EE292: Fundamentals of ECE

Fall 2012
TTh 10:00-11:15 SEB 1242

Lecture 19
121030

http://www.ee.unlv.edu/~b1morris/ee292/

Outline

- Review
  - Phasors
  - Complex Impedance
- Circuit Analysis with Complex Impedance

Phasors

- A representation of sinusoidal signals as vectors in the complex plane
  - Simplifies sinusoidal steady-state analysis

- Given
  - $v_1(t) = V_1 \cos(\omega t + \theta_1)$
- The phasor representation is
  - $\boldsymbol{V}_1 = V_1 \angle \theta_1$
- For consistency, use only cosine for the phasor
  - $v_2(t) = V_2 \sin(\omega t + \theta_2) = V_2 \cos(\omega t + \theta_2 - 90^{\circ}) =$
    $\boldsymbol{V}_2 = V_2 \angle(\theta_2 - 90^{\circ})$

Adding Sinusoids with Phasors

1. Get phasor representation of sinusoids
2. Convert phasors to rectangular form and add
3. Simplify result and convert into phasor form
4. Convert phasor into sinusoid
   - Remember that ω should be the same for each sinusoid and the result will have the same frequency

Example 5.3

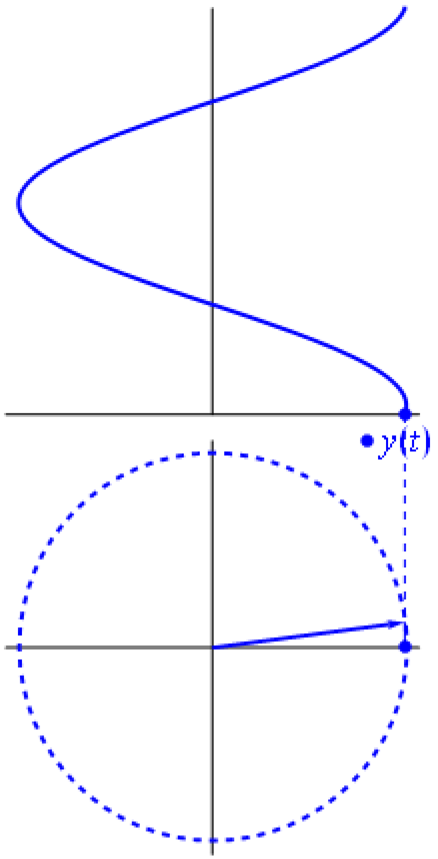
- $v_1(t) = 20\cos(\omega t - 45^\circ)$
  - $\boldsymbol{V}_1 = 20\angle(-45^\circ)$
- $v_2(t) = 10\sin(\omega t + 60^\circ)$
  - $\boldsymbol{V}_2 = 10\angle(60^\circ - 90^\circ) = 10\angle(-30^\circ)$
- Calculate
  - $\boldsymbol{V}_s = \boldsymbol{V}_1 + \boldsymbol{V}_2$

$$\begin{aligned}
\mathbf{V}_s &= \mathbf{V}_1 + \mathbf{V}_2 \\
&= 20\underline{/-45^\circ} + 10\underline{/-30^\circ} \\
&= 20\cos(-45^\circ) + j20\sin(-45^\circ) + 10\cos(-30^\circ) + j10\sin(-30^\circ) \\
&= \frac{20}{\sqrt{2}} - j\frac{20}{\sqrt{2}} + 10\frac{sqrt3}{2} - j\frac{10}{2} \\
&= 14.14 - j14.14 + 8.66 - j5 \\
&= 22.8 - j19.14 \\
&= \sqrt{22.8^2 + 19.14^2}\underline{/\arctan\left(\tfrac{-19.14}{22.8}\right)} \\
&= 29.77/-40.01^\circ
\end{aligned}$$

$$v_s(t) = 29.77\cos(\omega t - 40.01^\circ)$$

Phasor as a Rotating Vector

- $v(t) = V_m \cos(\omega t + \theta) = Re\{V_m e^{j(\omega t+\theta)}\}$
- $V_m e^{j(\omega t+\theta)}$ is a complex vector that rotates counter clockwise at ω rad/s
- $v(t)$ is the real part of the vector
  - The projection onto the real axis of the rotating complex vector

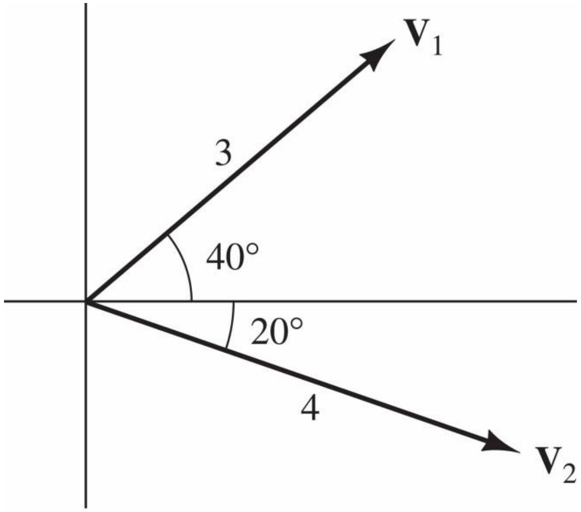


Source: Wikipedia

Phase Relationships

- Given
  - $v_1(t) = 3\cos(\omega t + 40^{\circ})$
  - $v_2(t) = 4\cos(\omega t - 20^{\circ})$
- In phasor notation
  - $\boldsymbol{V}_1 = 3\angle(40^{\circ})$
  - $\boldsymbol{V}_2 = 4\angle(-20^{\circ})$
- Since phasors rotate counter clockwise
  - $\boldsymbol{V}_1$ leads $\boldsymbol{V}_2$ by 60°
  - $\boldsymbol{V}_2$ lags $\boldsymbol{V}_1$ by 60°

- Phasor diagram



Complex Impedance

- Previously we saw that resistance was a measure of the opposition to current flow
  - Larger resistance → less current allowed to flow
- Impedance is the extension of resistance to AC circuits
  - Inductors oppose a change in current
  - Capacitors oppose a change in voltage
- Capacitors and inductors have imaginary impedance → called reactance

Resistance

- From Ohm's Law
  - $v(t) = Ri(t)$
- Extend to an impedance form for AC signals
  - $\boldsymbol{V} = Z\boldsymbol{I}$

- Converting to phasor notation for resistance
  - $\boldsymbol{V}_R = R\boldsymbol{I}_R$
  - $R = \frac{\boldsymbol{V}_R}{\boldsymbol{I}_R}$
- Comparison with the impedance form results in
  - $Z_R = R$
  - Since R is real, the impedance for a resistor is purely real

Inductance

- I/V relationship
  - $v_L(t) = L\frac{di_L(t)}{dt}$
- Assume current
  - $i_L(t) = I_m \sin(\omega t + \theta)$
  - $\boldsymbol{I}_L = I_m\angle\left(\theta - \frac{\pi}{2}\right)$
- Using the I/V relationship
  - $v_L(t) = L\omega I_m \cos(\omega t + \theta)$
  - $\boldsymbol{V}_L = \omega L I_m\angle(\theta)$
  - Notice current lags voltage by 90°
- Using the generalized Ohm's Law
  - $Z_L = \frac{\boldsymbol{V}_L}{\boldsymbol{I}_L} = \frac{\omega L I_m\angle(\theta)}{I_m\angle\left(\theta - \frac{\pi}{2}\right)} = \omega L\angle\left(\frac{\pi}{2}\right) = j\omega L$
  - Notice the impedance is completely imaginary

Capacitance

- I/V relationship
  - $i_C(t) = C\frac{dv_C(t)}{dt}$
- Assume voltage
  - $v_c(t) = V_m \sin(\omega t + \theta)$
  - $\boldsymbol{V}_C = V_m\angle\left(\theta - \frac{\pi}{2}\right)$
- Using the I/V relationship
  - $i_C(t) = C\omega V_m \cos(\omega t + \theta)$
  - $\boldsymbol{I}_C = \omega C V_m \angle(\theta)$
  - Notice current leads voltage by 90°
- Using the generalized Ohm's Law
  - $Z_C = \frac{\boldsymbol{V}_C}{\boldsymbol{I}_C} = \frac{V_m\angle\left(\theta - \frac{\pi}{2}\right)}{\omega C V_m\angle(\theta)} = \frac{1}{\omega C}\angle\left(-\frac{\pi}{2}\right) = -j\frac{1}{\omega C} = \frac{1}{j\omega C}$
  - Notice the impedance is completely imaginary

Circuit Analysis with Impedance

- KVL and KCL remain the same
  - Use phasor notation to setup equations

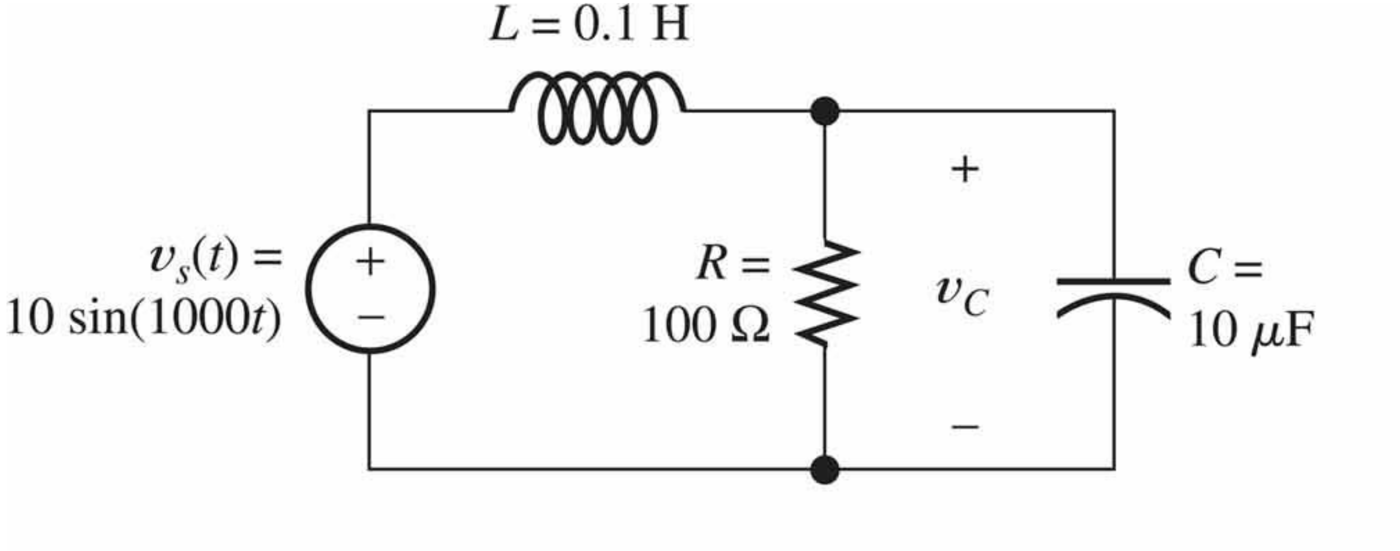
- E.g.
- $v_1(t) + v_2(t) - v_3(t) = 0$
- $\boldsymbol{V}_1 + \boldsymbol{V}_2 - \boldsymbol{V}_3 = 0$

Steps for Sinusoidal Steady-State Analysis

1. Replace time descriptions of voltage and current sources with corresponding phasors. (All sources must have the same frequency)
2. Replace inductances by their complex impedances $Z_L = \omega L \angle \left(\frac{\pi}{2}\right) = j\omega L$. Replace capacitances by their complex impedances $Z_C = \frac{1}{\omega C} \angle \left(-\frac{\pi}{2}\right) = \frac{1}{j\omega C}$. Resistances have impedances equal to their resistances.
3. Analyze the circuit by using any of the techniques studied in Chapter 2, and perform the calculations with complex arithmetic

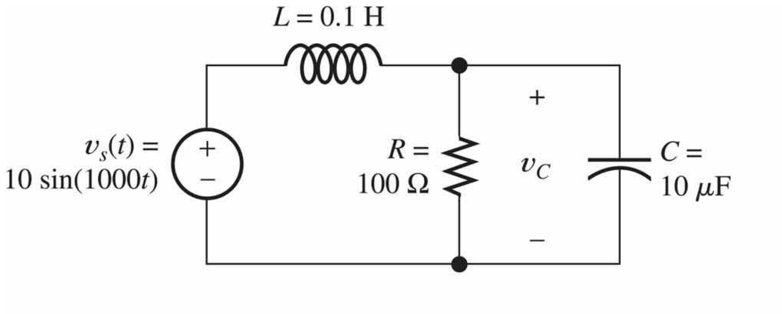
Example 5.5

- Find voltage $v_c(t)$ in steady-state
- Find the phasor current through each element
- Construct the phasor diagram showing currents and v_s



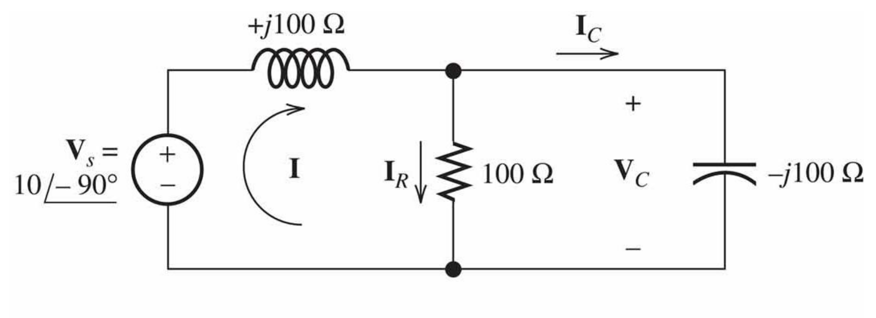
Convert Sources and Impedances

- Voltage source
  - $\boldsymbol{V}_s = 10\angle(-90^{\circ})$
  - $\omega = 1000$



- Inductance
  - $Z_L = j\omega L = \text{j}(1000)(0.1) = \text{j}100\ \Omega$

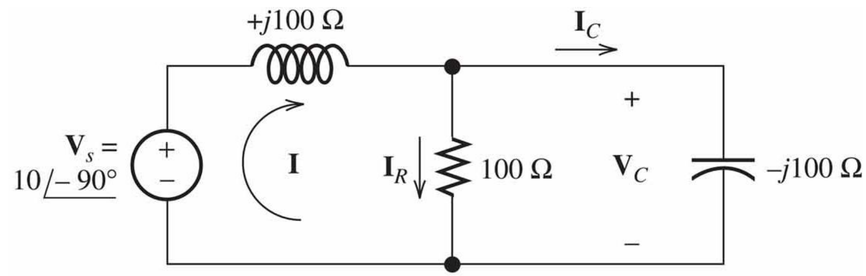
- Capacitance
  - $Z_C = \dfrac{1}{j\omega C} = \dfrac{1}{j1000(10\mu)} = \dfrac{10^6}{j10^4} = \dfrac{100}{j}\ \Omega$



Find Voltage $v_c(t)$

- Find voltage by voltage divider
- $\boldsymbol{V_C} = \boldsymbol{V_s}\left(\frac{Z_{eq}}{Z_{eq}+Z_L}\right)$

$$
\begin{aligned}
\mathbf{V}_C &= \mathbf{V}_s\left(\frac{Z_{eq}}{Z_{eq}+Z_L}\right)\\
&= 10\angle{-90^\circ}\left(\frac{\frac{100}{\sqrt{2}}\angle{-45^\circ}}{50-j50+j100}\right)\\
&= 10\angle{-90^\circ}\left(\frac{\frac{100}{\sqrt{2}}\angle{-45^\circ}}{50+j50}\right)\\
&= 10\angle{-90^\circ}\left(\frac{\frac{100}{\sqrt{2}}\angle{-45^\circ}}{\frac{100}{\sqrt{2}}\angle{45^\circ}}\right)\\
&= 10\angle{-180^\circ}
\end{aligned}
$$

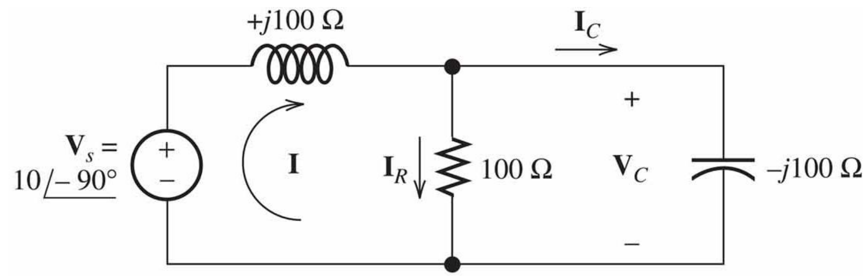


- Equivalent impedance
- $Z_{eq} = Z_R || Z_C$

$$
\begin{aligned}
Z_{eq} &= \frac{Z_C Z_R}{Z_C + Z_R} = \frac{100\angle{-\pi/2}(100\angle{0})}{-j100+100}\\
&= \frac{100^2\angle{-90^\circ}}{100\sqrt{2}\angle{-45^\circ}} = \frac{100}{\sqrt{2}}\angle{-45^\circ}\\
&= 70.71\angle{-45^\circ}\\
&= 50 - j50
\end{aligned}
$$

$$
v_c(t) = 10\cos(\omega t - 180^\circ) = 10\cos(1000t - \pi) = -10\cos(1000t)
$$

Phasor Diagram



- Source current

$$\mathbf{I} = \frac{\mathbf{V}_s}{Z_{eq} + Z_L} = \frac{10\underline{/-90^\circ}}{100/\sqrt{2}\underline{/45^\circ}} = \frac{\sqrt{2}}{10}\underline{/-135^\circ}$$

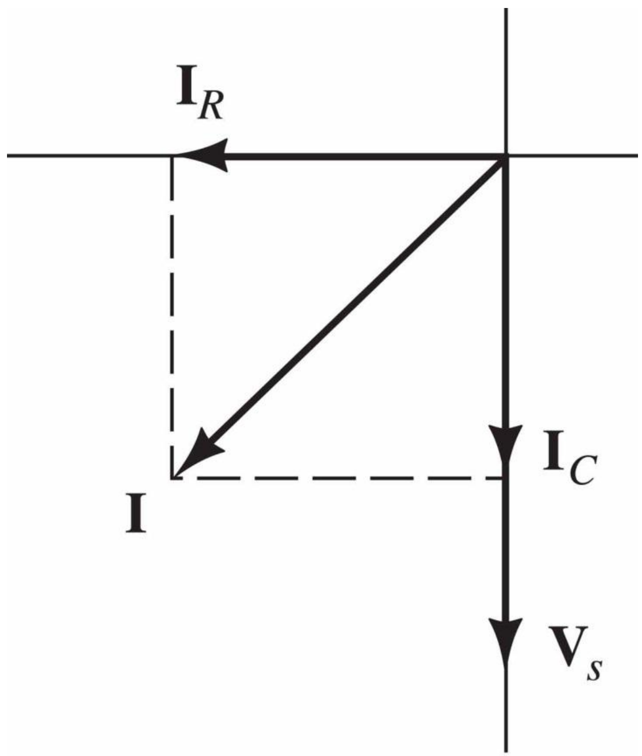
- Capacitor current

$$\mathbf{I}_C = \frac{\mathbf{V}_C}{Z_C} = \frac{10\underline{/-180^\circ}}{100\underline{/-90^\circ}} = \frac{1}{10}\underline{/-90^\circ}$$

- Resistor current

$$\mathbf{I}_R = \frac{\mathbf{V}_C}{Z_R} = \frac{10\underline{/-180^\circ}}{100\underline{/0^\circ}} = \frac{1}{10}\underline{/-180^\circ}$$

- Phasor diagram

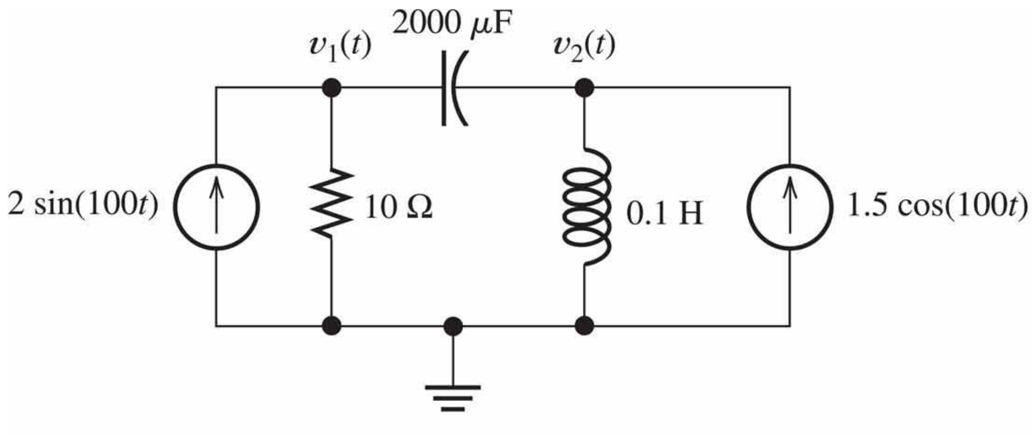


More AC Circuit Analysis

- Use impedance relationships and convert sinusoidal sources to phasors

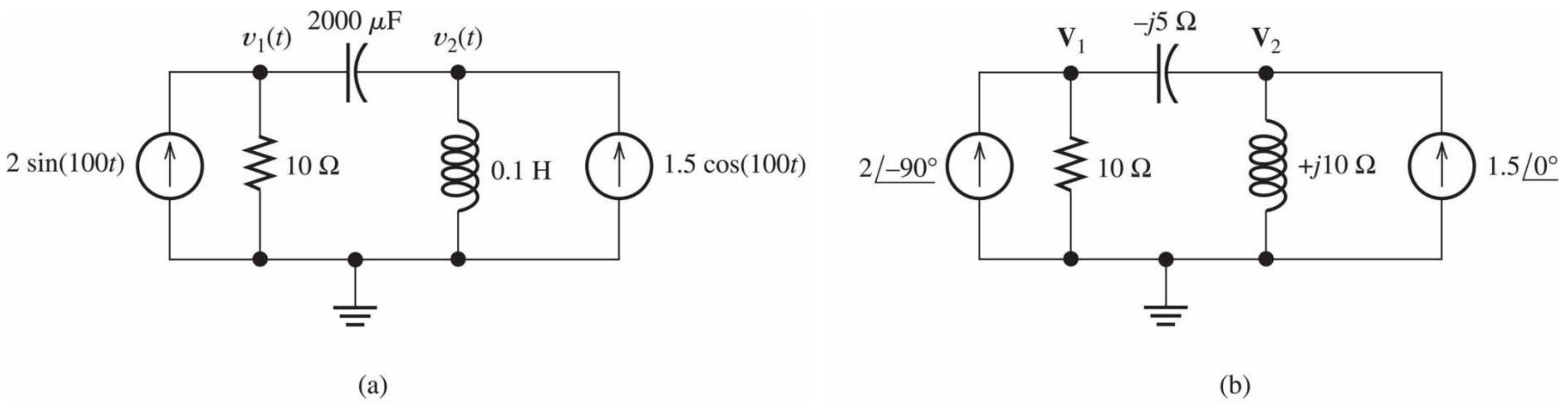
- Techniques such as node-voltage and mesh-current analysis remain the same
  - (Hambley Section 5.4)
- Thevenin and Norton equivalents are extended the same way
  - (Hambley Section 5.6)
  - Instead of a resistor and source, use an impedance
  - $Z_t = \frac{\mathbf{V}_{oc}}{\mathbf{I}_{sc}}$

Example 5.6



- Use node-voltage to find $v_1(t)$

Convert to Phasors



- Sources
  - $2\sin(100t) = 2\angle(-90^{\circ})$
  - -90° because of conversion to cosine from sine
  - $\omega = 100$
  - $1.5\cos(100t) = 1.5\angle(0^{\circ})$
- Inductor
  - $Z_L = \omega L\angle\left(\frac{\pi}{2}\right) = j\omega L = j100(0.1) = j10$
- Capacitor
  - $Z_C = \frac{1}{\omega C}\angle\left(-\frac{\pi}{2}\right) = \frac{1}{j\omega C} = -j\frac{1}{100(2000\mu)} = -j5$

Use Node-Voltage Analysis

- KCL @ 1
  - $\frac{\boldsymbol{V}_1}{10} + \frac{\boldsymbol{V}_1 - \boldsymbol{V}_2}{-j5} = 2\angle(-90^{\circ})$
- KCL @ 2
  - $\frac{\boldsymbol{V}_2}{j10} + \frac{\boldsymbol{V}_2 - \boldsymbol{V}_1}{-j5} = 1.5\angle(0^{\circ})$

- In standard form
- $(0.1 + j0.2)\boldsymbol{V}_1 - j0.2\boldsymbol{V}_2 = -j2$
- $-j0.2\boldsymbol{V}_1 + j0.1\boldsymbol{V}_2 = 1.5$

- Solving for $\boldsymbol{V}_1$
- $(0.1 - j0.2)\boldsymbol{V}_1 = 3 - j2$

- Converting to phasor
- $0.2236\angle(-63.4^{\circ})\boldsymbol{V}_1 = 3.6\angle(-33.7^{\circ})$

- $\boldsymbol{V}_1 = 16.1\angle(29.7^{\circ})$

- Convert back to sinusoid
- $v_1(t) = 16.1\cos(100t + 29.7^{\circ})$