EE292: Fundamentals of ECE

Fall 2012
TTh 10:00-11:15 SEB 1242

Lecture 14
121011

http://www.ee.unlv.edu/~b1morris/ee292/

Outline

- Review
  - Steady-State Analysis
  - RC Circuits
- RL Circuits

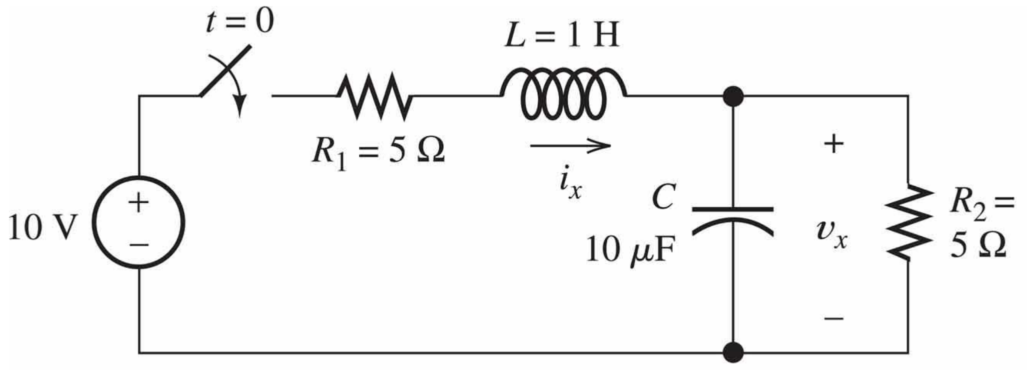
DC Steady-State Analysis

- Analysis of C, L circuits in DC operation
  - Steady-state – non-changing sources
  - A long time after switch event

- Capacitors $i = C\frac{dv}{dt}$
  - Voltage is constant → no current → open circuit
- Inductors $v = L\frac{di}{dt}$
  - Current is constant → no voltage → short circuit

Example

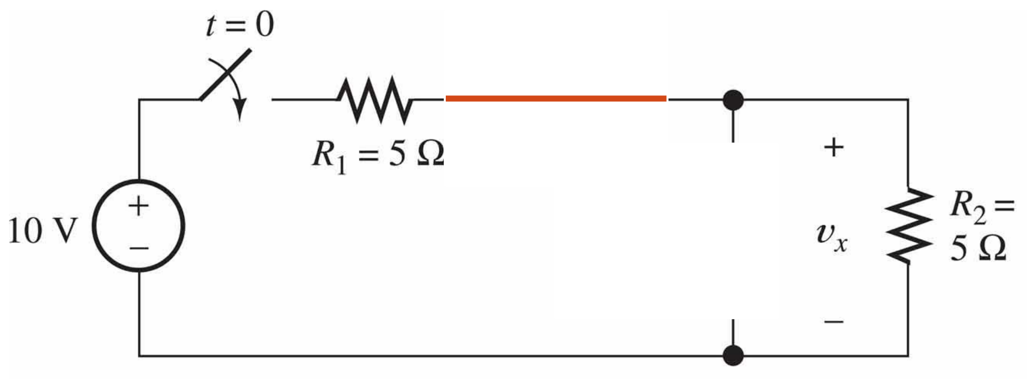
- What are initial conditions of this circuit



- Open switch → no current in circuit
  - $i_x(0^-) = 0$
  - No charge on capacitor → no voltage across it
    - $v_x(0^-) = 0$

Example

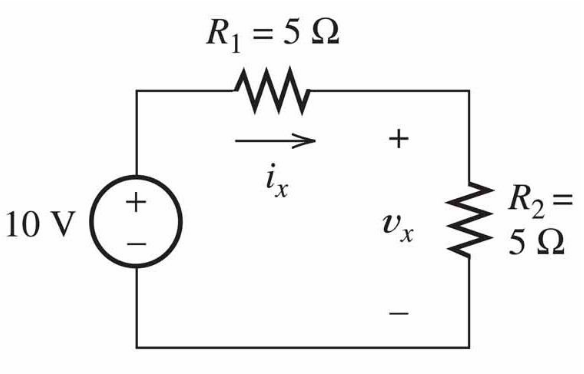
- Solve for the steady-state values



- Capacitors
  - Open circuit
- Inductors
  - Short circuit

Example

- Solve for the steady-state values



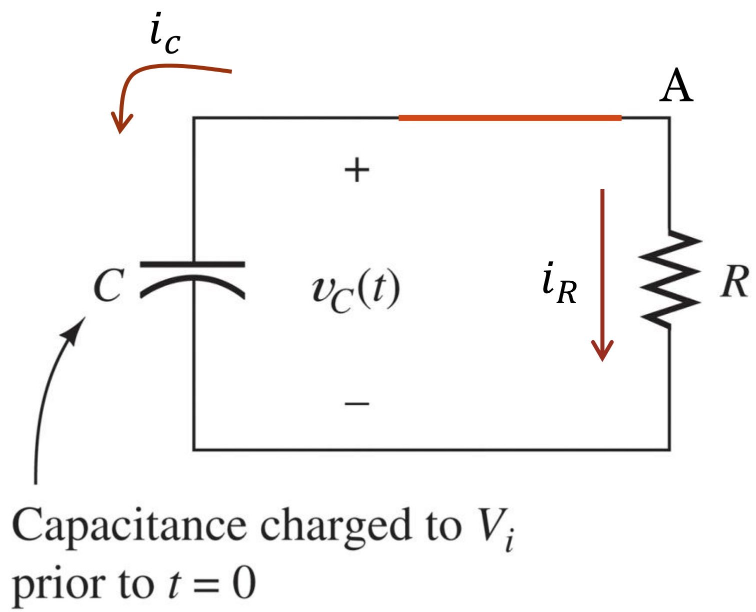
- Voltage v_x by voltage divider
  - $v_x = V_s\left(\frac{R_2}{R_1+R_2}\right) = 10\left(\frac{5}{5+5}\right) = 5\ V$
- Current i_x by Ohm's Law
  - $i_x = \frac{V}{R} = \frac{V}{R_{eq}} = \frac{10}{5+5} = 1\ A$
  - $v_x = i_x R_2 = 1(5) = 5V$

Transients

- The study of time-varying currents and voltages
  - Circuits contain sources, resistances, capacitances, inductances, and switches
- Studied using our basic analysis methods
  - KCL, KVL, node-voltage, mesh-current
  - But, more complex due to differential relationships between current and voltage with capacitors and inductors

Discharging a Capacitor

- KCL @ A
- $C\frac{dv_c(t)}{dt} + \frac{v_c(t)}{R} = 0$
- $RC\frac{dv_c(t)}{dt} + v_c(t) = 0$



- Differential equation describes the voltage across the capacitor over time
- Solution is of the exponential form
- $v_c(t) = Ke^{st}$

Discharging a Capacitor

- $RC\frac{dv_c(t)}{dt} + v_c(t) = 0$
  - Substitute $v_c(t) = Ke^{st}$ into differential equation

- $RCKse^{st} + Ke^{st} = 0$

- Solve for s
- $(RCs + 1)Ke^{st} = 0$
  - $(RCs + 1) = 0$
  - $s = -\frac{1}{RC} \rightarrow RC$ is known as the time constant

- $v_c(t) = Ke^{-t/(RC)}$

Discharging a Capacitor

- $v_c(t) = Ke^{-t/(RC)}$

- Solve for K
  - Voltage before and after switch are the same
    - $v_c(0^-) = v_c(0^+)$
  - $v_c(0^+) = V_i = Ke^0 = K$
    - V_i is initial charge on capacitor
    - $K = V_i$

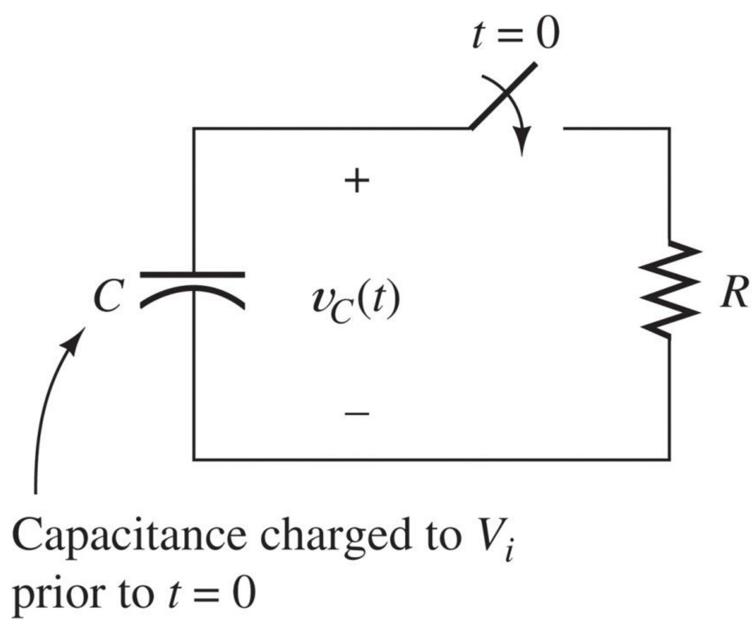
- $v_c(t) = V_i e^{-t/(RC)}$

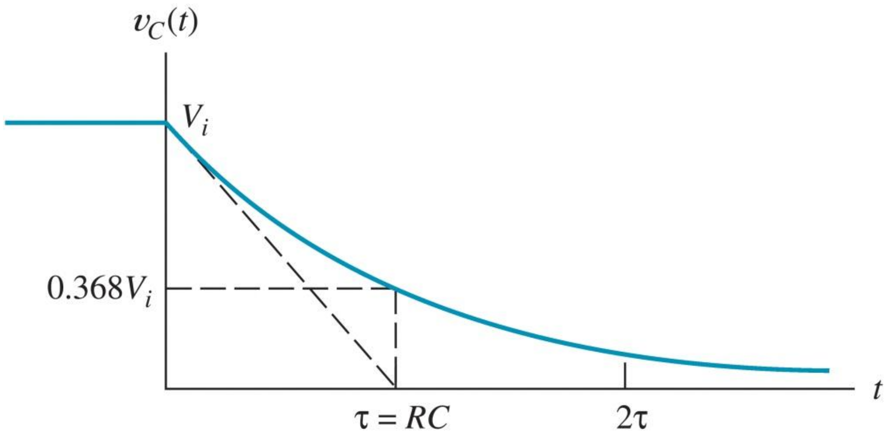
Voltage/Time Characteristics

- $v_c(t) = V_i e^{-t/(RC)}$

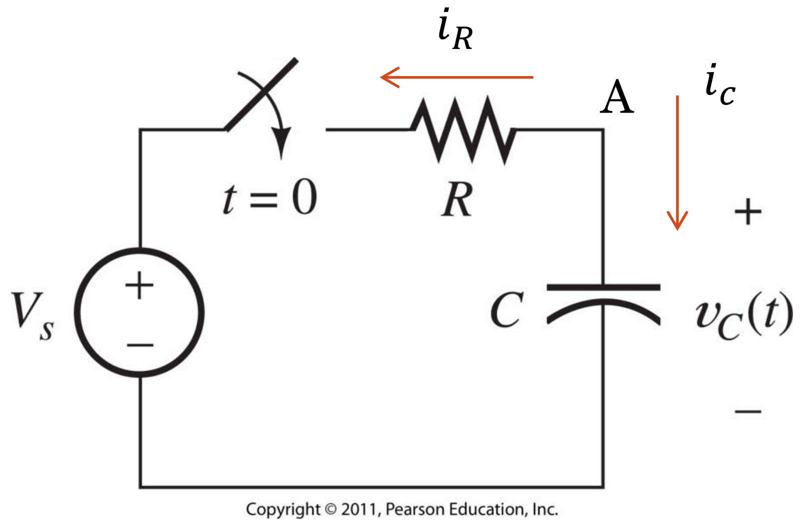
- $\tau = RC$
- Time constant of the circuit
- The amount of time for voltage to decay by a factor of $e^{-1} = 0.368$

- Decays to 0 in about five time constants (5τ)
- Large τ → longer decay time
  - Larger R → less current
  - Larger C → more charge





Charging a Capacitance



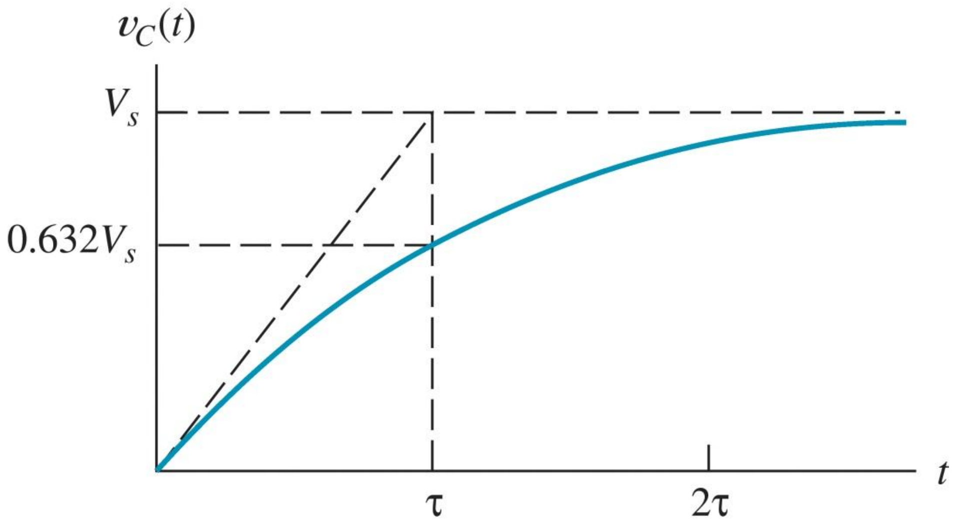
- Assume capacitor is fully discharged → no voltage across capacitor
  - $v_c(0^-) = 0$
- KCL @ A
- $C\frac{dv_c(t)}{dt} + \frac{v_c(t) - V_s}{R} = 0$
- $RC\frac{dv_c(t)}{dt} + v_c(t) = V_s$
- Assume solution of the form
  - $v_c(t) = K_1 + K_2e^{st}$
- Solve for s, K_1
- $RCK_2se^{st} + K_1 + K_2e^{st} = V_s$
- $(1 + RCs)K_2e^{st} + K_1 = V_s$
  - $s = -\frac{1}{RC}$
  - $K_1 = V_s$

Charging a Capacitance

- $v_c(t) = V_s + K_2 e^{-t/(RC)}$

- Solve for K_2
- $v_c(0^+) = 0 = V_s + K_2 e^0$
  - $K_2 = -V_s$

- Final solution
- $v_c(t) = V_s - V_s e^{-t/(RC)}$

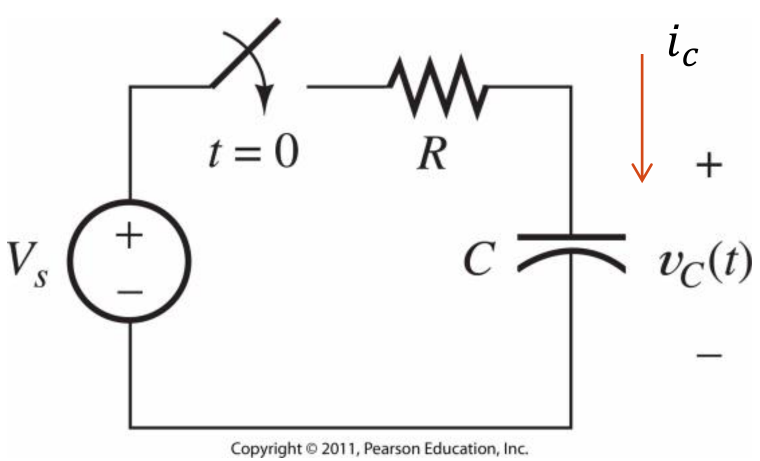


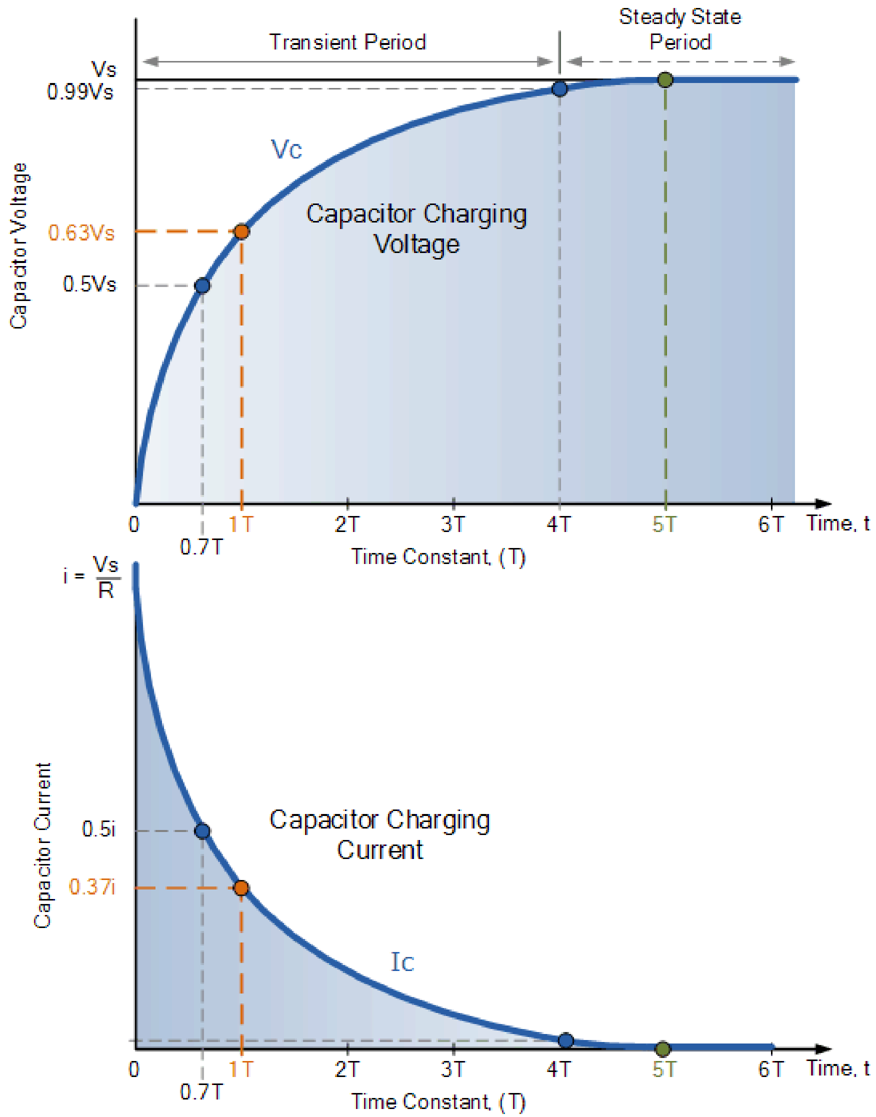
Transient response – eventually decays to a negligible value

Steady-state response or forced response

RC Current

- Voltage
  - $v_c(t) = V_s - V_s e^{-t/(RC)}$
- Current
  - $i_c = \frac{V_s - v_c(t)}{R} = C\frac{dv_c(t)}{dt}$
  - $i_c = C\left(\frac{V_s}{RC}e^{-t/(RC)}\right)$
  - $i_c = \frac{V_s}{R}e^{-t/(RC)}$





http://www.electronics-tutorials.ws/rc/rc_1.html

General 1st-Order RC Solution

- Notice both the current and voltage in an RC circuit has an exponential form

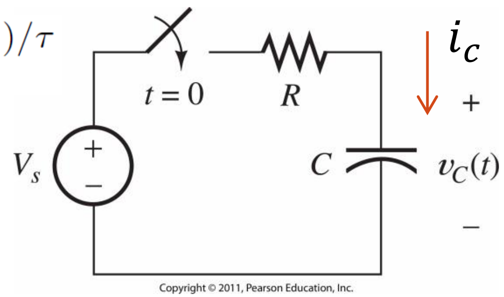
- The general solution for current/voltage is:

$$x(t) = x_f + \left[x(t_0^+) - x_f\right] e^{-(t-t_0)/\tau}$$

  - x – represents current or voltage
  - t_0 – represents time when source switches
  - x_f - final (asymptotic) value of current/voltage
  - τ – time constant (RC)
- Find values and plug into general solution

Example

$$x(t) = x_f + \left[x(t_0^+) - x_f\right] e^{-(t-t_0)/\tau}$$



- Solve for $v_c(t)$
  - $v_f = V_s$ steady-state analysis
  - $v_c(0^+) = 0$ no voltage when switch open
  - $\tau = RC$ equivalent resistance/capacitance
- $v_c(t) = V_s + [0 - V_s]e^{-t/(RC)} = V_s - V_s e^{-t/(RC)}$
- Solve for $i_c(t)$
  - $i_f = 0$ fully charged cap → no current
  - $i_c(0^+) = \frac{V_s - v_c(0^+)}{R} = \frac{V_s - 0}{R} = \frac{V_s}{R}$
- $i_c(t) = 0 + \left[\frac{V_s}{R} - 0\right] e^{-t/(RC)} = \frac{V_s}{R} e^{-t/(RC)}$

First-Order RL Circuits

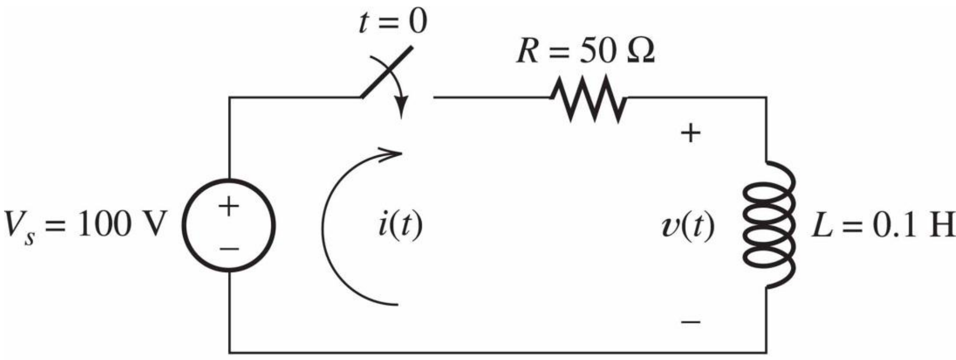
- Contains DC sources, resistors, and a single inductance

- Same technique to analyze as for RC circuits

1. Apply KCL and KVL to write circuit equations
2. If the equations contain integrals, differentiate each term in the equation to produce a pure differential equation
   - Use differential forms for I/V relationships for inductors and capacitors
3. Assume solution of the form $K_1 + K_2 e^{st}$
4. Substitute the solution into the differential equation to determine the values of K_1 and s
5. Use initial conditions to determine the value of K_2
6. Write the final solution

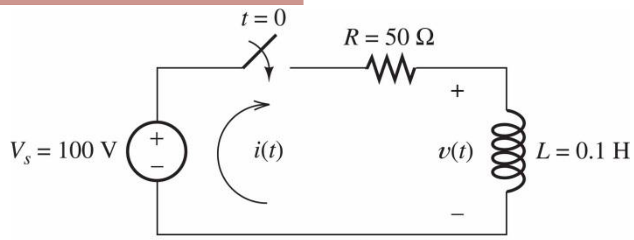
RL Example

- Before switch
  - $i(0^-) = 0$
- KVL around loop
  - $V_s - Ri(t) - L\frac{di(t)}{dt} = 0$
  - $i(t) + \frac{L}{R}\frac{di(t)}{dt} = \frac{V_s}{R}$
    - Notice this is the same equation form as the charging capacitor example
- Solution of the form
  - $i(t) = K_1 + K_2 e^{st}$
- Solving for K_1, s
  - $K_1 + K_2 e^{st} + \frac{L}{R} K_2 s e^{st} = \frac{V_s}{R}$
    - $K_1 = \frac{V_s}{R}$
    - $\left(1 + \frac{L}{R}s\right) = 0 \rightarrow S = -\frac{R}{L}$



- Solving for K_2
  - $i(0^+) = 0 = \frac{V_s}{R} + K_2 e^{-tR/L}$
  - $0 = \frac{V_s}{R} + K_2 e^0$
  - $K_2 = -\frac{V_s}{R}$
- Final Solution
  - $i(t) = \frac{V_s}{R} - \frac{V_s}{R} e^{-tR/L}$
  - $i(t) = 2 - 2e^{-500t}$

RL Example

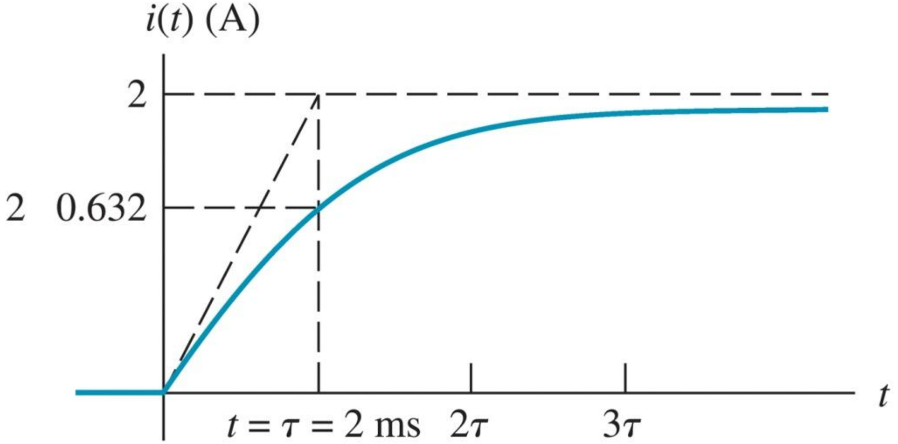


- $i(t) = 2 - 2e^{-500t}$
- Notice this is in the general form we used for RC circuits

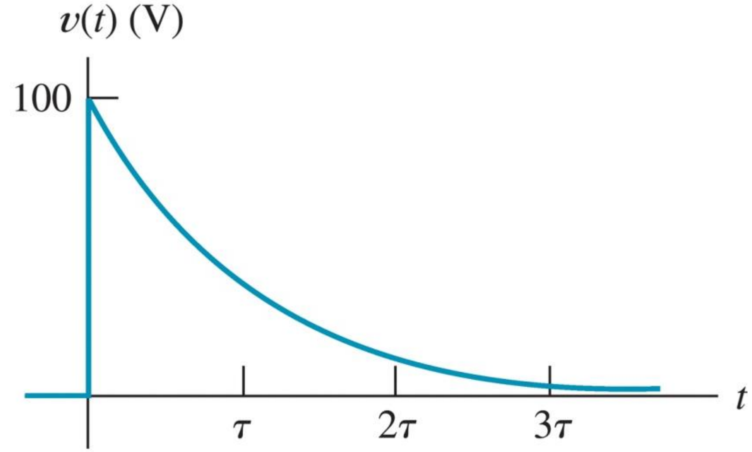
$$x(t) = x_f + \left[x(t_0^+) - x_f\right] e^{-(t-t_0)/\tau}$$

  - $\tau = \frac{L}{R}$

- Find voltage $v(t)$
  - $v_f = 0$, steady-state short
  - $v(0^+) = 100$
    - No current immediately through R, $v = L\frac{di(t)}{dt}$
- $v(t) = 100e^{-t/\tau}$

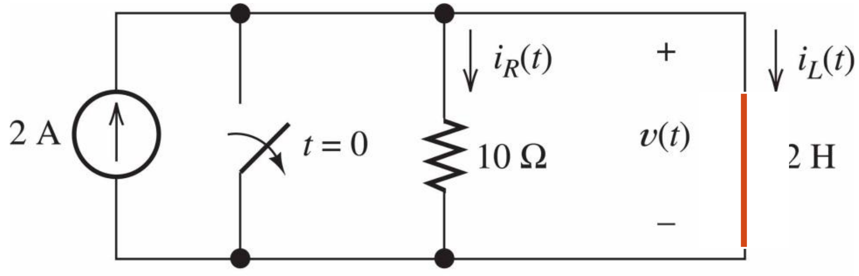


(a)



(b)

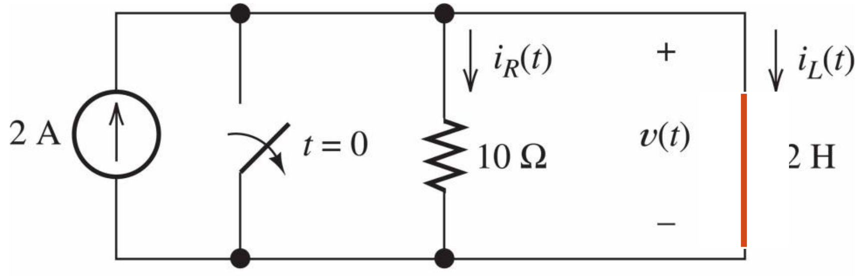
Exercise 4.5



- Initial conditions
- For $t < 0$
  - All source current goes through switched wire
  - $i_R(t) = i_L(t) = 0\ A$
  - $v(t) = i_R(t)\text{R} = 0\ \text{V}$
- For $t = 0^+$ (right after switch)
  - $i_L(t) = 0$
    - Current can't change immediately through an inductor
  - $i_R(t) = 2$ A, by KCL
  - $v(t) = i_R(t)\text{R} = 20\ \text{V}$

- Steady-state
  - Short inductor
- $v(t) = 0$
  - Short circuit across inductor
- $i_R = 0$
  - All current through short
- $i_L = 2\ A$
  - By KCL

Exercise 4.5



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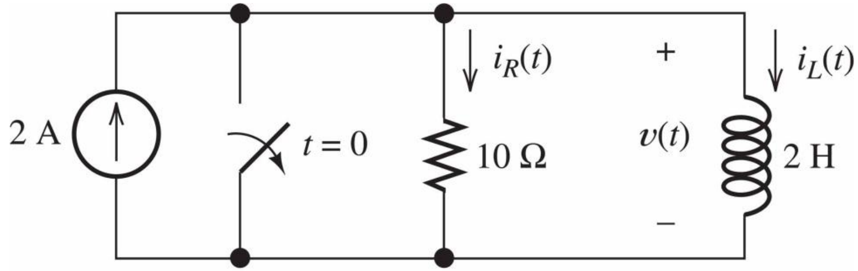
- For $t = 0^+$ (right after switch)
  - $i_L(t) = 0$
    - Current can't change immediately through an inductor
  - $i_R(t) = 2$ A, by KCL
  - $v(t) = i_R(t)\text{R} = 20\ \text{V}$

- Steady-state
  - Short inductor
- $v(t) = 0$
  - Short circuit across inductor
- $i_R = 0$
  - All current through short
- $i_L = 2\ A$
  - By KCL

Exercise 4.5

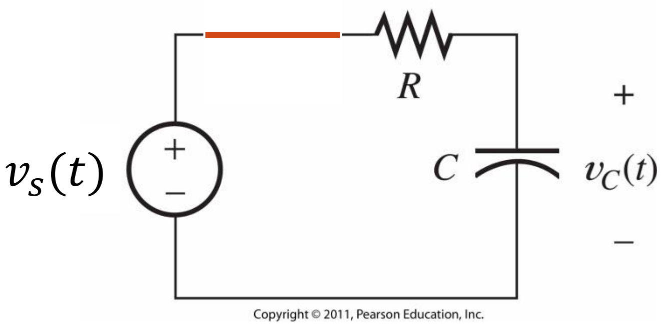


- Can use network analysis to come up with a differential equation, but you would need to solve it
- Instead, use the general 1st-order solution

$$x(t) = x_f + \left[x(t_0^+) - x_f\right] e^{-(t-t_0)/\tau}$$

- Time constant τ
  - $\tau = \frac{L}{R} = \frac{2}{10} = 0.2$
- Voltage $v(t)$
  - $v(t) = 0 + [20 - 0]e^{-t/0.2} = 20e^{-t/0.2}$ V
- Current $i_L(t)$, $i_R(t)$
  - $i_L(t) = 2 + [0 - 2]e^{-t/0.2} = 2 - 2e^{-t/0.2}$ A
  - $i_R(t) = 0 + [2 - 0]e^{-t/0.2} = 2e^{-t/0.2}$ A

RC/RL Circuits with General Sources



- Previously,
  - $RC\frac{dv_c(t)}{dt} + v_c(t) = V_s$
- What if V_s is not constant
  - $RC\frac{dv_c(t)}{dt} + v_c(t) = v_s(t)$
  - Now have a general source that is a function of time
- The solution is a differential equation of the form
  - $\tau\frac{dx(t)}{dt} + x(t) = f(t)$
  - Where $f(t)$ is known as the forcing function (the circuit source)

General Differential Equations

- General differential equation
  - $\tau \frac{dx(t)}{dt} + x(t) = f(t)$
- The solution to the diff equation is
  - $x(t) = x_p(t) + x_h(t)$

- $x_p(t)$ is the particular solution
- $x_h(t)$ is the homogeneous solution

Particular Solution

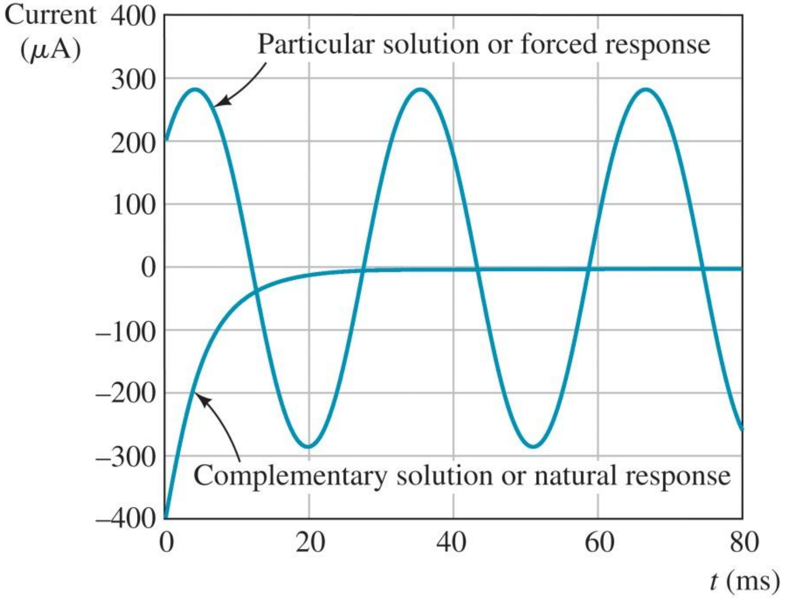
- $\tau \frac{dx_p(t)}{dt} + x_p(t) = f(t)$
- The solution $x_p(t)$ is called the forced response because it is the response of the circuit to a particular forcing input $f(t)$
- The solution $x_p(t)$ will be of the same functional form as the forcing function
  - E.g.
  - $f(t) = e^{st} \rightarrow x_p(t) = Ae^{st}$
  - $f(t) = \cos(\omega t) \rightarrow x_p(t) = A\cos(\omega t) + B\sin(\omega t)$

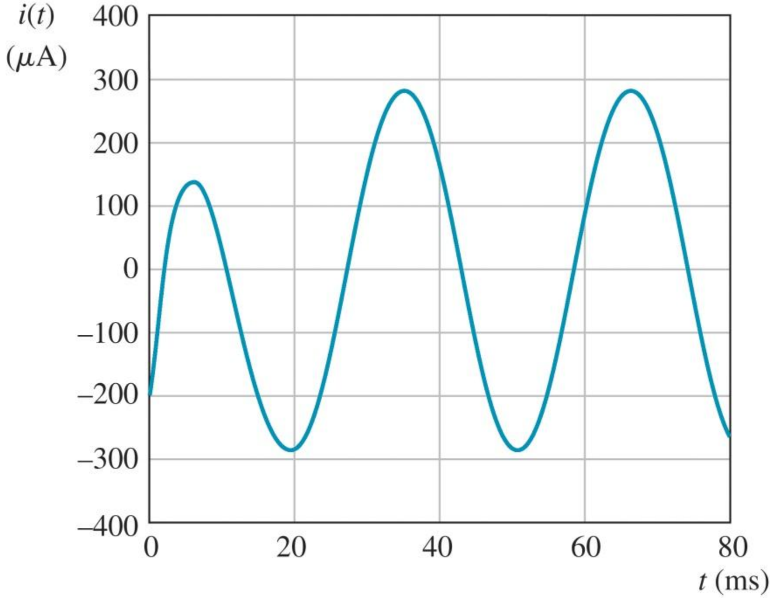
Homogeneous Solution

- $\tau \frac{dx_h(t)}{dt} + x_h(t) = 0$
- $x_h(t)$ is the solution to the differential equation when there is no forcing function
  - Does not depend on the sources
  - Dependent on initial conditions (capacitor voltage, current through inductor)
- $x_h(t)$ is also known as the natural response
- Solution is of the form
  - $x_h(t) = Ke^{-t/\tau}$

General Differential Solution

- Notice the final solution is the sum of the particular and homogeneous solutions
  - $x(t) = x_p(t) + x_h(t)$
- It has an exponential term due to $x_h(t)$ and a term $x_p(t)$ that matches the input source

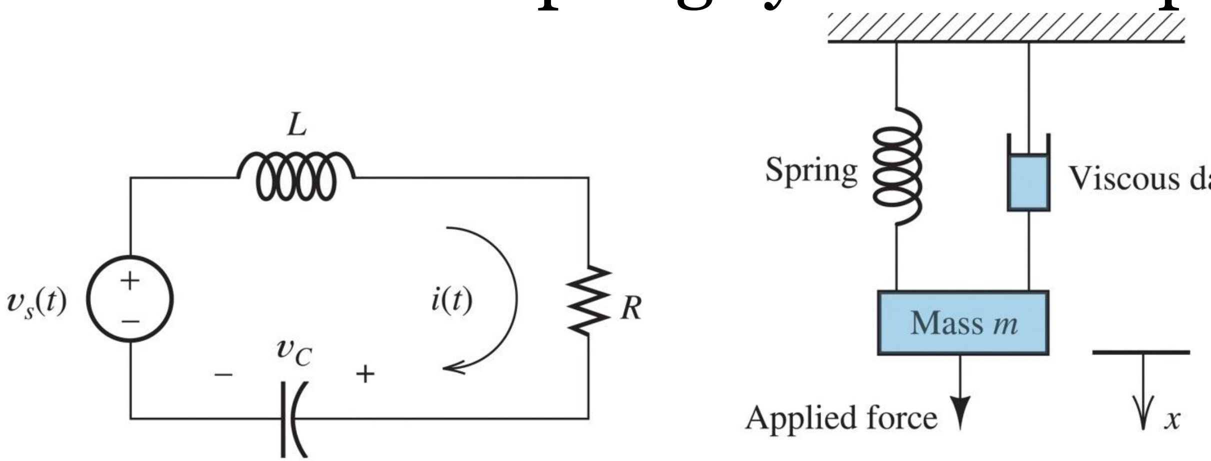




Second-Order Circuits

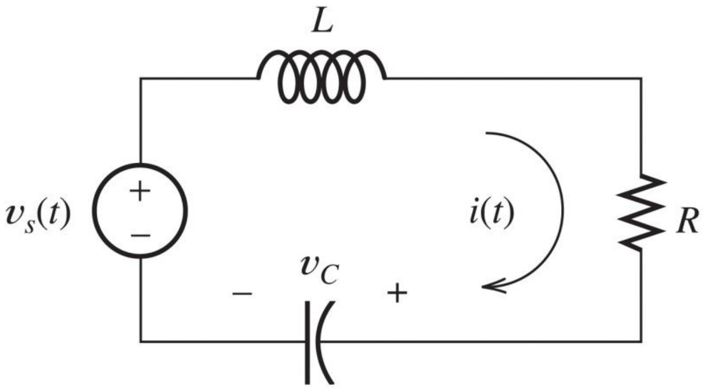
- RLC circuits contain two energy storage elements
  - This results in a differential equation of second order (has a second derivative term)

- This is like a mass spring system from physics



(a) Electrical circuit (b) Mechanical analog

RLC Series Circuit



- KVL around loop
  - $v_s(t) - L\frac{di(t)}{dt} - i(t)R - v_c(t) = 0$
- Solve for $v_c(t)$
  - $v_c(t) = v_s(t) - L\frac{di(t)}{dt} - i(t)R$
- Take derivative
  - $\frac{dv_c(t)}{dt} = \frac{dv_s(t)}{dt} - L\frac{d^2i(t)}{dt^2} - R\frac{di(t)}{dt}$

- Solve for current through capacitor
- $i(t) = C\frac{dv_c(t)}{dt}$
- $i(t) = C\left[\frac{dv_s(t)}{dt} - L\frac{d^2i(t)}{dt^2} - R\frac{di(t)}{dt}\right]$
- $\frac{d^2i(t)}{dt^2} - \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{LC}i(t) = \frac{1}{L}\frac{dv_s(t)}{dt}$

- The general 2nd-order constant coefficient equation

$$\frac{di^2(t)}{dt^2} + 2\alpha\frac{di(t)}{dt} + \omega_0^2 i(t) = f(t)$$

$$\alpha = \frac{R}{2L}, \qquad \omega_0 = \frac{1}{\sqrt{LC}}, \qquad f(t) = \frac{1}{L}\frac{dv_s(t)}{dt}$$