EE292: Fundamentals of ECE

Fall 2012
TTh 10:00-11:15 SEB 1242

Lecture 20
121101

http://www.ee.unlv.edu/~b1morris/ee292/
Outline

- Chapters 1-3
  - Circuit Analysis Techniques
- Chapter 10 – Diodes
  - Ideal Model
  - Offset Model
  - Zener Diodes
- Chapter 4 – Transient Analysis
  - Steady-State Analysis
  - 1st-Order Circuits
- Chapter 5 – Steady-State Sinusoidal Analysis
  - RMS Values
  - Phasors
  - Complex Impedance
  - Circuit Analysis with Complex Impedance
Chapter 10 - Diodes
Diode Voltage/Current Characteristics

- **Forward Bias ("On")**
  - Positive voltage $v_D$ supports large currents
  - Modeled as a battery (0.7 V for offset model)
- **Reverse Bias ("Off")**
  - Negative voltage $\rightarrow$ no current
  - Modeled as open circuit
- **Reverse-Breakdown**
  - Large negative voltage supports large negative currents
  - Similar operation as for forward bias
Diode Models

- Ideal model – simple
- Offset model – more realistic

- Two state model
- “On” State
  - Forward operation
  - Diode conducts current
    - Ideal model $\rightarrow$ short circuit
    - Offset model $\rightarrow$ battery
- “Off” State
  - Reverse biased
  - No current through diode $\rightarrow$ open circuit
Circuit Analysis with Diodes

- Assume state \{on, off\} for each ideal diode and check if the initial guess was correct
  - \(i_d > 0\) positive for “on” diode
  - \(v_d < v_{on}\) for “off” diode
    - These imply a correct guess
  - Otherwise adjust guess and try again

- Exhaustive search is daunting
  - \(2^n\) different combinations for \(n\) diodes
- Will require experience to make correct guess
Zener Diode

- Diode intended to be operated in breakdown
  - Constant voltage at breakdown
- Three state diode
  1. On – 0.7 V forward bias
  2. Off – reverse bias
  3. Breakdown
    \( v_{BD} \) reverse breakdown voltage
Chapter 4 - Transient Analysis
DC Steady-State Analysis

- Analysis of C, L circuits in DC operation
  - Steady-state – non-changing sources

- Capacitors $i = C \frac{dv}{dt}$
  - Voltage is constant $\rightarrow$ no current $\rightarrow$ open circuit

- Inductors $v = L \frac{di}{dt}$
  - Current is constant $\rightarrow$ no voltage $\rightarrow$ short circuit

- Use steady-state analysis to find initial and final conditions for transients
General 1\textsuperscript{st}-Order Solution

- Both the current and voltage in an 1\textsuperscript{st}-order circuit has an exponential form
  - RC and LR circuits

- The general solution for current/voltage is:
  \[ x(t) = x_f + [x(t_0^+) - x_f] e^{-(t-t_0)/\tau} \]
  - \( x \) – represents current or voltage
  - \( t_0 \) – represents time when source switches
  - \( x_f \) - final (asymptotic) value of current/voltage
  - \( \tau \) – time constant (\( RC \) or \( \frac{L}{R} \))
    - Transient is essentially zero after 5\( \tau \)

- Find values and plug into general solution
  - Steady-state for initial and final values
  - Two-port equivalents for \( \tau \)
Example Two-Port Equivalent

- Given a circuit with a parallel capacitor and inductor
  - Use Norton equivalent to make a parallel circuit equivalent
- Remember:
  - Capacitors add in parallel
  - Inductors add in series
RC/RL Circuits with General Sources

\[ \frac{RC}{dt} \frac{dv_c(t)}{} + v_c(t) = v_s(t) \]

- The solution is a differential equation of the form
  
  \[ \frac{dx(t)}{dt} + x(t) = f(t) \]

  Where \( f(t) \) the forcing function

- The full solution to the differential equation is composed of two terms
  
  \[ x(t) = x_p(t) + x_h(t) \]

- \( x_p(t) \) is the particular solution
  - The response to the particular forcing function
  - \( x_p(t) \) will be of the same functional form as the forcing function

  \[ f(t) = e^{st} \rightarrow x_p(t) = A e^{st}\]
  \[ f(t) = \cos(\omega t) \rightarrow x_p(t) = A \cos(\omega t) + B \sin(\omega t)\]

- \( x_h(t) \) is the homogeneous solution
  - “Natural” solution that is consistent with the differential equation for \( f(t) = 0 \)

  - The response to any initial conditions of the circuit

  - Solution of form
    
    \[ x_h(t) = Ke^{-t/\tau} \]
Second-Order Circuits

- RLC circuits contain two energy storage elements
  - This results in a differential equation of second order (has a second derivative term)
- Use circuit analysis techniques to develop a general 2nd-order differential equation of the form

\[
\frac{d^2i(t)}{dt^2} + 2\alpha \frac{di(t)}{dt} + \omega_0^2i(t) = f(t)
\]
  - Use KVL, KCL and I/V characteristics of inductance and capacitance to put equation into standard form
  - Must identify \(\alpha, \omega_0, f(t)\)
Useful I/V Relationships

- Inductor
  - \( v(t) = L \frac{di(t)}{dt} \)
  - \( i(t) = L \int_{t_0}^{t} v(t) \, dt + i(t_0) \)

- Capacitor
  - \( i(t) = C \frac{dv(t)}{dt} \)
  - \( v(t) = C \int_{t_0}^{t} i(t) \, dt + v(t_0) \)
Chapter 5 - Steady-State Sinusoidal Analysis
Steady-State Sinusoidal Analysis

- In Transient analysis, we saw response of circuit network had two parts
  \[ x(t) = x_p(t) + x_h(t) \]
- Natural response \( x_h(t) \) had an exponential form that decays to zero
- Forced response \( x_p(t) \) was the same form as forcing function
  - Sinusoidal source \( \rightarrow \) sinusoidal output
  - Output persists with the source \( \rightarrow \) at steady-state there is no transient so it is important to study the sinusoid response
Sinusoidal Currents and Voltages

• Sinusoidal voltage
  ▫ \( v(t) = V_m \cos(\omega_0 t + \theta) \)
  ▫ \( V_m \) - peak value of voltage
  ▫ \( \omega_0 \) - angular frequency in radians/sec
  ▫ \( \theta \) – phase angle in radians
• This is a periodic signal described by
  ▫ \( T \) – the period in seconds
    • \( \omega_0 = \frac{2\pi}{T} \)
  ▫ \( f \) – the frequency in Hz = 1/sec
    • \( \omega_0 = 2\pi f \)

• Note: Assuming that \( \theta \) is in degrees, we have
  ▫ \( t_{\text{max}} = -\theta/360 \times T. \)
Root-Mean-Square Values

\[ P_{avg} = \frac{\left[ \frac{1}{T} \int_0^T v^2(t) dt \right]^2}{R} \]

• Define rms voltage
  \[ V_{rms} = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt} \]
  \[ P_{avg} = \frac{V_{rms}^2}{R} \]

• Similarly define rms current
  \[ I_{rms} = \sqrt{\frac{1}{T} \int_0^T i^2(t) dt} \]
  \[ P_{avg} = I_{rms}^2 R \]
**RMS Value of a Sinusoid**

- **Given a sinusoidal source**
  - \( v(t) = V_m \cos(\omega_0 t + \theta) \)
  - \( V_{rms} = \sqrt{\frac{1}{T} \int_0^T v^2(t)dt} \)

- **The rms value is an “effective” value for the signal**
  - E.g. in homes we have 60Hz 115 V rms power
  - \( V_m = \sqrt{2} \cdot V_{rms} = 163 \text{ V} \)

\[
V_{rms} = \sqrt{\frac{1}{T} \int_0^T V_m^2 \cos^2(\omega_0 t + \theta)dt} \\
\text{using} \quad \cos^2(x) = \frac{1}{2} + \frac{1}{2} \cos(2x) \\
= \sqrt{\frac{V_m^2}{2T} \int_0^T [1 + \cos(2\omega_0 t + 2\theta)] dt} \\
= \frac{V_m}{\sqrt{2}}
\]
Conversion Between Forms

- Rectangular to polar form
  - $r^2 = x^2 + y^2$
  - $\tan \theta = \frac{y}{x}$

- Polar to rectangular form
  - $x = r \cos \theta$
  - $y = r \sin \theta$

- Convert to polar form
  - $z = 4 - j4$
  - $r = \sqrt{4^2 + 4^2} = 4\sqrt{2}$
  - $\theta = \arctan \left( \frac{y}{x} \right) = -\frac{\pi}{4}$
  - $z = 4\sqrt{2} e^{-j\pi/4}$

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Phasors

- A representation of sinusoidal signals as vectors in the complex plane
  - Simplifies sinusoidal steady-state analysis

- Given
  - \( v_1(t) = V_1 \cos(\omega t + \theta_1) \)
- The phasor representation is
  - \( V_1 = V_1 \angle \theta_1 \)
- For consistency, use only cosine for the phasor
  - \( v_2(t) = V_2 \sin(\omega t + \theta_2) = V_2 \cos(\omega t + \theta_2 - 90^\circ) \)
  - \( V_2 = V_2 \angle (\theta_2 - 90^\circ) \)

- Phasor diagram
  - \( V_1 = 3 \angle (40^\circ) \)
  - \( V_2 = 4 \angle (-20^\circ) \)
- Phasors rotate counter clockwise
  - \( V_1 \) leads \( V_2 \) by 60°
  - \( V_2 \) lags \( V_1 \) by 60°
Complex Impedance

• Impedance is the extension of resistance to AC circuits
  ▫ Extend Ohm’s Law to an impedance form for AC signals
    • \( V = ZI \)
• Inductors oppose a change in current
  ▫ \( Z_L = \omega L \angle \left( \frac{\pi}{2} \right) = j\omega L \)
  ▫ Current lags voltage by 90°
• Capacitors oppose a change in voltage
  ▫ \( Z_C = \frac{1}{\omega C} \angle \left( -\frac{\pi}{2} \right) = -j\frac{1}{\omega C} = \frac{1}{j\omega C} \)
  ▫ Current leads voltage by 90°
• Resistor impedance the same as resistance
  ▫ \( Z_R = R \)
Circuit Analysis with Impedance

- KVL and KCL remain the same
  - Use phasor notation to setup equations

- Replace sources by phasor notation
- Replace inductors, capacitors, and resistances by impedance value
  - This value is dependent on the source frequency $\omega$
- Use your favorite circuit analysis techniques to solve for voltage or current
  - Reverse phasor conversion to get sinusoidal signal in time