EE292: Fundamentals of ECE

Fall 2012
TTh 10:00-11:15 SEB 1242

Lecture 16
121018

http://www.ee.unlv.edu/~b1morris/ee292/

Outline

- Review
  - 1^{st}-Order Transients
  - General Sources
  - 2^{nd}-Order Circuits

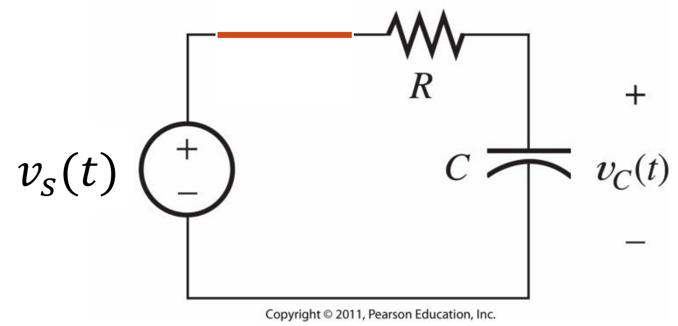
General 1st-Order Solution

- Both the current and voltage in an 1st-order circuit has an exponential form
  - RC and LR circuits
- The general solution for current/voltage is:

$$x(t) = x_f + \left[x(t_0^+) - x_f\right] e^{-(t-t_0)/\tau}$$

  - x – represents current or voltage
  - t_0 – represents time when source switches
  - x_f - final (asymptotic) value of current/voltage
  - τ – time constant (RC or $\frac{L}{R}$)
- Find values and plug into general solution

RC/RL Circuits with General Sources



- Previously,
  - $RC\dfrac{dv_c(t)}{dt} + v_c(t) = V_s$
- What if V_s is not constant
  - $RC\dfrac{dv_c(t)}{dt} + v_c(t) = v_s(t)$
  - Now have a general source that is a function of time
- The solution is a differential equation of the form
  - $\tau\dfrac{dx(t)}{dt} + x(t) = f(t)$
  - Where $f(t)$ is known as the forcing function (the circuit source)

General Differential Equations

- General differential equation
  - $\tau \frac{dx(t)}{dt} + x(t) = f(t)$
- The full solution to the diff equation is composed of two terms
  - $x(t) = x_p(t) + x_h(t)$

- $x_p(t)$ is the particular solution
  - The response to the particular forcing function
- $x_h(t)$ is the homogeneous solution
  - Another solution that is consistent with the differential equation for $f(t) = 0$
  - The response to any initial conditions of the circuit

Particular Solution

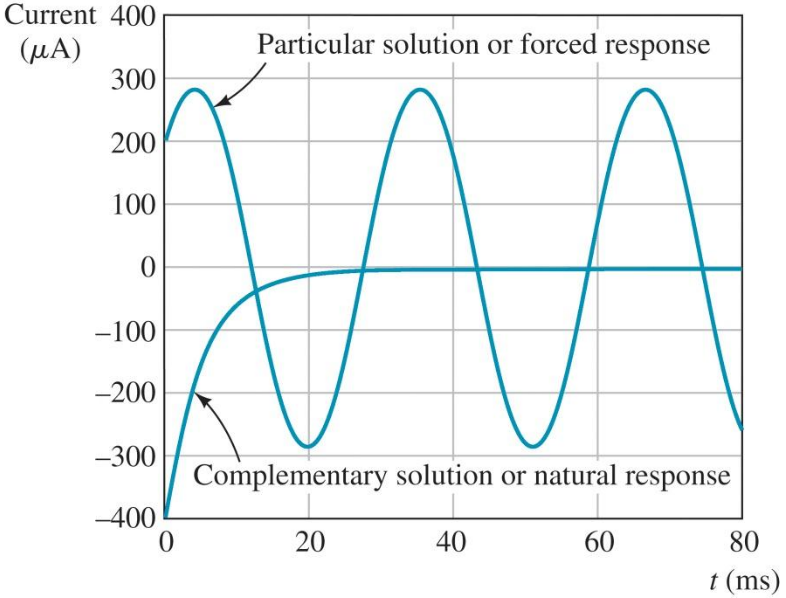
- $\tau \frac{dx_p(t)}{dt} + x_p(t) = f(t)$
- The solution $x_p(t)$ is called the forced response because it is the response of the circuit to a particular forcing input $f(t)$
- The solution $x_p(t)$ will be of the same functional form as the forcing function
  - E.g.
  - $f(t) = e^{st} \rightarrow x_p(t) = Ae^{st}$
  - $f(t) = \cos(\omega t) \rightarrow x_p(t) = A\cos(\omega t) + B\sin(\omega t)$

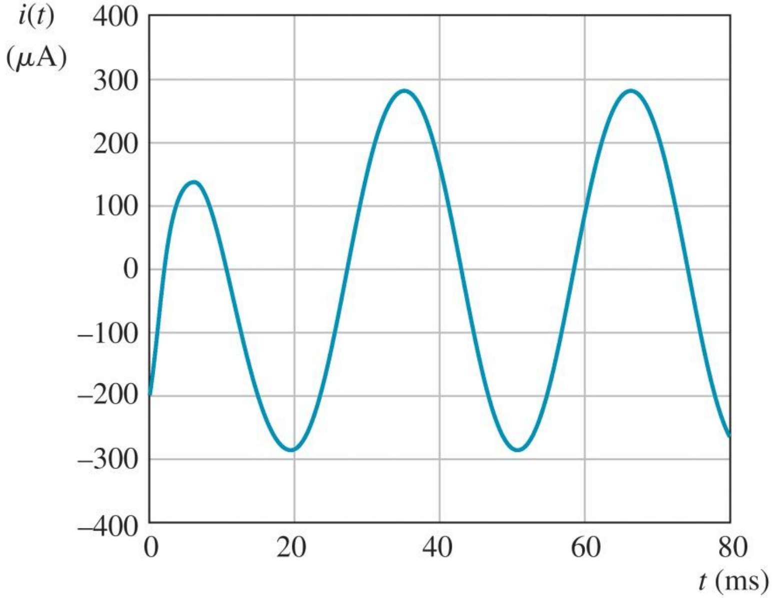
Homogeneous Solution

- $\tau \frac{dx_h(t)}{dt} + x_h(t) = 0$
- $x_h(t)$ is the solution to the differential equation when there is no forcing function
  - Does not depend on the sources
  - Dependent on initial conditions (capacitor voltage, current through inductor)
- $x_h(t)$ is also known as the natural response
- Solution is of the form
  - $x_h(t) = Ke^{-t/\tau}$

General Differential Solution

- Notice the final solution is the sum of the particular and homogeneous solutions
  - $x(t) = x_p(t) + x_h(t)$
- It has an exponential term due to $x_h(t)$ and a term $x_p(t)$ that matches the input source

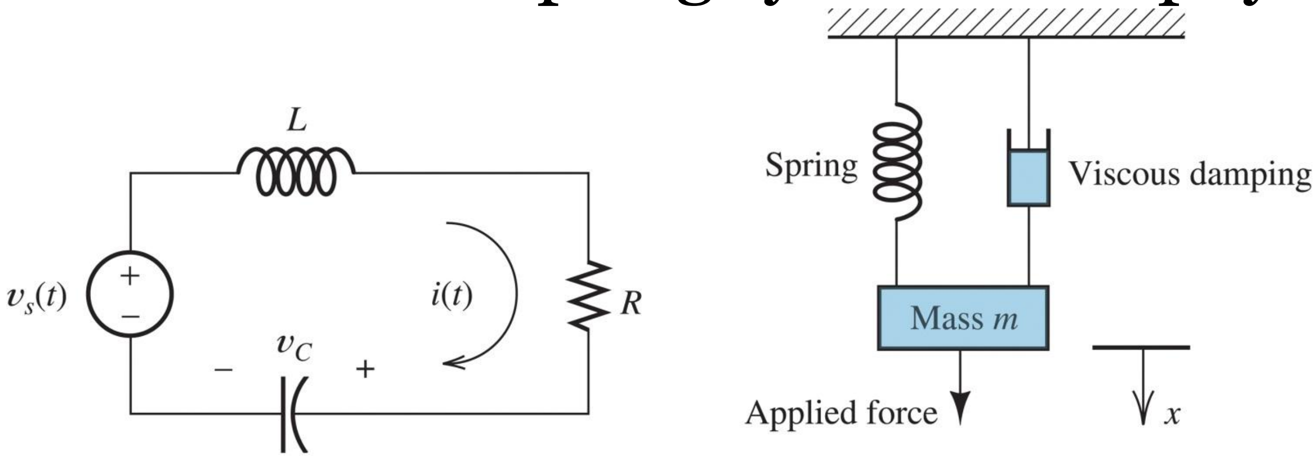




Second-Order Circuits

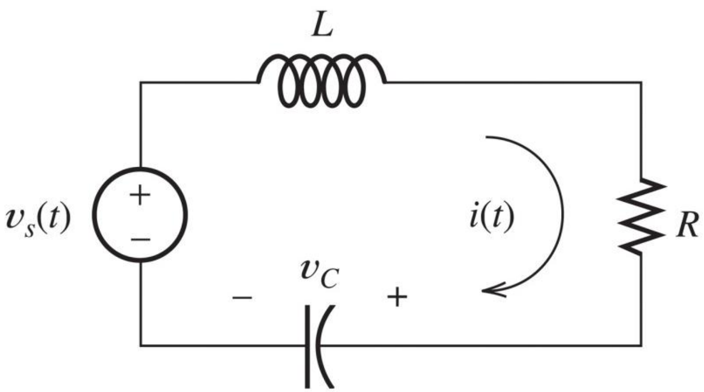
- RLC circuits contain two energy storage elements
  - This results in a differential equation of second order (has a second derivative term)

- This is like a mass spring system from physics



(a) Electrical circuit (b) Mechanical analog

RLC Series Circuit



- KVL around loop
  - $v_s(t) - L\frac{di(t)}{dt} - i(t)R - v_c(t) = 0$
- Solve for $v_c(t)$
  - $v_c(t) = v_s(t) - L\frac{di(t)}{dt} - i(t)R$
- Take derivative
  - $\frac{dv_c(t)}{dt} = \frac{dv_s(t)}{dt} - L\frac{d^2i(t)}{dt^2} - R\frac{di(t)}{dt}$

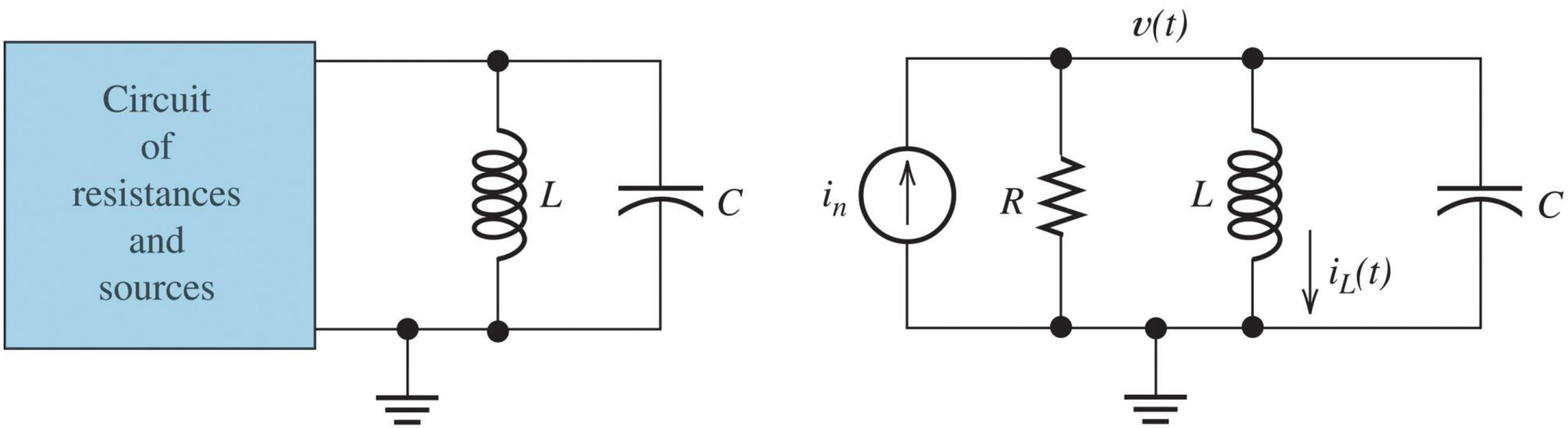
- Solve for current through capacitor
- $i(t) = C\frac{dv_c(t)}{dt}$
- $i(t) = C\left[\frac{dv_s(t)}{dt} - L\frac{d^2i(t)}{dt^2} - R\frac{di(t)}{dt}\right]$
- $\frac{d^2i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{LC}i(t) = \frac{1}{L}\frac{dv_s(t)}{dt}$

- The general 2nd-order constant coefficient equation

$$\frac{di^2(t)}{dt^2} + 2\alpha\frac{di(t)}{dt} + \omega_0^2 i(t) = f(t)$$

$$\alpha = \frac{R}{2L}, \qquad \omega_0 = \frac{1}{\sqrt{LC}}, \qquad f(t) = \frac{1}{L}\frac{dv_s(t)}{dt}$$

Circuits with Parallel L and C

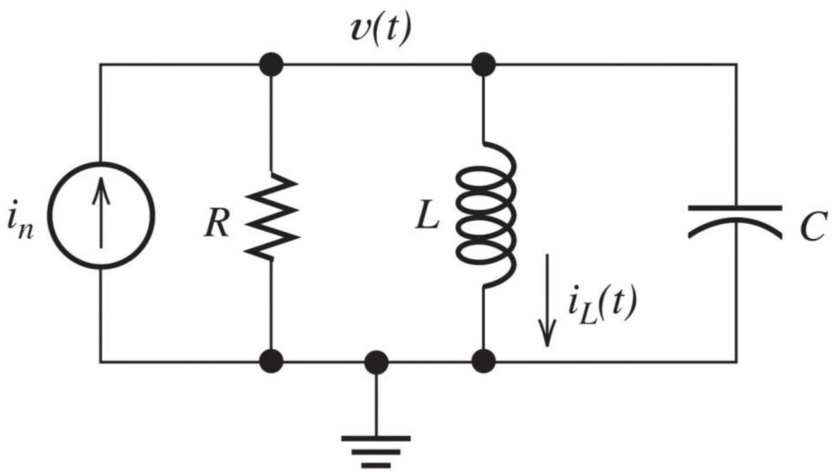


(a) (b)

- Given a circuit with a parallel capacitor and inductor
  - Use Norton equivalent to make a parallel circuit equivalent

Circuits with Parallel L and C

- Find current in inductor $i_L(t)$
- KCL @ top node
- $i_n(t) = \frac{v(t)}{R} + i_L(t) + C\frac{dv(t)}{dt}$



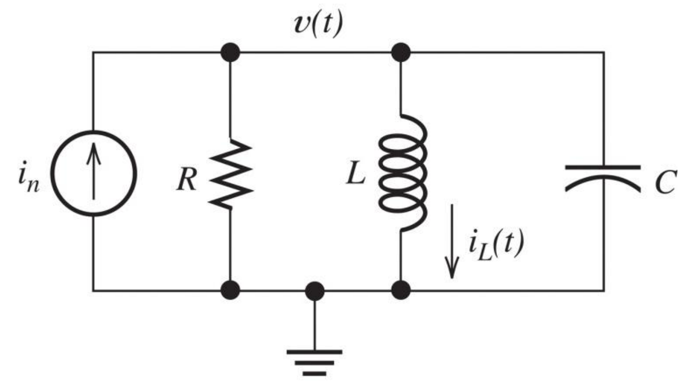
- $i_n(t) = \frac{L}{R}\frac{di(t)}{dt} + i(t) + CL\frac{d^2i(t)}{dt^2}$
- $\frac{d^2i(t)}{dt^2} + \frac{1}{RC}\frac{di(t)}{dt} + \frac{1}{LC}i(t) = \frac{1}{CL}i_n(t)$

$$\frac{di^2(t)}{dt^2} + 2\alpha\frac{di(t)}{dt} + \omega_0^2 i(t) = f(t)$$

- $f(t) = \frac{1}{LC}i_n(t)$
- $\alpha = \frac{1}{2RC}$
- $\omega_0 = \sqrt{\frac{1}{LC}}$

- $v(t) = L\frac{di(t)}{dt}$
- $\frac{dv(t)}{dt} = L\frac{d^2i(t)}{dt^2}$

Circuits with Parallel L and C



- Find parallel voltage $v(t)$
- KCL @ top node
- $i_n(t) = \frac{v(t)}{R} + i_L(t) + C\frac{dv(t)}{dt}$
- $i_n(t) = \frac{v(t)}{R} + \frac{1}{L}\int_{t_0}^{t} v(t)dt + i(t_0) + C\frac{dv(t)}{dt}$
- Take derivative of both sides with respect to time
- $\frac{di_n(t)}{dt} = \frac{1}{R}\frac{dv(t)}{dt} + \frac{1}{L}v(t) + C\frac{d^2v(t)}{dt^2}$
- $\frac{d^2v(t)}{dt^2} + \frac{1}{RC}\frac{dv(t)}{dt} + \frac{1}{LC}v(t) = \frac{1}{C}\frac{di_n(t)}{dt}$

- $v(t) = L\frac{di(t)}{dt}$
- $i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)dt + i(t_0)$

- $f(t) = \frac{1}{C}\frac{di_n(t)}{dt}$
- $\alpha = \frac{1}{2RC}$
- $\omega_0 = \sqrt{\frac{1}{LC}}$

$$\frac{di^2(t)}{dt^2} + 2\alpha\frac{di(t)}{dt} + \omega_0^2 i(t) = f(t)$$