EE360: Signals and System I

Schaum’s Signals and Systems
Chapter 7
State Space Analysis

http://www.ee.unlv.edu/~b1morris/ee360/
Outline

- Concept of State
- DT State Space Representations
- DT State Equation Solutions
- CT State Space Representations
- CT State Equation Solutions
Introduction to State Space

• So far, we have studied LTI systems based on input-output relationships
  ▫ Known as external description of a system
• Now will examine state space representation of systems
  ▫ Known as internal description of systems

• Consists of two parts
  ▫ State equations – set of equations relating state variables to inputs
  ▫ Output equations – set of equations relating outputs to state variables and inputs
Advantages of State Space

• Provides new insight into system behavior
  ▫ Use of matrix linear algebra

• Can handle multiple-input multiple-output (MIMO) systems
  ▫ Generalize from single-input single-output

• Can be extended to non-linear and time-varying systems
  ▫ General mathematical model

• State equations can be implemented efficiently on computers
  ▫ Enables solving/simulation of complex systems
Definition of State

- State – state of a system @ time $t_0$ is defined as the minimal information that is sufficient to determine the state and output of a system for all times $t > t_0$ when the input is also known for $t > t_0$

- State variables ($q_i$) - variables that contain all state information (memory)

- Note: definition only applies to causal systems
Motivation Example

- **Input:** $v(t)$
- **Output:** $i(t)$
- **State:**
  - $v_L(t) = L \frac{di}{dt}$
  - $i_c(t) = C \frac{dv_c}{dt}$

- Knowing $x(t) = v(t)$ over $[-\infty, t]$ is sufficient to determine $y(t) = i(t)$ over the same interval.

- If $x(t)$ is only known between $[t_0, t]$ then the output cannot be determined without knowledge of:
  - Current through inductor
  - Voltage across capacitor

- Imagine being handed a circuit (system) that was in operation at time $t_0$
  - Initial condition problem
Selection of State Variables

• Need to determine “memory elements” of a system

• DT: Select outputs of delay elements
• CT: Select outputs of integrators or energy-storing elements (capacitors, inductors)

• However, state-variable choice is not unique
  ▫ Transformations of variables will result in same state space analysis
DT State Space Representation I

- Consider a single-input single-output (SISO) DT LTI system
  - \( y[n] + a_1 y[n - 1] + \cdots + a_N y[n - N] = x[n] \)
- To uniquely determine a complete solution (output), requires \( N \) initial conditions
  - \( y[-1], y[-2], \ldots, y[-N] \)
- Define state variables (outputs of delay elements)
  - \( q_1[n] = y[n - N] \)
  - \( q_2[n] = y[n - (N - 1)] = y[n - N + 1] \)
  - \( \cdots \)
  - \( q_N[n] = y[n - 1] \)
DT State Space Representation II

• Find next (step-ahead) state
  ▫ By definition of delay or using signal flow graph

• $q_1[n + 1] = y[n + 1 - N] = q_2[N]$
• $q_2[n + 1] = y[n + 1 - N + 1] = y[n - N + 2] = q_3[n]$
• ...
• $q_N[n + 1] = y[n + 1 - 1] = y[n] = -a_1y[n - 1] + \cdots + -a_Ny[n - N]$ (recursive form)

• $q_N[n + 1] = -a_1q_N[n] - a_2q_{N-1}[n] + \cdots + -a_Nq_1[n]$
These relationships can be compactly expressed in matrix form

\[
\begin{bmatrix}
q_1[n+1] \\
q_2[n+1] \\
\vdots \\
q_N[n+1]
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 & \cdots & 0 \\
0 & 0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 0 & 1 & \cdots & 0 \\
-a_N & -a_{N-1} & -a_{N-2} & -a_{N-3} & \cdots & -a_1
\end{bmatrix}
\begin{bmatrix}
q_1[n] \\
q_2[n] \\
\vdots \\
q_N[n]
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
\vdots \\
1
\end{bmatrix}
x[n]
\]

**State equation** – next state from past state and input
\[
q[n + 1] = Aq[n] + Bx[n]
\]

**Output equation** – output based on state and input
\[
y[n] = Cq[n] + Dx[n]
\]

Note: generalized form for MIMO systems with vector \(y[n], x[n]\)
DT State Space Representation IV

- Previous example:
  - Defined state variables as outputs of delay elements
  - Rewrote state relationships using a vectorized form of state $q[n]$

- Goal: build state-equations given either a difference equation or block-diagram

- Note: previous example had no delayed input $x[n]$. How would delayed inputs change state space representation?
  - Consider DFII structure and develop state equations
Similarity Transformation

- Choice of state-variable is not unique

- Can have another choice of state variables as a transformation
- If $\nu[n] = Tq[n]$
  - $T$ is $N \times N$ non-singular transformation matrix
- Then, $q[n] = T^{-1}\nu[n]$

- You and your friend could have different (valid) state variable choices for same state space representation
Solution to DT State Equations

- Two approaches
  - Time-domain solution
  - Z-transform solution