EE292: Fundamentals of ECE

Fall 2012
TTh 10:00-11:15 SEB 1242

Lecture 20
121101

http://www.ee.unlv.edu/~b1morris/ee292/

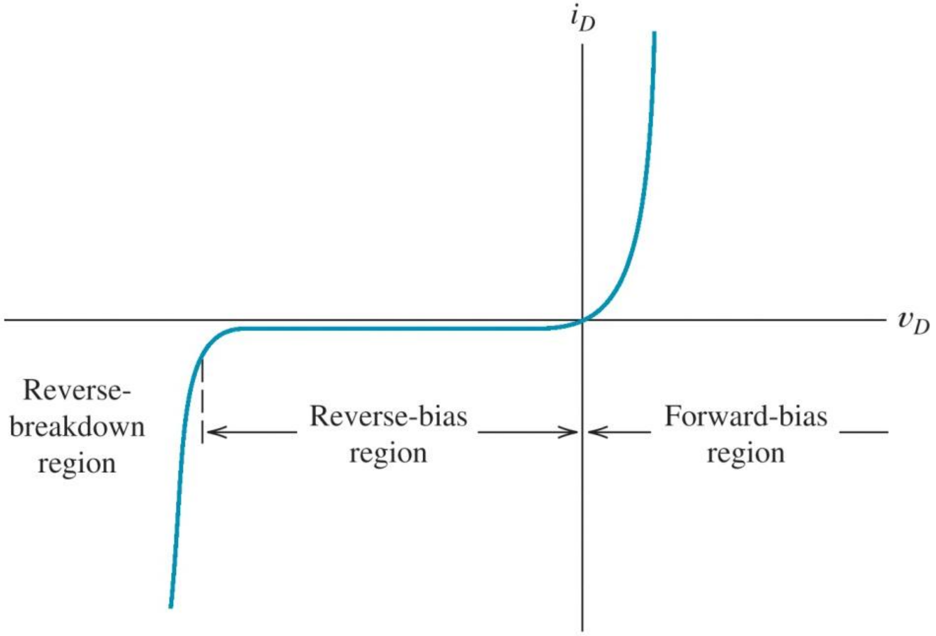
Outline

- Chapters 1-3
  - Circuit Analysis Techniques
- Chapter 10 – Diodes
  - Ideal Model
  - Offset Model
  - Zener Diodes
- Chapter 4 – Transient Analysis
  - Steady-State Analysis
  - 1^{st}-Order Circuits
- Chapter 5 – Steady-State Sinusoidal Analysis
  - RMS Values
  - Phasors
  - Complex Impedance
  - Circuit Analysis with Complex Impedance

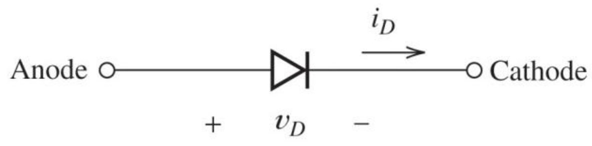
Chapter 10 – Diodes

Diode Voltage/Current Characteristics

- Forward Bias ("On")
  - Positive voltage v_D supports large currents
  - Modeled as a battery (0.7 V for offset model)
- Reverse Bias ("Off")
  - Negative voltage → no current
  - Modeled as open circuit
- Reverse-Breakdown
  - Large negative voltage supports large negative currents
  - Similar operation as for forward bias



(b) Volt–ampere characteristic



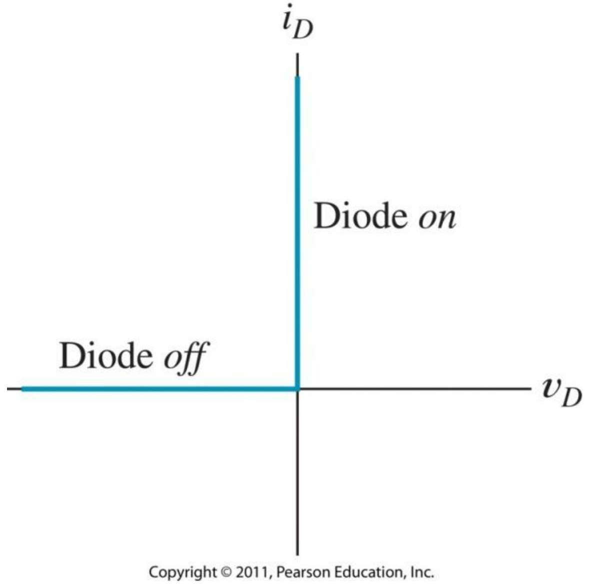
(a) Circuit symbol

Diode Models

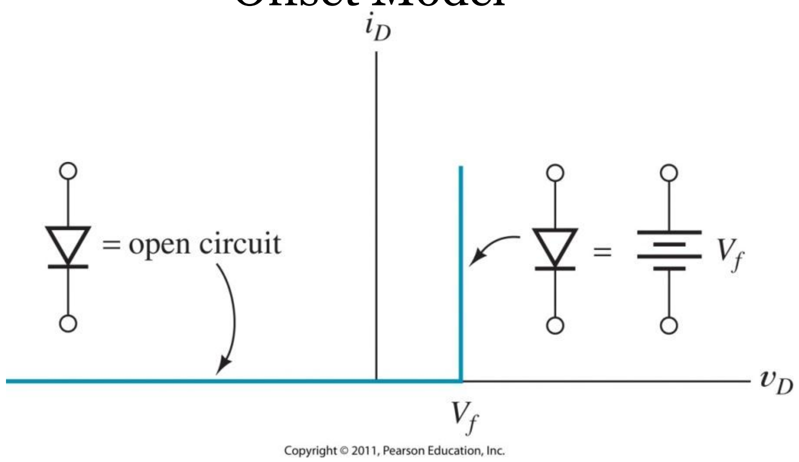
- Ideal model – simple
- Offset model – more realistic

- Two state model
- "On" State
  - Forward operation
  - Diode conducts current
    - Ideal model → short circuit
    - Offset model → battery
- "Off" State
  - Reverse biased
  - No current through diode → open circuit

Ideal Model



Offset Model



Circuit Analysis with Diodes

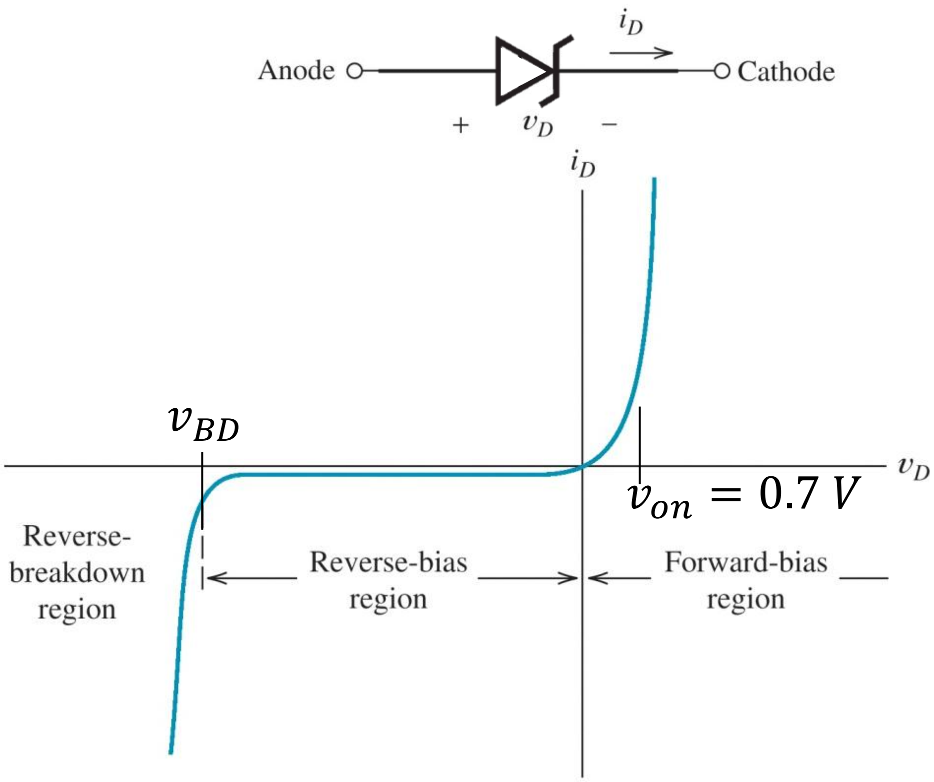
- Assume state {on, off} for each ideal diode and check if the initial guess was correct
  - $i_d > 0$ positive for "on" diode
  - $v_d < v_{on}$ for "off" diode
    - These imply a correct guess
  - Otherwise adjust guess and try again

- Exhaustive search is daunting
  - 2^n different combinations for n diodes
- Will require experience to make correct guess

Zener Diode

- Diode intended to be operated in breakdown
  - Constant voltage at breakdown
- Three state diode

1. On – 0.7 V forward bias
2. Off – reverse bias
3. Breakdown
   v_{BD} reverse breakdown voltage



(b) Volt–ampere characteristic

Chapter 4 – Transient Analysis

DC Steady-State Analysis

- Analysis of C, L circuits in DC operation
  - Steady-state – non-changing sources

- Capacitors $i = C\frac{dv}{dt}$
  - Voltage is constant → no current → open circuit
- Inductors $v = L\frac{di}{dt}$
  - Current is constant → no voltage → short circuit

- Use steady-state analysis to find initial and final conditions for transients

General 1st-Order Solution

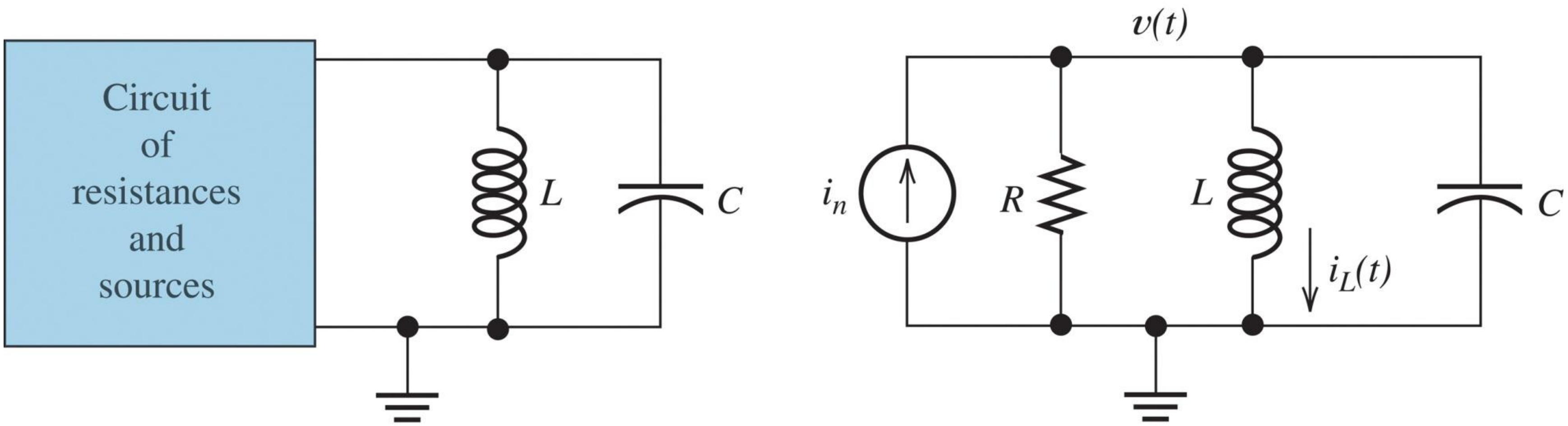
- Both the current and voltage in an 1st-order circuit has an exponential form
  - RC and LR circuits

- The general solution for current/voltage is:

$$x(t) = x_f + \left[x(t_0^+) - x_f\right] e^{-(t-t_0)/\tau}$$

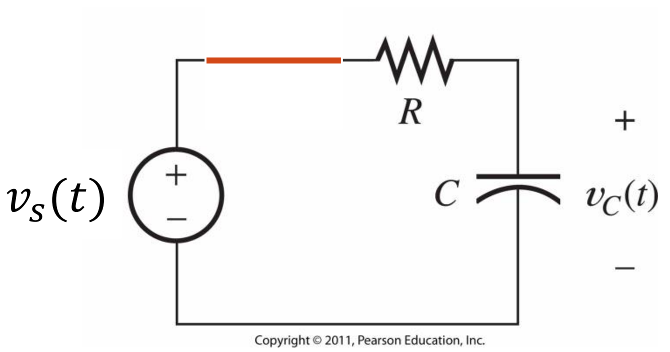
  - x – represents current or voltage
  - t_0 – represents time when source switches
  - x_f - final (asymptotic) value of current/voltage
  - τ – time constant (RC or $\frac{L}{R}$)
    - Transient is essentially zero after 5τ
- Find values and plug into general solution
  - Steady-state for initial and final values
  - Two-port equivalents for τ

Example Two-Port Equivalent



- Given a circuit with a parallel capacitor and inductor
  - Use Norton equivalent to make a parallel circuit equivalent
- Remember:
  - Capacitors add in parallel
  - Inductors add in series

RC/RL Circuits with General Sources



- $RC\frac{dv_c(t)}{dt} + v_c(t) = v_s(t)$
- The solution is a differential equation of the form
  - $\tau\frac{dx(t)}{dt} + x(t) = f(t)$
  - Where $f(t)$ the forcing function
- The full solution to the diff equation is composed of two terms
  - $x(t) = x_p(t) + x_h(t)$
- $x_p(t)$ is the particular solution
  - The response to the particular forcing function
  - $x_p(t)$ will be of the same functional form as the forcing function
  - $f(t) = e^{st} \rightarrow x_p(t) = Ae^{st}$
  - $f(t) = \cos(\omega t) \rightarrow x_p(t) = A\cos(\omega t) + B\sin(\omega t)$
- $x_h(t)$ is the homogeneous solution
  - "Natural" solution that is consistent with the differential equation for $f(t) = 0$
  - The response to any initial conditions of the circuit
  - Solution of form
    - $x_h(t) = Ke^{-t/\tau}$

Second-Order Circuits

- RLC circuits contain two energy storage elements
  - This results in a differential equation of second order (has a second derivative term)
- Use circuit analysis techniques to develop a general 2nd-order differential equation of the form

$$\frac{di^2(t)}{dt^2} + 2\alpha\frac{di(t)}{dt} + \omega_0^2 i(t) = f(t)$$

  - Use KVL, KCL and I/V characteristics of inductance and capacitance to put equation into standard form
  - Must identify $\alpha, \omega_0, f(t)$

Useful I/V Relationships

- Inductor
  - $v(t) = L\frac{di(t)}{dt}$
  - $i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)dt + i(t_0)$
- Capacitor
  - $i(t) = C\frac{dv(t)}{dt}$
  - $v(t) = \frac{1}{C}\int_{t_0}^{t} i(t)dt + v(t_0)$

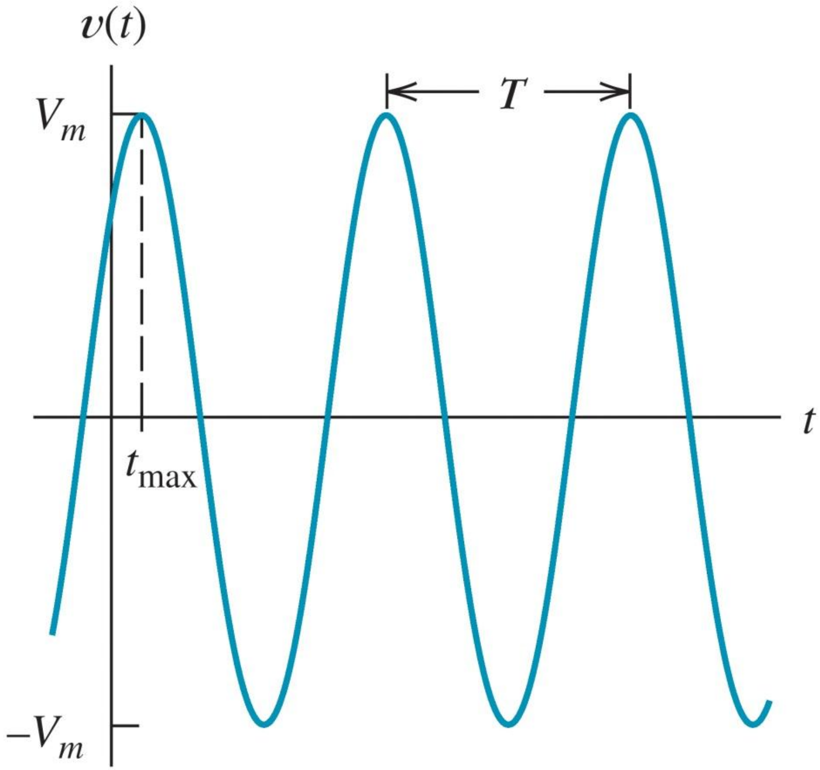
Chapter 5 – Steady-State Sinusoidal Analysis

Steady-State Sinusoidal Analysis

- In Transient analysis, we saw response of circuit network had two parts
  - $x(t) = x_p(t) + x_h(t)$
- Natural response $x_h(t)$ had an exponential form that decays to zero
- Forced response $x_p(t)$ was the same form as forcing function
  - Sinusoidal source → sinusoidal output
  - Output persists with the source → at steady-state there is no transient so it is important to study the sinusoid response

Sinusoidal Currents and Voltages

- Sinusoidal voltage
  - $v(t) = V_m \cos(\omega_0 t + \theta)$
  - V_m - peak value of voltage
  - ω_0 - angular frequency in radians/sec
  - θ – phase angle in radians
- This is a periodic signal described by
  - T – the period in seconds
    - $\omega_0 = \frac{2\pi}{T}$
  - f – the frequency in Hz = 1/sec
    - $\omega_0 = 2\pi f$



- Note: Assuming that θ is in degrees, we have
  - tmax = –θ/360 × T.

Root-Mean-Square Values

- $P_{avg} = \frac{\left[\sqrt{\frac{1}{T}\int_0^T v^2(t)dt}\right]^2}{R}$
- Define rms voltage
  - $V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt}$

  $$P_{avg} = \frac{V_{rms}^2}{R}$$
- Similarly define rms current
  - $I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt}$

  $$P_{avg} = I_{rms}^2 R$$

RMS Value of a Sinusoid

- Given a sinusoidal source
  - $v(t) = V_m \cos(\omega_0 t + \theta)$
- $V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt}$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T V_m^2 \cos^2(\omega_0 t + \theta)dt}$$

using $\cos^2(x) = 1/2 + 1/2\cos(2x)$

$$= \sqrt{\frac{V_m^2}{2T}\int_0^T \left[1 + \cos(2\omega_0 t + 2\theta)\right] dt}$$

$$\vdots$$

$$= \frac{V_m}{\sqrt{2}}$$

- The rms value is an “effective” value for the signal
  - E.g. in homes we have 60Hz 115 V rms power
  - $V_m = \sqrt{2} \cdot V_{rms} = 163\ V$

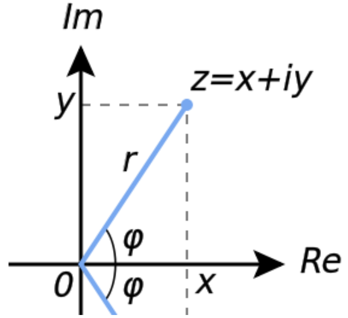
Conversion Between Forms

- Rectangular to polar form
- $r^2 = x^2 + y^2$
- $\tan\theta = \frac{y}{x}$

- Polar to rectangular form
- $x = r\cos\theta$
- $y = r\sin\theta$

- Convert to polar form
- $z = 4 - j4$
- $r = \sqrt{4^2 + 4^2} = 4\sqrt{2}$
- $\theta = \arctan\left(\frac{y}{x}\right) = \arctan(-1) = -\frac{\pi}{4}$
- $z = 4\sqrt{2}e^{-j\pi/4}$

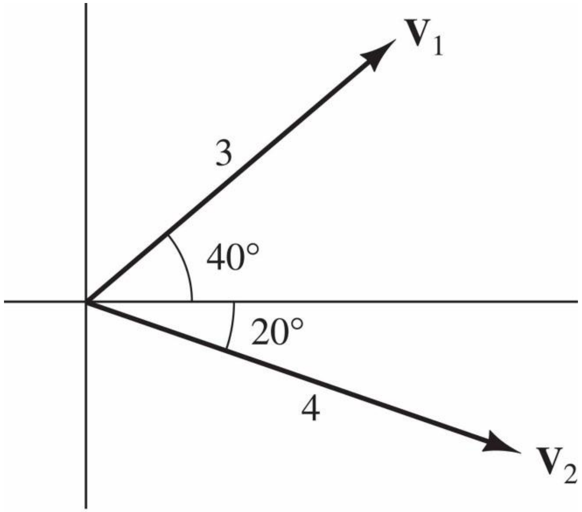
x **(degrees)**	x **(radians)**	$\sin(x)$	$\cos(x)$	$\tan(x)$
0	0	0	1	0
15	$\frac{\pi}{12}$	$\frac{-1+\sqrt{3}}{2\sqrt{2}}$	$\frac{1+\sqrt{3}}{2\sqrt{2}}$	$2-\sqrt{3}$
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
75	$\frac{5\pi}{12}$	$\frac{1+\sqrt{3}}{2\sqrt{2}}$	$\frac{-1+\sqrt{3}}{2\sqrt{2}}$	$2+\sqrt{3}$
90	$\frac{\pi}{2}$	1	0	NaN



Source: Wikipedia

Phasors

- A representation of sinusoidal signals as vectors in the complex plane
  - Simplifies sinusoidal steady-state analysis
- Given
  - $v_1(t) = V_1 \cos(\omega t + \theta_1)$
- The phasor representation is
  - $\boldsymbol{V}_1 = V_1 \angle \theta_1$
- For consistency, use only cosine for the phasor
  - $v_2(t) = V_2 \sin(\omega t + \theta_2) = V_2 \cos\left(\omega t + \theta_2 - 90^{\circ}\right)$
  - $\boldsymbol{V}_2 = V_2 \angle(\theta_2 - 90^{\circ})$
- Phasor diagram



  - $\boldsymbol{V}_1 = 3\angle\left(40^{\circ}\right)$
  - $\boldsymbol{V}_2 = 4\angle\left(-20^{\circ}\right)$
- Phasors rotate counter clockwise
  - $\boldsymbol{V}_1$ leads $\boldsymbol{V}_2$ by 60°
  - $\boldsymbol{V}_2$ lags $\boldsymbol{V}_1$ by 60°

Complex Impedance

- Impedance is the extension of resistance to AC circuits
  - Extend Ohm's Law to an impedance form for AC signals
    - $\boldsymbol{V} = Z\boldsymbol{I}$
- Inductors oppose a change in current
  - $Z_L = \omega L \angle \left(\frac{\pi}{2}\right) = j\omega L$
  - Current lags voltage by 90°
- Capacitors oppose a change in voltage
  - $Z_C = \frac{1}{\omega C} \angle \left(-\frac{\pi}{2}\right) = -j\frac{1}{\omega C} = \frac{1}{j\omega C}$
  - Current leads voltage by 90°
- Resistor impendence the same as resistance
  - $Z_R = R$

Circuit Analysis with Impedance

- KVL and KCL remain the same
  - Use phasor notation to setup equations

- Replace sources by phasor notation
- Replace inductors, capacitors, and resistances by impedance value
  - This value is dependent on the source frequency ω
- Use your favorite circuit analysis techniques to solve for voltage or current
  - Reverse phasor conversion to get sinusoidal signal in time