Homework #1
Due We. 1/30

1. (OW 1.21)

   Solution

   See Fig. 1 for plots.

   (a) Shift right by 1.

   (b) Shift left by 2 and flip.

   (c) Shift left 1 and squeeze by factor of 2.

   (d) Shift left by 4, flip, and stretch by factor of 2.

   (e) $x(t)u(t) + x(-t)u(t)$

   (f) $x(t)\delta(t+\frac{3}{2}) - x(t)\delta(t-\frac{3}{2}) = x(\frac{-3}{2})\delta(t+\frac{3}{2}) - x(\frac{3}{2})\delta(t-\frac{3}{2})) = -0.5\delta(t+\frac{3}{2}) - 0.5\delta(t-\frac{3}{2}))$

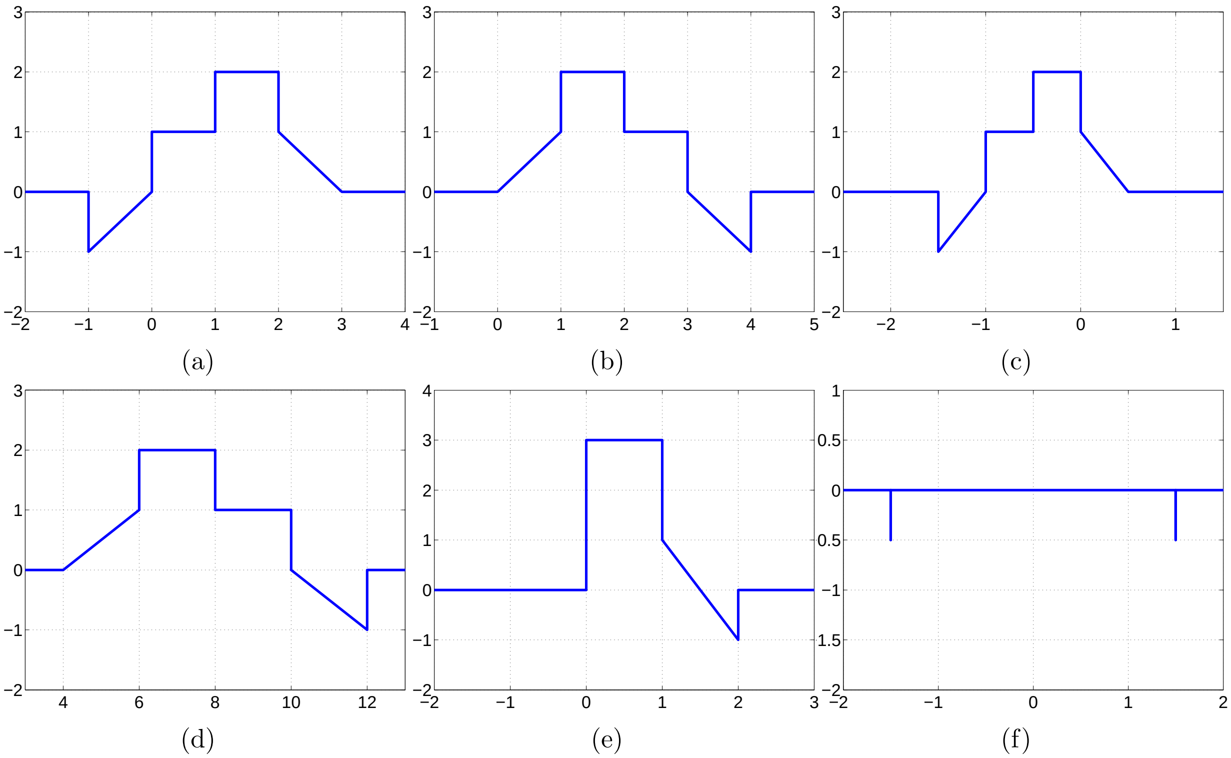

Figure 1: OW 1.21 Solutions

2. (OW 1.22 (a)-(f))

   Solution

   See Fig. 2 for plots.

   (a) Shift right by 4.

   (b) Shift left by 3 and flip.

   (c) Decimate (squeeze) by 3. Only keep integer indicies.

   (d) Shift left by 1 and decimate by 3.

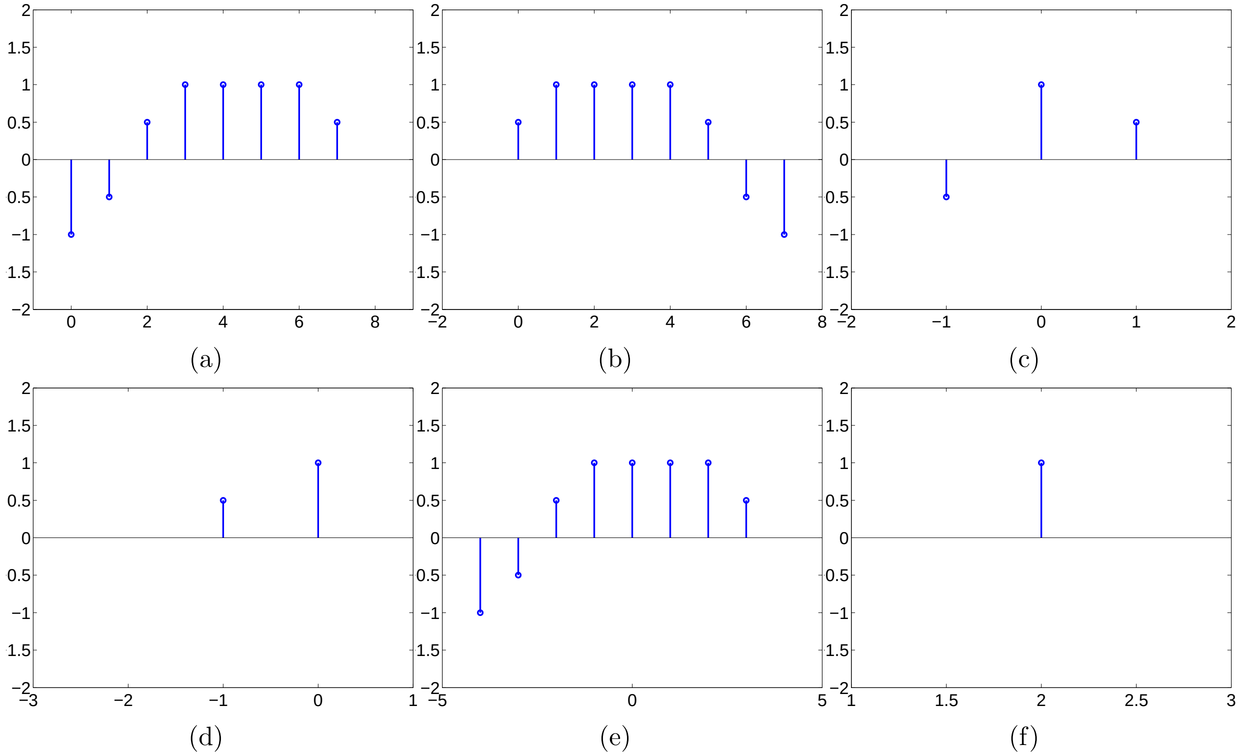

Figure 2: OW 1.22 Solutions

(e) $x[n]u[3-n] = x[n]$ since $u[3-n] = 1$ for $n \leq 3$.

(f) $x[n-2]\delta[n-2] = x[2-2]\delta[n-2] = x[0]\delta[n-2] = \delta[n-2]$

3. (OW 1.31)

Solution

See Fig. 3 for plots.

(a) $x_2(t) = x_1(t) - x_1(t-2) \longrightarrow y_2(t) = y_1(t) - y_1(t-2)$

(b) $x_3(t) = x_1(t+1) + x_1(t) \longrightarrow y_3(t) = y_1(t+1) + y_1(t)$

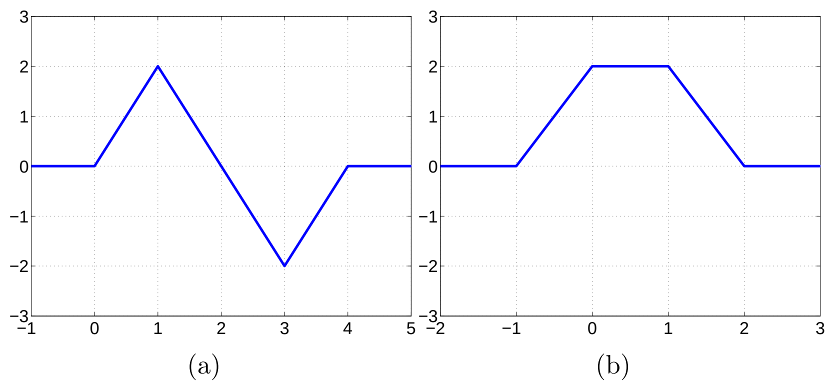

Figure 3: OW 1.31 Solutions

4. (OW 1.49 (a)-(f), (i), (k), (l))

Solution

Convert the Cartesian coordinates ($z = x + jy$) into polar using the formulas

$$r = \sqrt{x^2 + y^2} \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

The points are plotted in the complex plane in Fig. 4.

(a) $1 + j\sqrt{3} = \sqrt{1 + (\sqrt{3})^2} e^{j\tan^{-1}(\frac{\sqrt{3}}{1})} = 2e^{j\pi/3}$

(b) $-5 = 5e^{j\pi}$

(c) $-5 - 5j = 5\sqrt{2}e^{j5\pi/4}$

(d) $3 + 4j = 5e^{j\tan^{-1}(\frac{4}{3})}$

(e) $(1 - j\sqrt{3})^3 = (1 - j2\sqrt{3} - 3)(1 - j\sqrt{3}) = -2(1 + j\sqrt{3})(1 - j\sqrt{3}) = -2(1 + 3) = -8 = 8e^{j\pi}$

(f) $(1 - j)^5 = (1 - j)^2(1 - j)^2(1 - j) = (-2j)^2(1 - j) = 4j^2(1 - j) = -4(1 - j) = -4 + 4j = 4\sqrt{2}e^{j3\pi/4}$

(i) $\dfrac{1 + j\sqrt{3}}{\sqrt{3} + j} = \dfrac{\sqrt{3} + 3j - j + \sqrt{3}}{4} = \dfrac{2\sqrt{3} + 2j}{4} = \dfrac{1}{2}(\sqrt{3} + j) = e^{j\pi/6}$

(k) Using results from (i)
$(\sqrt{3} + j)2\sqrt{2}e^{-j\pi/4} = (2e^{j\pi/6})2\sqrt{2}e^{-j\pi/4} = 4\sqrt{2}e^{-j\pi/12}$

(l) Using results from (i)
$\dfrac{e^{j\pi/3} - 1}{1 + j\sqrt{3}} = \dfrac{(2 + j2\sqrt{3}) - 1}{1 + j\sqrt{3}} = \dfrac{1}{2}\left(\dfrac{-1 + j\sqrt{3}}{1 + j\sqrt{3}}\right) = \dfrac{1}{4}(1 + j\sqrt{3}) = \dfrac{1}{2}e^{j\pi/3}$

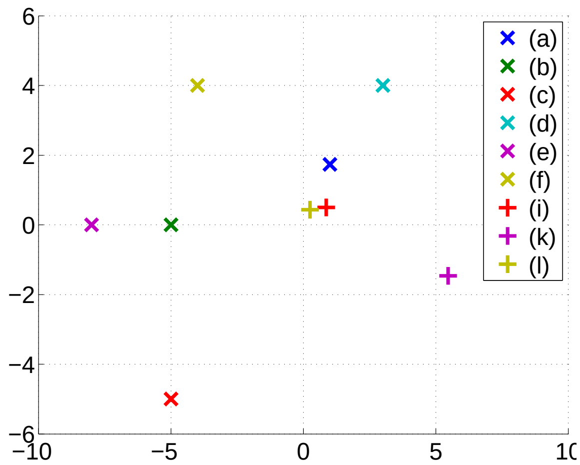

Figure 4: OW 1.49 Solutions

5. (OW 1.51 (a)-(c))

Solution

(a)

$$\begin{aligned}\cos\theta &= \frac{1}{2}(e^{j\theta}+e^{-j\theta})\\ &= \frac{1}{2}(\cos\theta + j\sin\theta + \cos\theta - j\sin\theta)\\ &= \frac{1}{2}(2\cos\theta)\\ &= \cos\theta\end{aligned}$$

(b)

$$\begin{aligned}\sin\theta &= \frac{1}{2j}(e^{j\theta}-e^{-j\theta})\\ &= \frac{1}{2j}(\cos\theta + j\sin\theta - (\cos\theta - j\sin\theta))\\ &= \frac{1}{2j}(2j\sin\theta)\\ &= \sin\theta\end{aligned}$$

(c)

$$\begin{aligned}\cos^2\theta &= \left(\frac{1}{2}\left(e^{j\theta}+e^{-j\theta}\right)\right)^2\\ &= \frac{1}{4}\left(e^{j2\theta}+e^{j\theta}e^{-j\theta}+e^{-j\theta}e^{j\theta}+e^{-j2\theta}\right)\\ &= \frac{1}{4}\left(e^{j2\theta}+e^{-j2\theta}\right)\frac{1}{4}\left(e^{j0}+e^{j0}\right)\\ &= \frac{1}{2}\cdot\frac{1}{2}\left(e^{j2\theta}+e^{-j2\theta}\right)+\frac{1}{4}(2)\\ &= \frac{1}{2}\cos 2\theta + \frac{1}{2}\\ &= \frac{1}{2}(1+\cos 2\theta)\end{aligned}$$

6. (OW 1.55)

Solution

Use the results from Problem 1.54 for summation formulas.

(a)

$$\sum_{n=0}^{9} e^{j\pi n/2} = \frac{1-e^{j\pi(10)/2}}{1-e^{j\pi/2}} = \frac{1-e^{j5\pi}}{1-j} = \frac{1-(-1)}{1-j} = \frac{2}{1-j} = 1+j$$

(b) Using results from (a)

$$\sum_{n=-2}^{7} e^{j\pi n/2} = \sum_{l=0}^{9} e^{j\pi(l+2)/2} = e^{-j\pi}\sum_{l=0}^{9} e^{j\pi l/2} = -1\times\sum_{l=0}^{9} e^{j\pi l/2} = -1(1+j) = -1-j$$

(c)
$$\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n e^{j\pi n/2} = \frac{1}{1-\frac{1}{2}e^{j\pi/2}} = \frac{1}{1-\frac{1}{2}j} = \frac{1}{5}(4+2j)$$

(d) Using results from (c)
$$\sum_{n=2}^{\infty}\left(\frac{1}{2}\right)^n e^{j\pi n/2} = \frac{(\frac{1}{2}e^{j\pi/2})}{1-\frac{1}{2}j} = \frac{j^2}{4}\{\frac{1}{5}(4+2j)\} = -\frac{1}{4}\{\frac{1}{5}(4+2j)\} = -\frac{1}{5} - \frac{j}{10}$$

(e) Using results from (a)
$$\sum_{n=0}^{9}\cos(\frac{\pi}{2}n) = \sum_{n=0}^{9}\frac{1}{2}e^{j\pi n/2} + \frac{1}{2}e^{-j\pi n/2} = \frac{1}{2}(1+j) + \frac{1}{2}(1-j) = 1$$

(f) Using results from (c)
$$\sum_{n=0}^{\infty}\frac{1}{2}^n\cos(\frac{\pi}{2}n) = \frac{1}{2}\sum_{n=0}^{\infty}(\frac{1}{2})^n e^{j\pi n/2} + (\frac{1}{2})^n e^{-j\pi n/2} = \frac{1}{2}\{\frac{1}{5}(4+2j)\} + \frac{1}{2}\{\frac{1}{5}(4-2j)\} = \frac{4}{5}$$

7. (OW 1.56)

Solution

(a)
$$\int_0^4 e^{j\pi t/2}dt = \left[\frac{1}{j\pi/2}e^{j\pi t/2}\right]_0^4 = \frac{1}{j\pi/2}\left[e^{j2\pi}-1\right] = 0$$

(b)
$$\int_0^6 e^{j\pi t/2}dt = \left[\frac{1}{j\pi/2}e^{j\pi t/2}\right]_0^6 = \frac{1}{j\pi/2}\left[e^{j3\pi}-1\right] = \frac{1}{j\pi/2}\left[-2\right] = -\frac{4}{j\pi} = \frac{4j}{\pi}$$

(c)
$$\int_2^8 e^{j\pi t/2}dt = \left[\frac{1}{j\pi/2}e^{j\pi t/2}\right]_2^8 = \frac{1}{j\pi/2}\left[e^{j4\pi}-e^{j\pi}\right] = \frac{1}{j\pi/2}\left[1-(-1)\right] = \frac{4}{j\pi} = -\frac{4j}{\pi}$$

(d)
$$\int_0^\infty e^{-(1+j)t}dt = \left[-\frac{1}{1+j}e^{-(1+j)t}\right]_0^\infty = -\frac{1}{1+j}\left[0-1\right] = \frac{1}{1+j} = \frac{1-j}{2}$$

(e) Using result from part (d)
$$\int_0^\infty e^t\cos(t)dt = \frac{1}{2}\int_0^\infty e^{-(1+j)t} + e^{-(1-j)t}dt = \frac{1}{2}\left[\frac{1+j}{2}+\frac{1-j}{2}\right] = \frac{1}{2}$$

(f) Using a similar approach as used in part (e)

$$\begin{aligned}\int_0^\infty e^t\sin(t)dt &= \frac{1}{2j}\int_0^\infty e^{-(2-3j)t} + e^{-(2+3j)t}dt \\ &= \frac{1}{2j}\left\{\left[-\frac{1}{2-3j}e^{-(2-3j)t}\right]_0^\infty + \left[\frac{1}{2+3j}e^{-(2+3j)t}\right]_0^\infty\right\} \\ &= \frac{1}{2j}\left[\frac{1}{2-3j} - \frac{1}{2+3j}\right] \\ &= \frac{3}{13}\end{aligned}$$

8. Using expressions in OW 1.54 and for any $0 < N_1, N_2 < \infty$,

(a) For $a \neq 1$, find a closed form expression for

$$\sum_{n=N_1}^{N_2} a^n$$

(b) For $|a| < 1$, find a closed form expression for

$$\sum_{n=N_1}^{\infty} a^n.$$

Solution

(a) Let $k = n - N_1$,

$$\begin{aligned}
\sum_{n=N_1}^{N_2} a^n &= \sum_{k=0}^{N_2-N_1} a^{k+N_1} \\
&= \sum_{k=0}^{N_2-N_1} a^k a^{N_1} = a^{N_1} \sum_{k=0}^{N_2-N_1} a^k \\
&= a^{N_1} \frac{1 - a^{N_2-N_1+1}}{1-a} \\
&= \frac{a^{N_1} - a^{N_2+1}}{1-a}
\end{aligned}$$

(b) Let $k = n - N_1$,

$$\begin{aligned}
\sum_{n=N_1}^{\infty} a^n &= \sum_{k=0}^{\infty} a^{k+N_1} \\
&= a^{N_1} \sum_{k=0}^{\infty} a^k \\
&= a^{N_1} \frac{1}{1-a} \\
&= \frac{a^{N_1}}{1-a}
\end{aligned}$$