SIGNALS AND SYSTEMS I
Computer Assignment 6

Implementing State Systems

Because linear time invariant state systems can be described by the state equations,

$$\mathbf{q}(n+1) = \mathbf{A}\mathbf{q}(n) + \mathbf{B}\mathbf{x}(n)$$
$$\mathbf{y}(n) = \mathbf{C}\mathbf{q}(n) + \mathbf{D}\mathbf{x}(n)$$

which are a set of first order difference equations, state systems can also be implemented using MATLAB's *for* loop and *do* loop constructs. For example, the first 10 output samples of the state equations

$$\begin{bmatrix} q_1(n+1) \\ q_2(n+1) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} q_1(n) \\ q_2(n) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} x(n)$$
$$y(n) = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} q_1(n) \\ q_2(n) \end{bmatrix} + dx(n)$$

where

$$x(n) = u(n)$$

can be calculated as follows:

```
x = ones (11,1);
A = [a11 a12 ; a21 a22];
B = [b1 ; b2];
C = [c1 c2];
D = d
q = zeros(2,1)
for n = 2:11,
    y(n) =C*q + D*x(n);
    q = A*q + B*x(n)
end
y = y(2:11)
```

Outputs of state equations can also be calculated using built-in MATLAB functions. Before using these functions, the state system must be defined. MATLAB's built-in function, **ss**, can be used to define a state state system. For example, the discrete state system

$$\mathbf{q}(n+1) = \mathbf{A}\mathbf{q}(n) + \mathbf{B}\mathbf{x}(n)$$
$$\mathbf{y}(n) = \mathbf{C}\mathbf{q}(n) + \mathbf{D}\mathbf{x}(n)$$

is defined as

```
sys = ss(A,B,C,D,1)
```

After the system has been defined, the **impulse** function, the **step** function and the **lsim** function can be used to calculate the system's impulse response, step response and response to a user defined input signal, respectively. For example,

```
[h,n] = impulse(sys,10)
```

calculates the vector, *h*, which contains the first 11 samples of the system's impulse response and the vector, *n*, which contains the corresponding sample numbers for the elements in *h*. Similarly,

```
[s,n] = step(sys,10)
```

calculates the vector, *s*, which contains the first 11 samples of the system's step response and the vector, *n*, which contains the corresponding sample numbers for the elements in *s*. The **lsim** function calculates the system's response to a user defined input signal, $x(n)$. For example

```
[y,n] = lsim(sys,x)
```

calculates the system's first *N* output samples in response to the input signal, *x*, where *N* is the length of the input signal, *x*.

The **ss**, **impulse**, **step**, and **lsim** functions are part of MATLAB's control toolbox which should be installed on the college systems. If MATLAB reports back that

function.m not found

MATLAB's control toolbox has not been installed. Please E-mail the system administrator at staff@egr.unlv.edu and inform him of this problem. He might inform you that you need to use the college UNIX systems.

Exercises

Consider the discrete system described by the difference equation,

$$y(n)+a_1 y(n-1)+a_2 y(n-2)+a_3 y(n-3)=b_0 x(n)+b_1(n-1)+b_2(n-2)+b_3(n-3)$$

where $x(n)$ is the system's input, $y(n)$ is the system's output and

$a_0 = 1$	$a_1 = -0.18902544$	$a_2 = 0.71974192$	$a_3 = -0.15739157$
$b_0 = 0.25445939$	$b_2 = 0.43220307$	$b_2 = 0.43220307$	$b_3 = 0.25445939$

1. Draw

   a) a Direct Form I block diagram of the system.
   b) a Direct Form II block diagram of the system.
   c) a transposed Direct Form II block diagram of the system.

   Indicate the number of delays (memory registers) required to implement each block diagram.

2. Generate state equations for the Direct Form II and the Transposed Direct Form II block diagrams that you drew in Exercise 1.

3. Using a *for* loop or a *while* loop, write programs that implement each of the block diagrams that you drew in Exercise 4. Using both programs, calculate the system's first 51 outputs when the system's input, $x(n)$, is

   a) $x(n) = \delta(n)$.
   b) $x(n) = u(n)$.
   c) $x(n) = \cos(0.2\pi n)u(n)$
   d) $x(n) = \cos(0.2\pi n)u(n) + \cos(0.7\pi n)u(n)$

   Plot the inputs and outputs using the **stem**, **title** and **subplot** functions. (You should generate 12 plots on 4 pages, that is, three plots per page.)

4. Using MATLAB's built-in functions and one of your state space models, calculate the system's first 51 outputs when the system's input, $x(n)$, is

   a) $x(n) = \delta(n)$.
   b) $x(n) = u(n)$.
   c) $x(n) = \cos(0.2\pi n)u(n)$
   d) $x(n) = \cos(0.2\pi n)u(n) + \cos(0.7\pi n)u(n)$

   Plot the inputs and outputs using the **stem**, **title** and **subplot** functions. (You should generate 8 plots on 2 pages, that is, four plots per page.)