EE292: Fundamentals of ECE

Fall 2012
TTh 10:00-11:15 SEB 1242

Lecture 21
121113

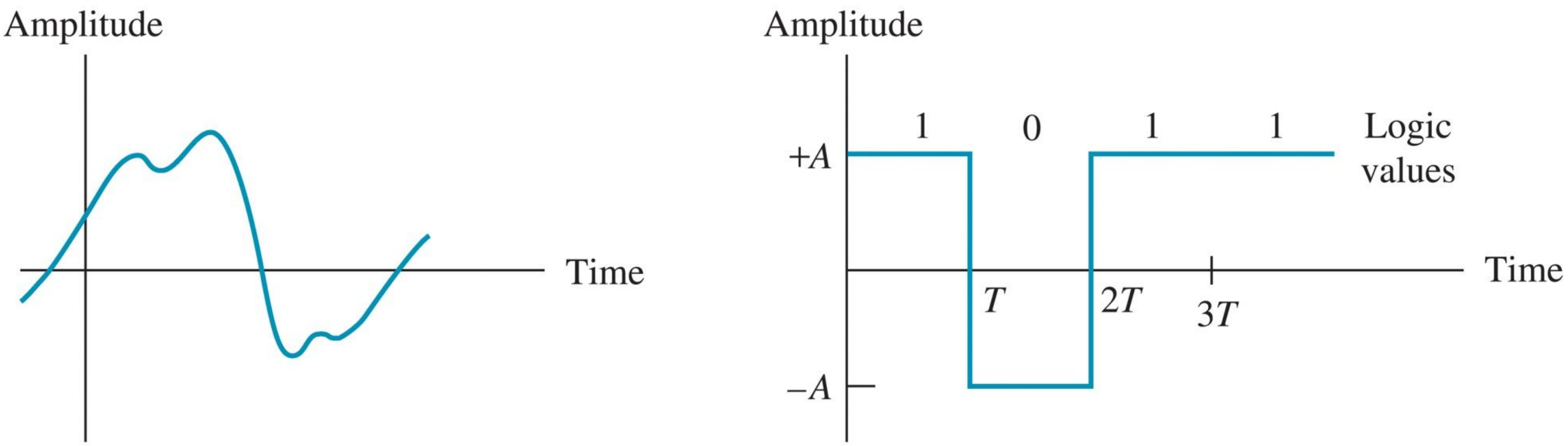
http://www.ee.unlv.edu/~b1morris/ee292/

Outline

- Chapter 7 - Logic Circuits
- Binary Number Representation
- Binary Arithmetic
- Combinatorial Logic

Logic Circuits

- Analog signal – signal of continuous "time" variable with a continuous range of outputs
  - The signal has an infinite range of values at any time
  - E.g. a speech signal
- Digital signal – a signal with discrete "time" variable and only a few restricted amplitude values



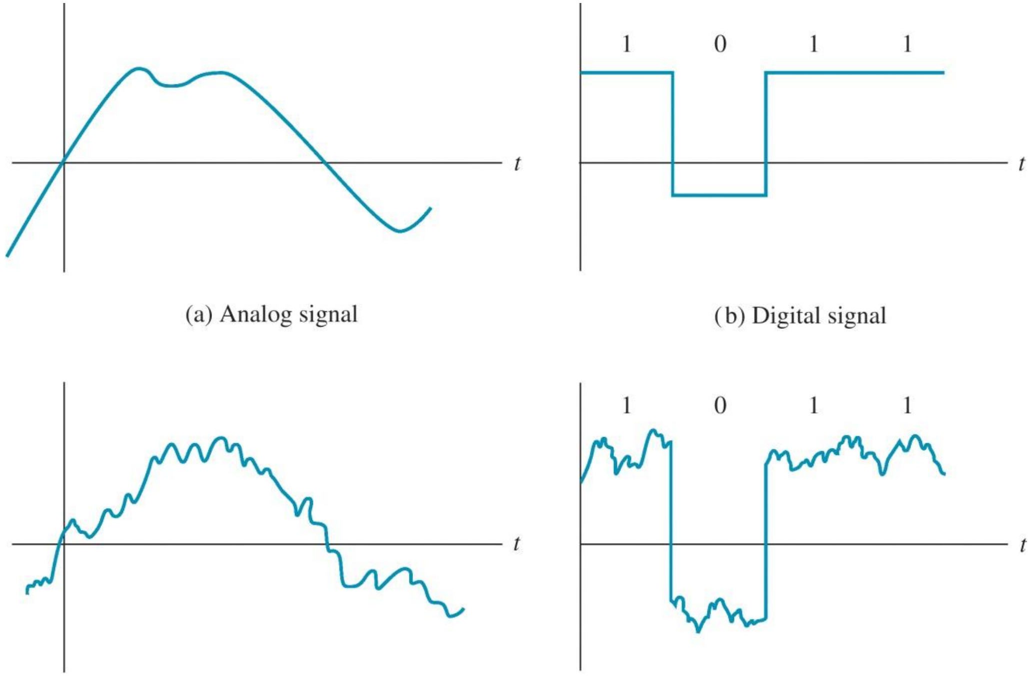
(a) Analog signal (b) Digital signal

Digital Signals

- Computers are examples of digital circuits
  - They operate on digital signals
- Binary signals are the most common type of signal
  - The output of a binary signal takes only two possible values
  - The two output values are often given “logical values” of a 1 or 0
- Often digital signals often come from physical analog processes
  - The analog signal is converted into a digital form for processing in a computer

Digital Noise Advantage

- Digital signals are robust to noise
  - The exact signal value is not required
  - Rely on “logic” values
- Today it is possible to manufacture large numbers of digital logic circuits on integrated circuits because of this simplification



(a) Analog signal (b) Digital signal

(c) Analog signal plus noise (d) Digital signal plus noise

Positive Logic

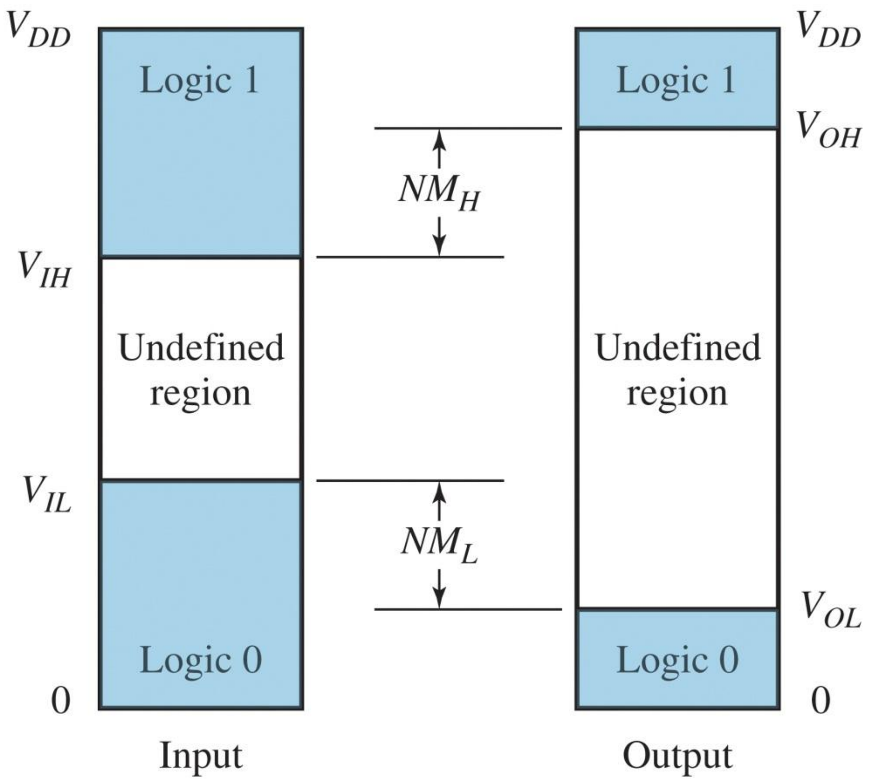
- Logical 1
  - The higher amplitude value in a binary system
    - E.g. 5 volts
  - Also known as “high”, “true”, or “on”

- Logical 0
  - The lower amplitude in a binary system
    - E.g. 0 volts
  - Also known as “low”, “false”, or “off”

- Logic variables – signals in logic systems that switch between high and low
  - Will be denoted by uppercase letters (E.g. A, B, C)

Logic Ranges and Noise Margins

- Logic circuits are designed to have a range of input voltages map to a logical "high" or "low"
  - V_{IL} - largest input value for logic 0 at input
  - V_{IH} - smallest input value for logic 1 at input
  - V_{OL} - largest output value for logic 0 at input
  - V_{OH} - smallest output value for logic 1 at input
- Input and output have different logical ranges due to noise
  - The difference is known as the noise margin



Digital Words

- Bit – a single binary digit
  - Smallest amount of information that can be represented in a digital system
  - Represents a yes/no for a digital variable
  - E.g. $R = 0$, represents not raining while $R = 1$, represents raining
- In order to represent more complex information, bits can be combined into digital words
  - A byte is 8 bits and a nibble is 4 bits (used often in computers, e.g. a byte to represent each key on a keyboard)
- Example RWS
  - R for rain, W for wind, S for sunny
  - $RWS = 110$ indicates it is raining, with winds, and cloudy (e.g. not sunny)

Representation of Numerical Data

- Digital words allow representation of more complex values by concatenating digital variables
  - Only binary yes/no results were allowed
  - RWS allowed 2^3 different combinations of weather conditions
- Need a way to represent the wide range of values encountered in the physical world
  - Must be able to convert real numbers into a binary form for computation in a digital fashion

Decimal Representation of Numbers

- Consider a decimal number (base 10)
  - This is what we as humans are familiar with

- Example 743.2_{10}
  - This is interpreted as
  - $7 \times 10^2 + 4 \times 10^1 + 3 \times 10^0 + 2 \times 10^{-1}$
  - Each digit is a multiplier by 10^d
    - d is the digit location
    - Positive to the left of decimal point and negative to the right

Binary Representation of Numbers

- Use the same technique as for decimal but instead use base 2 numbers

- Example $1101.1_{\ 2}$
- $1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + +1 \times 2^0 + 1 \times 2^{-1}$
  - $1 \times 2^3 = 8$
  - $1 \times 2^2 = 4$
  - $1 \times 2^0 = 1$
  - $1 \times 2^{-1} = 0.5$
- $1101.1 = 13.5$

- Notice the subscript is used to indicate what the base to use for the number interpretation

Numerical Binary Words

- Enumerate all combinations of values for binary word
  - An N bit word can represent 2^N different numbers

- Let $N = 4$, then there are $2^4 = 16$ different values that can be represented

Binary	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

- The leading zeros are presented in binary form because the digital circuits typically operate on fixed size words

Positional Notation for Numbers

- Base B number → B symbols per digit
  - Base 10 (Decimal): 0, 1, 2, 3, 4, 5, 6 , 7, 8, 9
  - Base 2 (binary) 0, 1
- Number representation
  - $d_{31}d_{30}\ldots d_2d_1d_0$ is 32 digit number
  - Value = $d_{31}\times B^{31} + d_{30}\times B^{30} + \ldots + d_1\times B^1 + d_0\times B^0$
- Examples
  - (Decimal): 90
    - $= 9\times10^1 + 0\times10^0$
  - (Binary): 1011010
    - $= 1\times2^6 + 0\times2^5 + 1\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0$
    - $= 64 + 16 + 8 + 2$
    - $= 90$
  - 7 binary digits needed for 2 digit decimal number

Hexadecimal Number: Base 16

- More human readable than binary
- Base with easy conversion to binary
  - Any multiple of 2 base could work (e.g. octal)
- Hexadecimal digits

Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Binary	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
Hex (16)	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
octal (8)	0	1	2	3	4	5	6	7								

  - 1 hex digit represents 16 decimal values or 4 binary digits
    - Will use `0x` to indicate hex digit

Hex/Binary Conversion

Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
binary	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
hex	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F

- Convert between 4-bits and a hex digit using the conversion table above
- Examples
  - 1010 1100 0101 (binary)
    - = 0xAC5
  - 10111 (binary)
    - = 0001 0111 (binary)
    - = 0x17
  - 0x3F9
    - = 0011 1111 1001 (binary)
    - = 11 1111 1001 (binary)

Signed Numbers

- N bits represents 2^N values
- Unsigned integers
  - Range `[0, 2^32-1]`
- How can negative values be indicated?
  - Use a sign-bit
  - Boolean indicator bit (flag)

Sign and Magnitude

- 16-bit numbers
  - +1 (decimal) = `0000 0000 0000 0001 = 0x0001`
  - -1 (decimal) = `1000 0000 0000 0001 = 0x8001`
- Problems
  - Two zeros
    - `0x0000`
    - `0x8000`
  - Complicated arithmetic
    - Special steps needed to handle when signs are same or different (must check sign bit)

Ones Complement

- Complement the bits of a number
  - +1 (decimal) = `0000 0000 0000 0001 = 0x0001`
  - -1 (decimal) = `1111 1111 1111 1110 = 0xFFFE`
- Positive number have leading zeros
- Negative number have leading ones
- Arithmetic not too difficult
- Still have two zeros

Two's Complement

- Subtract large number from a smaller one
  - Borrow from leading zeros
  - Result has leading ones
- Unbalanced representation
  - Leading zeros for positive
    - 2^{N-1} non-negatives
  - Leading ones for negative number
    - 2^{N-1} negative number
  - One zero representation
- First bit is sign-bit (must indicate width)
  - Value = $d_{31} \times -2^{31} + d_{30} \times 2^{30} + \ldots + d_1 \times 2^1 + d_0 \times 2^0$

Negative value for sign bit

Binary	Decimal
… 0011	3
… 0100	4
… 1111	-1

Two's Complement Negation

- Shortcut = invert bits and add 1
  - Number + complement = 0xF..F = -1
    - $x + \bar{x} = -1$
    - $\bar{x} + 1 = -x$
- Example
x	1111 1110
$\bar{x}$	0000 0001
$\bar{x} + 1$	0000 0010

Two's Complement Sign Extension

- Machine's have fixed width (e.g. 32-bits)
  - Real numbers have infinite width (invisible extension)
    - Positive has infinite 0's
    - Negative has infinite 1's
- Replicate sign bit (msb) of smaller container to fill new bits in larger container
- Example
  - 1111 1111 1111 1111 1111 1111 1111 1110
    1111 1111 1111 1111 1111 1111 1111 1110

Overflow

- Fixed bit width limits number representation
- Occurs if result of arithmetic operation cannot be represented by hardware bits
- Example
  - 8-bit: 127 + 127

Binary	Decimal
0111 1111	127
0111 1111	127
1111 1110	**-2 (254)**