EE292: Fundamentals of ECE

Fall 2012
TTh 10:00-11:15 SEB 1242

Lecture 17
121023

http://www.ee.unlv.edu/~b1morris/ee292/

Outline

- Review
  - 1st-Order Transients
  - General Sources
  - 2nd-Order Circuits
- Steady-State Sinusoidal Analysis
- Root-Mean-Square Values
- Complex Number Review
- Phasors

General 1st-Order Solution

- Both the current and voltage in an 1st-order circuit has an exponential form
  - RC and LR circuits

- The general solution for current/voltage is:

$$x(t) = x_f + \left[x(t_0^+) - x_f\right] e^{-(t-t_0)/\tau}$$

  - x – represents current or voltage
  - t_0 – represents time when source switches
  - x_f - final (asymptotic) value of current/voltage
  - τ – time constant (RC or $\frac{L}{R}$)
- Find values and plug into general solution

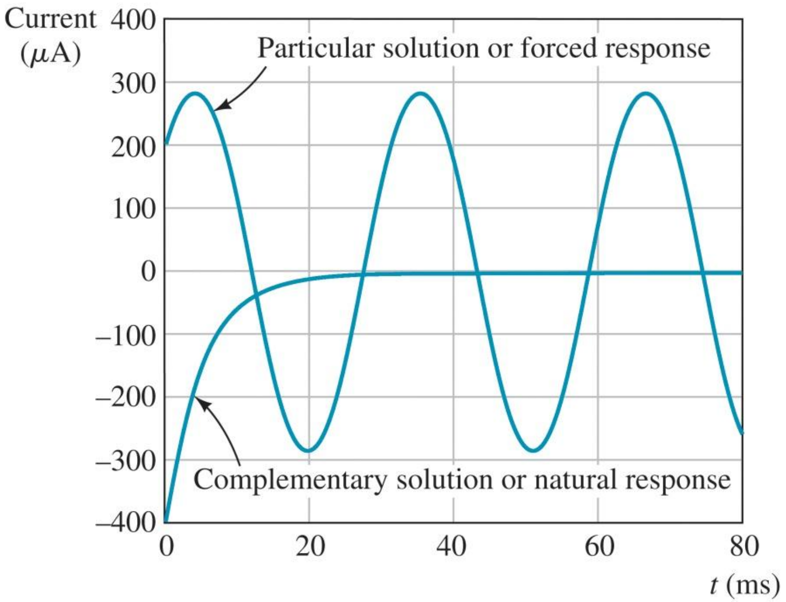
General Differential Equations

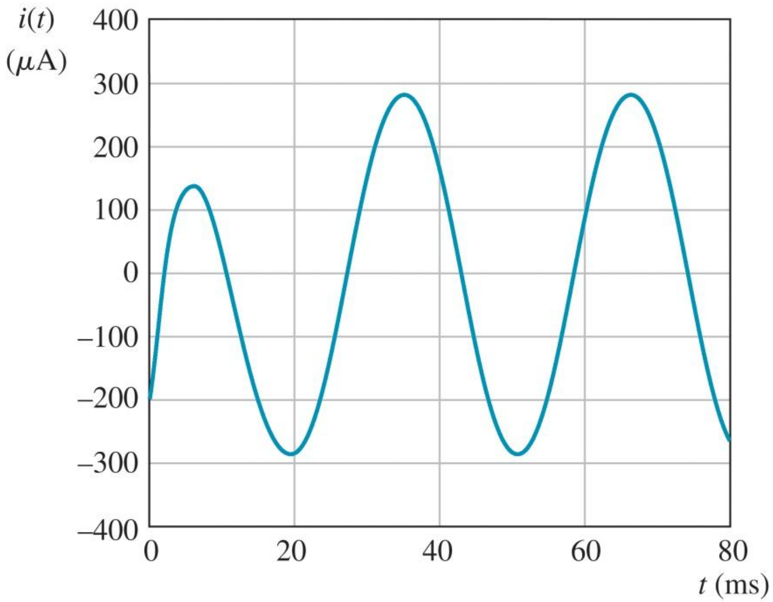
- General differential equation
  - $\tau \frac{dx(t)}{dt} + x(t) = f(t)$
- The full solution to the diff equation is composed of two terms
  - $x(t) = x_p(t) + x_h(t)$

- $x_p(t)$ is the particular solution
  - The response to the particular forcing function
  - Solution has the same form as the forcing function
- $x_h(t)$ is the homogeneous solution (natural response)
  - Another solution that is consistent with the differential equation for $f(t) = 0$
  - The response to any initial conditions of the circuit
  - Exponential form for the solution
    - $x_h(t) = Ke^{-t/\tau}$

General Differential Solution

- Notice the final solution is the sum of the particular and homogeneous solutions
  - $x(t) = x_p(t) + x_h(t)$
- It has an exponential term due to $x_h(t)$ and a term $x_p(t)$ that matches the input source





Second-Order Circuits

- RLC circuits contain two energy storage elements
  - This results in a differential equation of second order (has a second derivative term)

- Differential equation is of the form

$$\frac{dx^2(t)}{dt^2} + 2\alpha\frac{dx(t)}{dt} + \omega_0^2 x(t) = f(t)$$

  - Use KVL, KCL and I/V characteristics of inductance and capacitance to put equation into standard form

Chapter 5 - Steady-State Sinusoidal Analysis

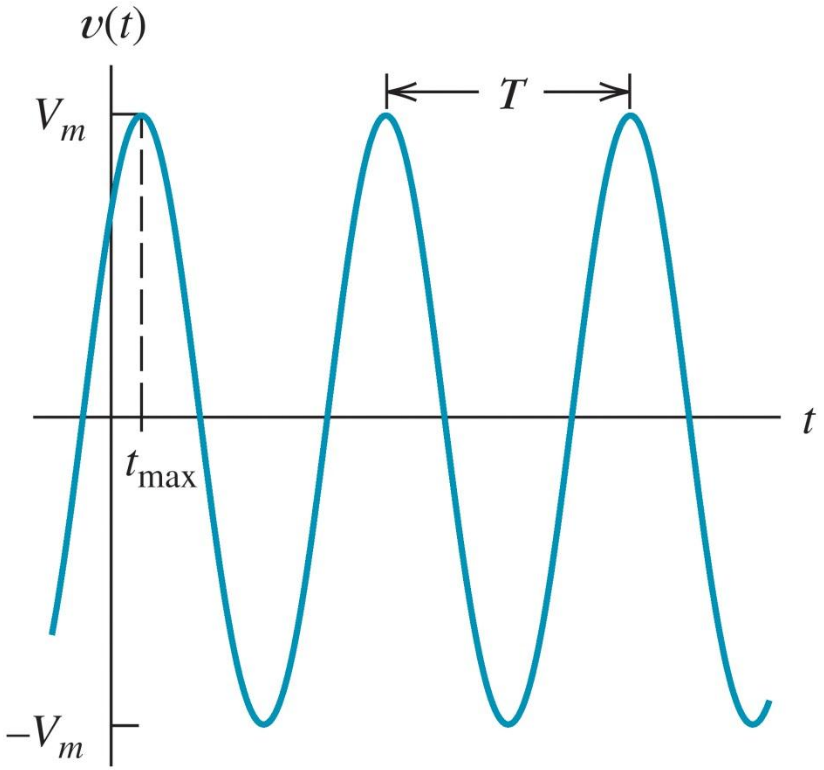
- Sinusoidal sources have many applications
  - Power distribution (current and voltages) in homes
  - Radio communications
  - Fourier analysis – signals can be comprised of linear combinations of sinusoids

Steady-State Sinusoidal Analysis

- In Chapter 4, transient analysis, we saw response of circuit network had two parts
  - $x(t) = x_p(t) + x_h(t)$
- Natural response $x_h(t)$ had an exponential form that decays to zero
- Forced response $x_p(t)$ was the same form as forcing function
  - Sinusoidal source → sinusoidal output
  - Output persists with the source → at steady-state there is no transient so it is important to study the sinusoid response

Sinusoidal Currents and Voltages

- Sinusoidal voltage
  - $v(t) = V_m \cos(\omega_0 t + \theta)$
  - V_m - peak value of voltage
  - ω_0 - angular frequency in radians/sec
  - θ – phase angle in radians
- This is a periodic signal described by
  - T – the period in seconds
    - $\omega_0 = \frac{2\pi}{T}$
  - f – the frequency in Hz = 1/sec
    - $\omega_0 = 2\pi f$



- Note: Assuming that θ is in degrees, we have
  - tmax = –θ/360 × T.

Sinusoidal Currents and Voltages

- For consistency/uniformity always express a sinusoid as a cosine
- Convert between sine and cosine
- In Degrees
  - $\sin(x) = \cos(x - 90^{\circ})$
- In Radians
  - $\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$

- $v(t) = 10\sin(200t + 30^{\circ})$
- $v(t) = 10\cos(200t + 30^{\circ} - 90^{\circ})$
- $v(t) = 10\cos(200t - 60^{\circ})$

Root-Mean-Square Values

- Apply a sinusoidal source to a resistance
- Power absorbed
  - $p(t) = \frac{v^2(t)}{R}$
- Energy in a single period
  - $E_T = \int_0^T p(t)dt$
- Average power (power absorbed in a single period)
  - $P_{avg} = \frac{E_T}{T} = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T \frac{v^2(t)}{R}dt$
  - $P_{avg} = \frac{\left[\sqrt{\frac{1}{T}\int_0^T v^2(t)dt}\right]^2}{R}$

Root-Mean-Square Values

- $P_{avg} = \frac{\left[\sqrt{\frac{1}{T}\int_0^T v^2(t)dt}\right]^2}{R}$
- Define rms voltage
  - $V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt}$

  $$P_{avg} = \frac{V_{rms}^2}{R}$$
- Similarly define rms current
  - $I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt}$

  $$P_{avg} = I_{rms}^2 R$$

RMS Value of a Sinusoid

- Given a sinusoidal source
  - $v(t) = V_m \cos(\omega_0 t + \theta)$
- $V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt}$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T V_m^2 \cos^2(\omega_0 t + \theta)dt}$$

using $\cos^2(x) = 1/2 + 1/2\cos(2x)$

$$= \sqrt{\frac{V_m^2}{2T}\int_0^T \left[1 + \cos(2\omega_0 t + 2\theta)\right] dt}$$

$$\vdots$$

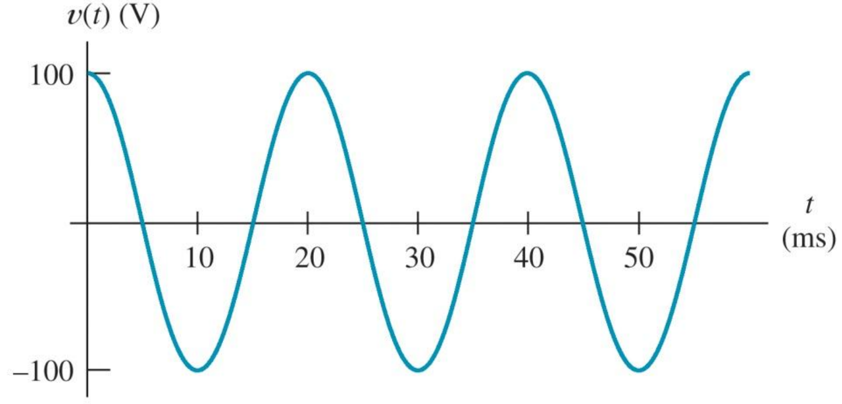
$$= \frac{V_m}{\sqrt{2}}$$

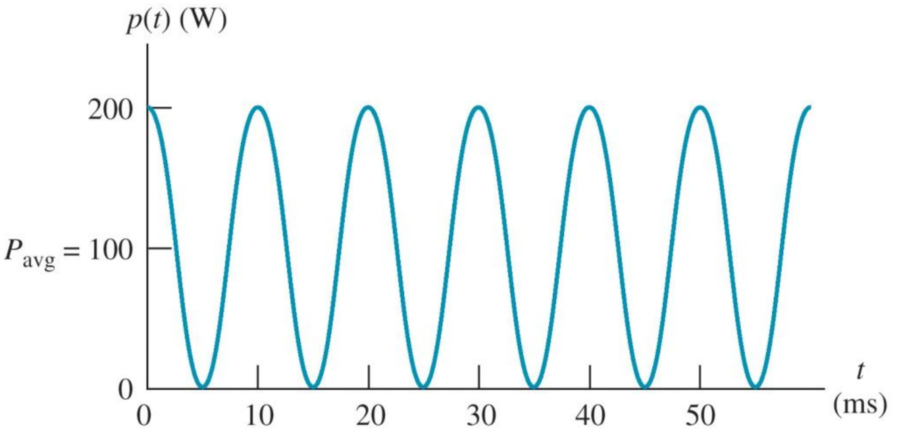
- The rms value is an “effective” value for the signal
  - E.g. in homes we have 60Hz 115 V rms power
  - $V_m = \sqrt{2} \cdot V_{rms} = 163\ V$

Example 5.1

- Voltage
  - $v(t) = 100\cos(100\pi t)$ V
- Applied to a 50Ω resistance

- Find the rms voltage and average power and plot power
- $\omega_0 = 100\pi$
- $f = \frac{\omega}{2\pi} = 50$ Hz
- $T = \frac{1}{f} = \frac{1}{50} = 20\text{msec}$
- $p(t) = \frac{v^2(t)}{R} = \frac{1}{50}100^2\cos^2(100\pi t) = 200\cos^2(100\pi t)$ W
- $V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.71\ V$
- $P_{avg} = \quad \frac{V_{rms}^2}{R} \quad \frac{70.71^2}{50} = 100$ W





Appendix A – Complex Numbers

- Complex numbers involve the imaginary number $j = \sqrt{-1}$
  - Not i as used in math because we know that i is already used for currents in circuits

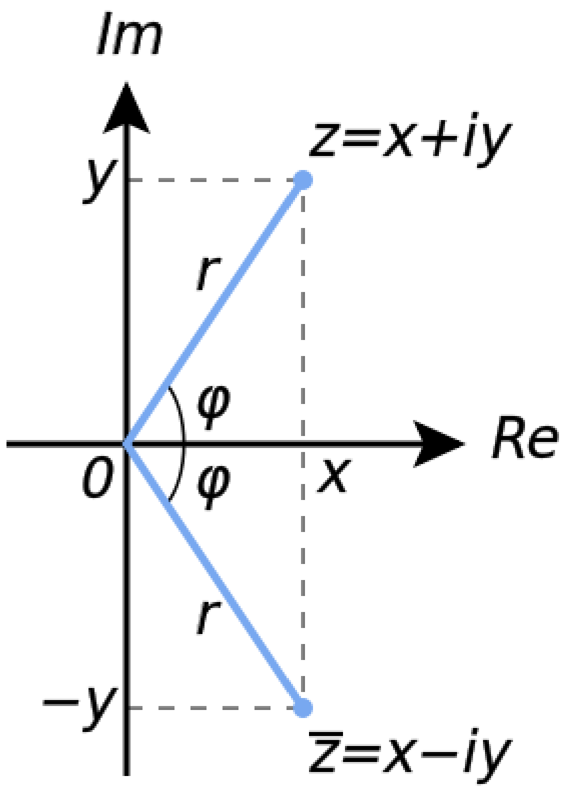
Complex Numbers in Rectangular Form

- A complex number in rectangular form

$$z = x + jy$$

  - x – real part
  - y – imaginary part

- A vector in the complex plane with
  - x – horizontal coordinate
  - y – vertical coordinate



Source: Wikipedia

Complex Arithmetic in Rectangular Form

- $z_1 = 5 + j5, \quad z_2 = 3 - j4$

- Summation
  - Add the real and complex parts separately

- $z_1 + z_2 = (5 + j5) + (3 - j4)$
- $z_1 + z_2 = (8 + j)$

- $z_1 - z_2 = (5 + j5) - (3 - j4)$
- $z_1 - z_2 = (2 + j9)$

- Multiplication
  - Use $j^2 = -1$
- $z_1 z_2 = (5 + j5)(3 - j4)$
- $\quad = 15 - j20 + j15 - j^2 20$
- $\quad = 15 - j5 - (-1)20$
- $\quad = 35 - j5$

- Division
  - Multiply num/den by complex conjugate term to remove imaginary term in denominator
- $\frac{z_1}{z_2} = \frac{(5+j5)}{(3-j4)} \cdot$

Complex Arithmetic in Rectangular Form

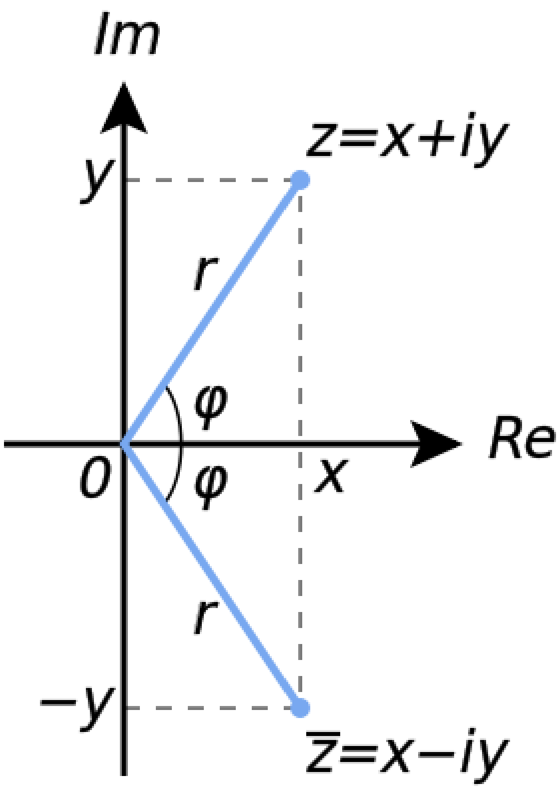
- Division
  - Multiply num/den by complex conjugate term to remove imaginary term in denominator

$$\begin{aligned}\frac{z_1}{z_2} &= \frac{5+j5}{3-j4}\cdot\frac{z_2^*}{z_2^*}\\ &= \frac{5+j5}{3-j4}\cdot\frac{3+j4}{3+j4}\\ &= \frac{15+j20+j15+j^2(20)}{9-j12+j12-j^2(16)}\\ &= \frac{-5+j35}{25}\\ &= -0.2+j1.4\end{aligned}$$

Complex Numbers in Polar Form

- Number represented by magnitude and phase
- Magnitude – the length of the complex vector
- Phase – the angle between the real axis and the vector

- Polar notation
- $z = re^{j\theta}$
- Phasor notation
- $z = r\angle\theta$



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Conversion Between Forms

- Rectangular to polar form
- $r^2 = x^2 + y^2$
- $\tan\theta = \frac{y}{x}$

- Polar to rectangular form
- $x = r\cos\theta$
- $y = r\sin\theta$

- Convert to polar form
- $z = 4 - j4$
- $r = \sqrt{4^2 + 4^2} = 4\sqrt{2}$
- $\theta = \arctan\left(\frac{y}{x}\right) = \arctan(-1) = -\frac{\pi}{4}$
- $z = 4\sqrt{2}e^{-j\pi/4}$