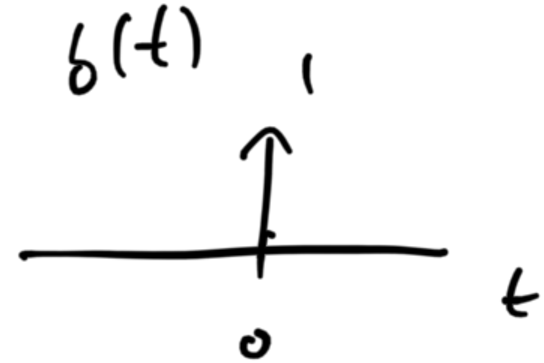
+



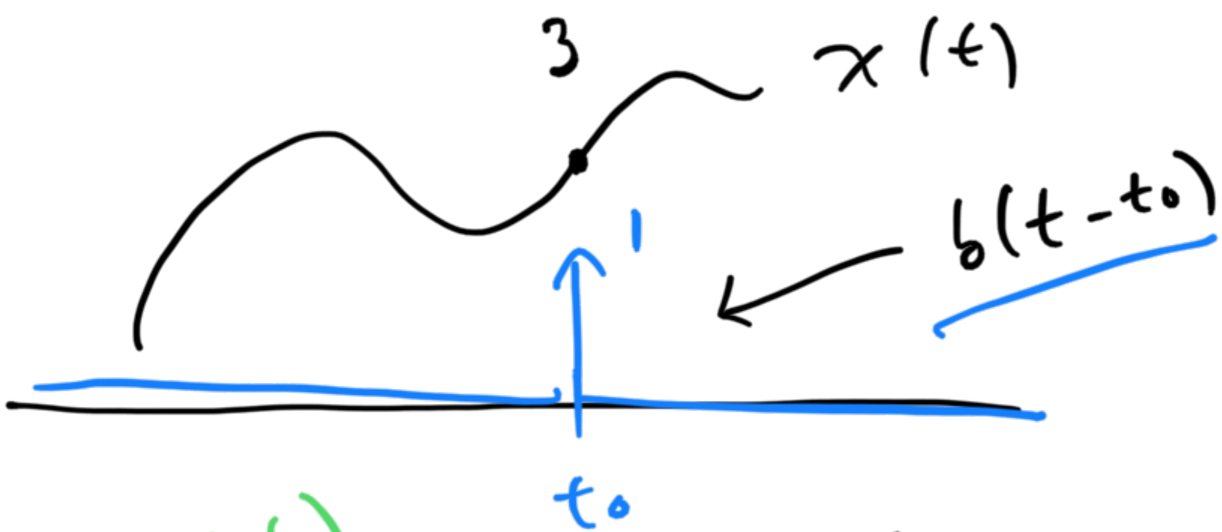


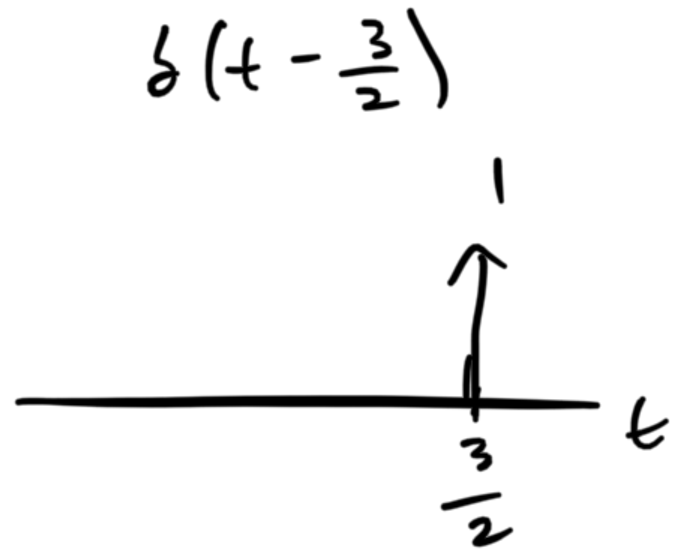






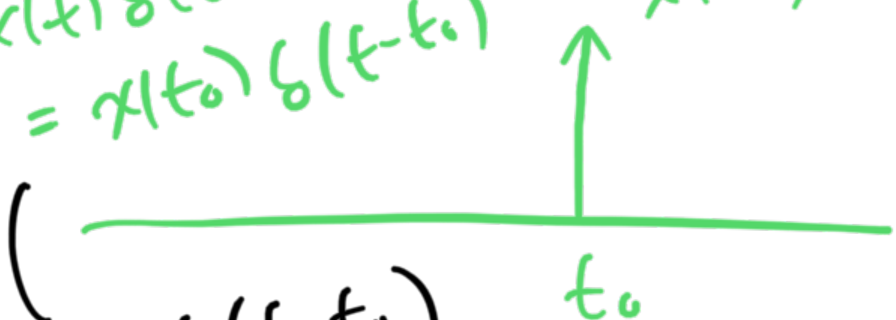
f) $x(t)\,\delta(t+\frac{3}{2}) - x(t)\,\delta(t-\frac{3}{2})$





$x(t)\,\delta(t-t_0)$
$= x(t_0)\,\delta(t-t_0)$

$x(t_0) = 3$



$= 3\,\delta(t-t_0)$

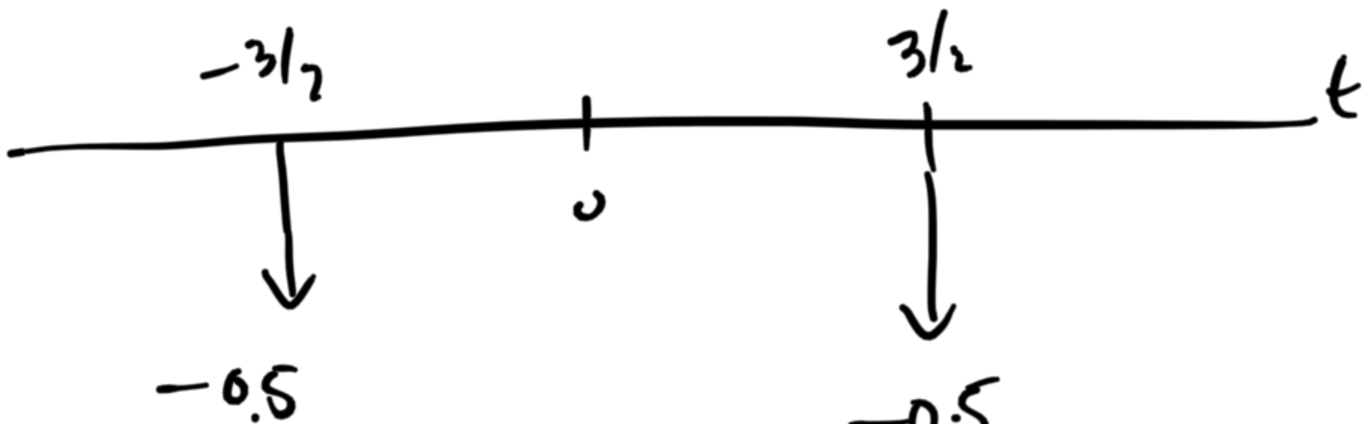
$x(t)\,\delta(t+\frac{3}{2}) - x(t)\,\delta(t-\frac{3}{2})$

$$x\left(-\tfrac{3}{2}\right)\delta\left(t+\tfrac{3}{2}\right) - x\left(\tfrac{3}{2}\right)\delta\left(t-\tfrac{3}{2}\right)$$

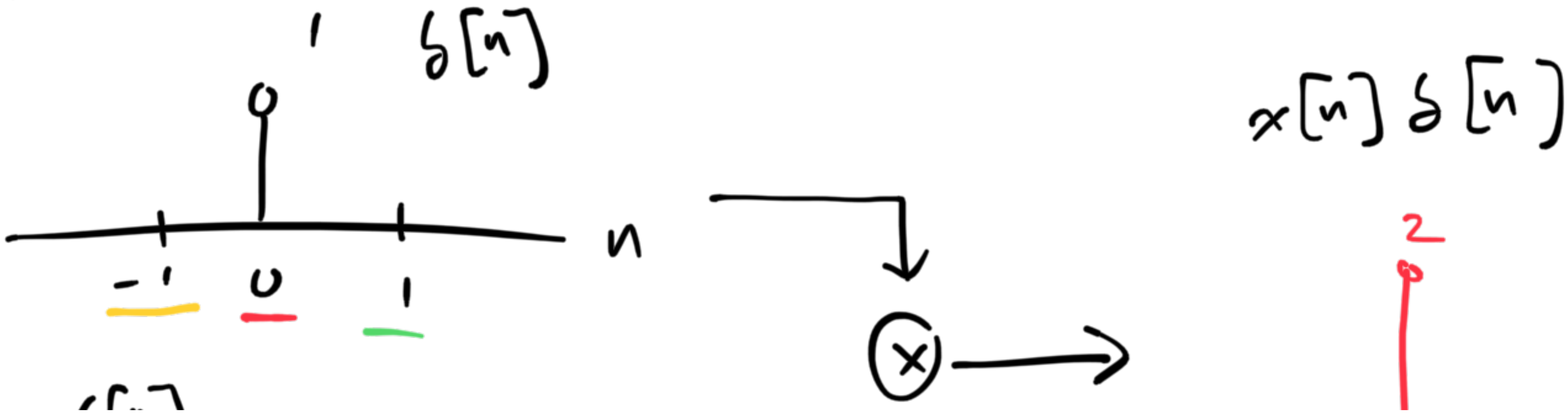
$$x\left(-\tfrac{3}{2}\right) = -0.5$$

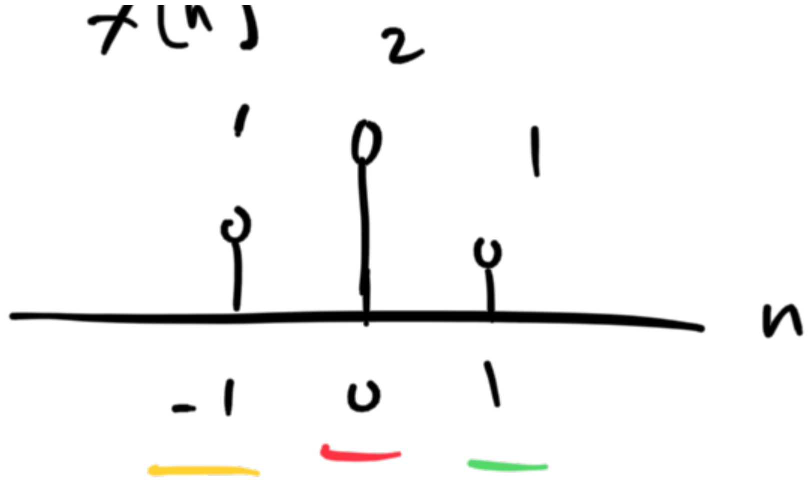
$$x\left(\tfrac{3}{2}\right) = 0.5$$

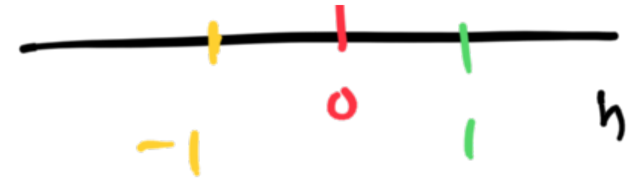
$$-0.5\,\delta\left(t+\tfrac{3}{2}\right) - 0.5\,\delta\left(t-\tfrac{3}{2}\right)$$



DT version







d)* $x\left(4 - \frac{t}{2}\right) = x\left(\frac{8 - t}{2}\right)$

① shift left by 4

② flip

③ $\frac{t}{2}$ stretch by factor of 2

$p(t) = g(-t)$

$q(t) = p\left(\frac{t}{2}\right)$

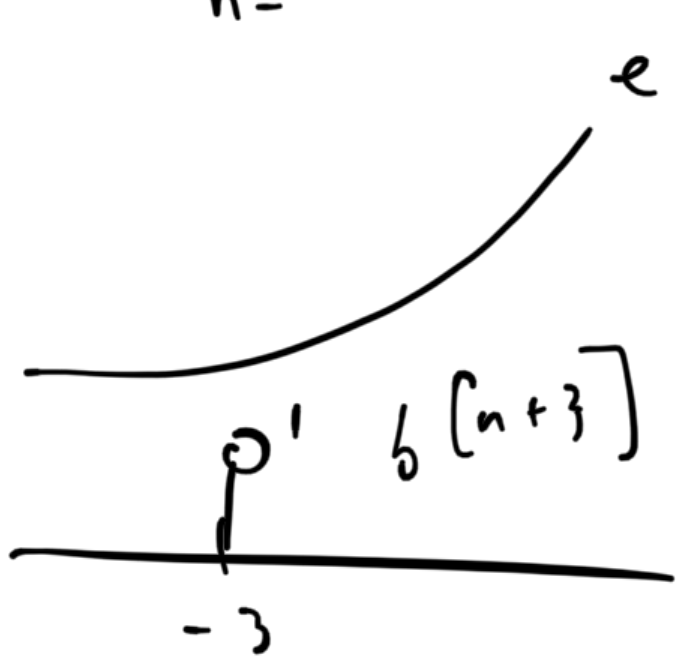
$g(t) = x(t+4)$

---

class 02

Q5 $\sum^{\infty} e^{n}\,\delta[n+3] = \sum e^{-3}\,\delta[n+3]$

$\sum_{n=-\infty} \qquad \sum_{n}$

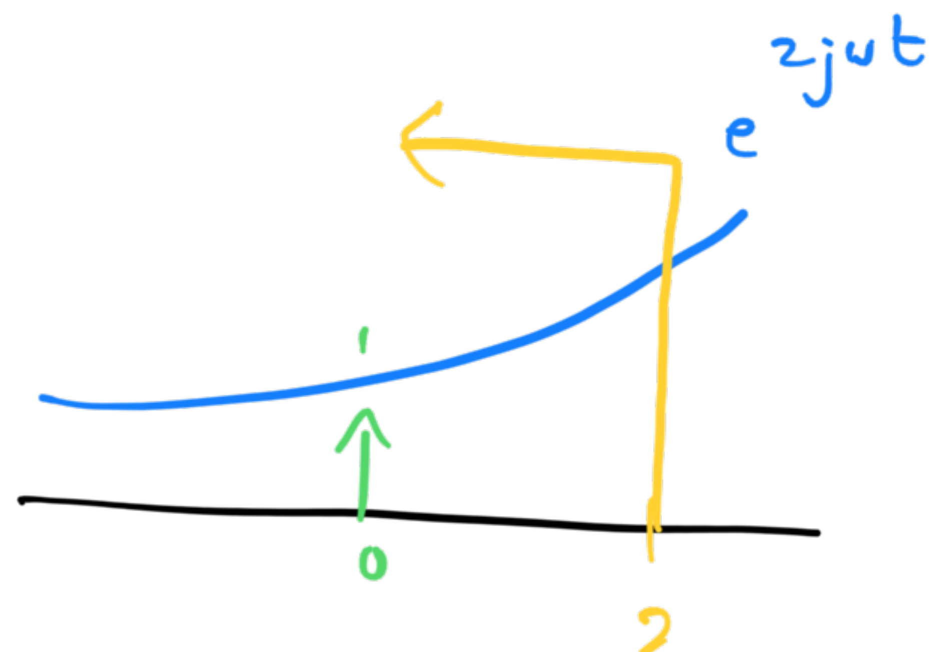


$$= e^{-3} \underbrace{\sum_{n} \delta[n+3]}_{=1}$$

$$= e^{-3}$$

Q4

$$\int_{-\infty}^{2} e^{2j\omega t}\, \delta(t)\, dt$$



$$= \int_{-\infty}^{2} e^{2j\omega(0)}\, \delta(t)\, dt$$

t=0 time of delta

$$= \int^{2} \delta(t)\, dt \quad 1$$

$$= \underbrace{\int_{-\infty} \delta(t)}_{=1} e^{\cdots} = 1$$

---

sys properties

1) memoryless
2) causality
3) stability – BIBO
4) linearity
5) time-invariance

ex. time invariance

$$y[n] = n\,x[n]$$

check

$\leftarrow$ replace n by $n-n_0$

1) $y_1[n] = y[n-n_0]$

$$\boxed{\text{for TI } y_1[n] = y_2[n]}$$

2) $\ldots[n] = f\left(x[n-n_0]\right)$

2) $y_2[n] = f(x[n-n_0])$ ← replace $x[n]$ by $x[n-n_0]$

1) $y_1[n] = (n-n_0)\, x[n-n_0]$

let $g[n] = x[n-n_0]$

2) $y_2[n] = f(x[n-n_0]) = \underline{f(g[n])}$

$= n\, g[n] = n\, x[n-n_0]$

$y_1[n] \neq y_2[n]$ not TI

$y(t) = \sin(x(t))$

1) $y_1(t) = \sin(x(t-t_0))$

2) $y_2(t) = f(g(t)) = \sin(g(t))$

$= \sin(x(t-t_0))$

$= \sin(x(t-t_0))$

$y_1(t) = y_2(t)$ TI ✓

---

linear sys.

$x(t) \longrightarrow y(t)$

$x_3(t) = a x_1(t) + b x_2(t) \longrightarrow y_3(t) =$

$a y_1(t) + b y_2(t)$

$y(t) = t\, x(t)$

$x_3 = a x_1(t) + b x_2(t)$

$y_3(t) = t\, x_3(t) = t\left[a x_1(t) + b x_2(t)\right]$

$$= a\,\underbrace{t\,x_1(t)} + b\,\underbrace{t\,x_2(t)}$$

$$= a\,y_1(t) + b\,y_2(t)$$

L ✓

$$y[n] = 2\big(x[n]\big)^2$$

$$y_3[n] = 2\big(x_3[n]\big)^2 = 2\big(a x_1[n] + b x_2[n]\big)^2$$

$$= 2\big(a^2 x_1^2[n] + b^2 x_2^2[n] + 2ab\, x_1[n]\, x_2[n]\big)$$

$$\neq a y_1[n] + b y_2[n]$$

$$a\,2\big(x_1[n]\big)^2 + b\,2\big(x_2[n]\big)^2$$