EE292: Fundamentals of ECE

Fall 2012
TTh 10:00-11:15 SEB 1242

Lecture 13
121009

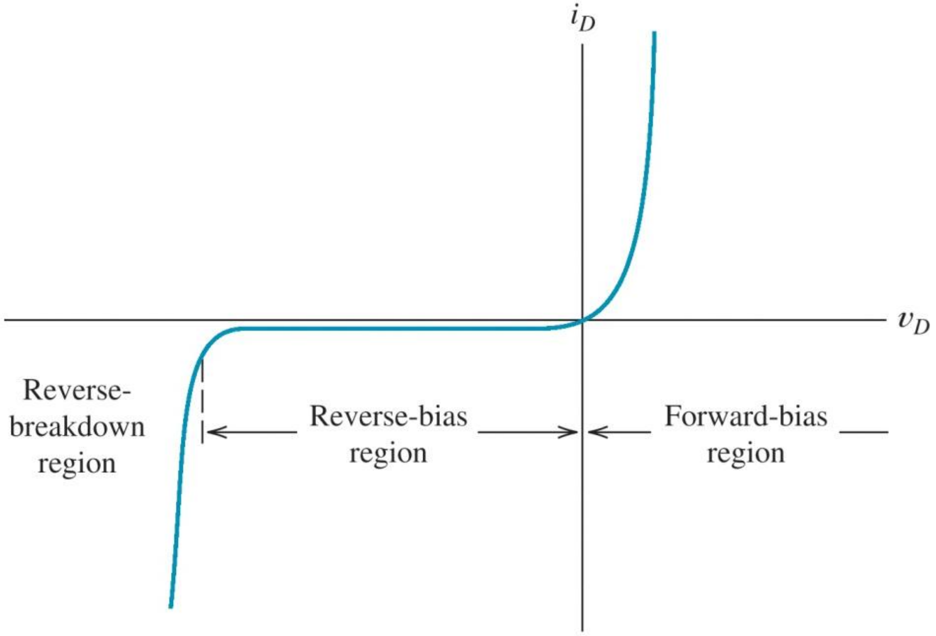
http://www.ee.unlv.edu/~b1morris/ee292/

Outline

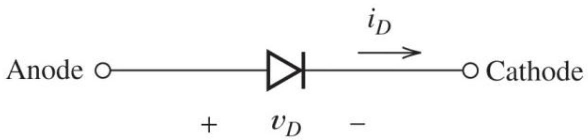
- Review Diodes
- DC Steady State Analysis
- Transient Analysis
- RC Circuits
- RL Circuits

Diode Voltage/Current Characteristics

- Forward Bias ("On")
  - Positive voltage v_D supports large currents
  - Modeled as a battery (0.7 V for offset model)
- Reverse Bias ("Off")
  - Negative voltage → no current
  - Modeled as open circuit
- Reverse-Breakdown
  - Large negative voltage supports large negative currents
  - Similar operation as for forward bias



(b) Volt–ampere characteristic



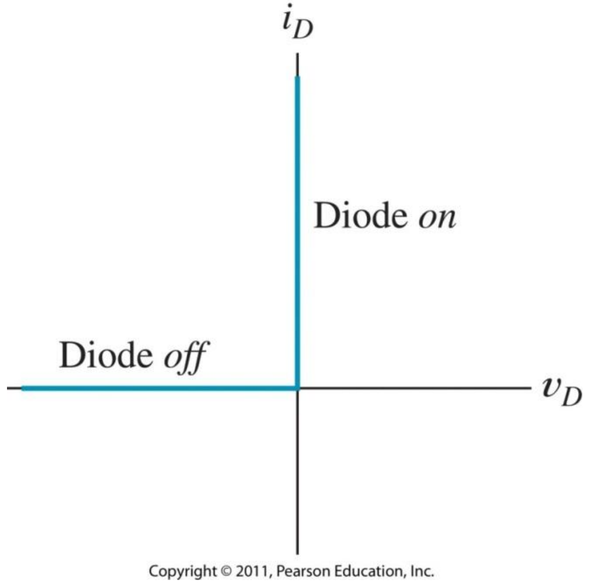
(a) Circuit symbol

Diode Models

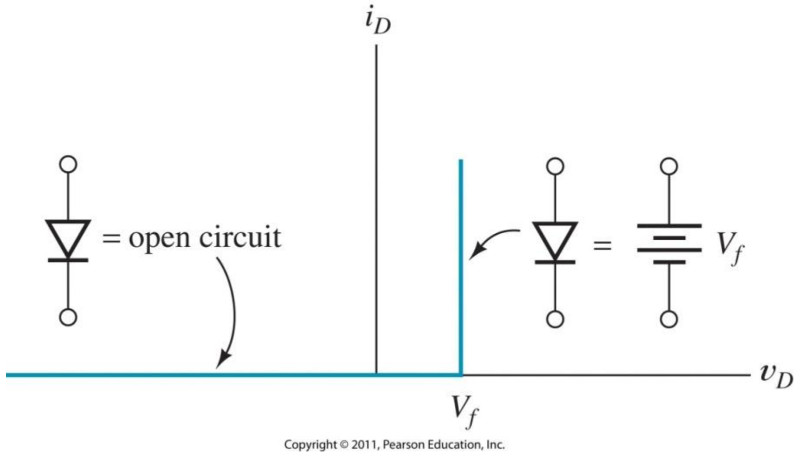
- Ideal model – simple
- Offset model – more realistic

- Two state model
- "On" State
  - Forward operation
  - Diode conducts current
    - Ideal model → short circuit
    - Offset model → battery
- "Off" State
  - Reverse biased
  - No current through diode → open circuit

Ideal Model



Offset Model



Circuit Analysis with Diodes

- Assume state {on, off} for each ideal diode and check if the initial guess was correct
  - $i_d > 0$ positive for “on” diode
  - $v_d < 0$ negative for “off” diode
    - These imply a correct guess
  - Otherwise adjust guess and try again

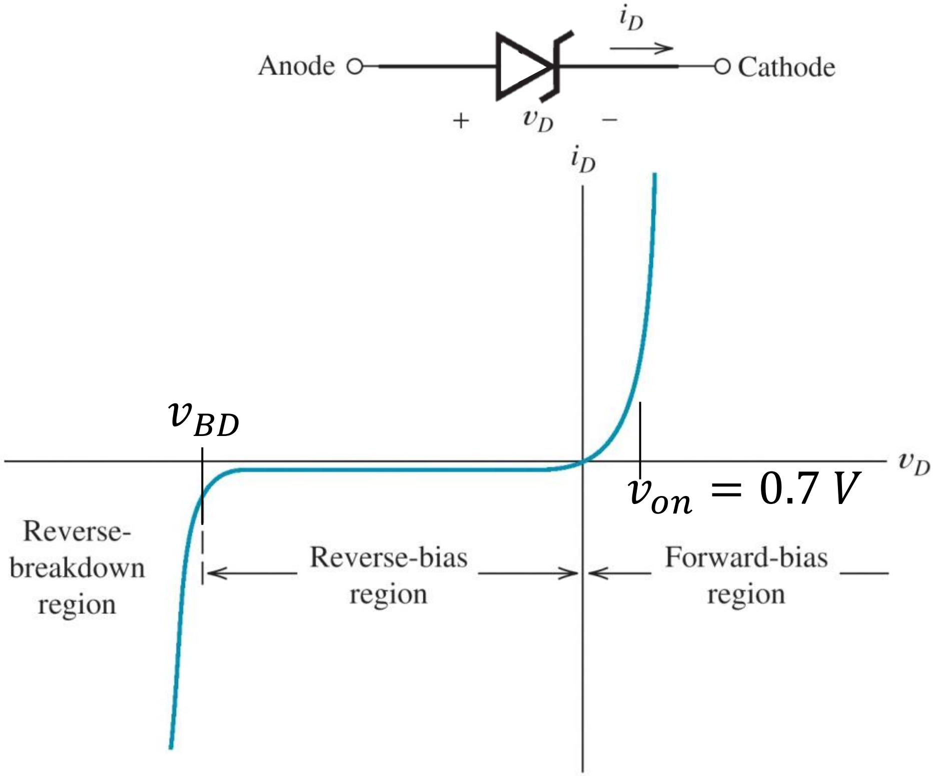
- Exhaustive search is daunting
  - 2^n different combinations for n diodes
- Will require experience to make correct guess

Zener Diode

- Diode intended to be operated in breakdown
  - Constant voltage at breakdown
- Three state diode

1. On – 0.7 V forward bias
2. Off – reverse bias
3. Breakdown

   v_{BD} reverse breakdown voltage



(b) Volt–ampere characteristic

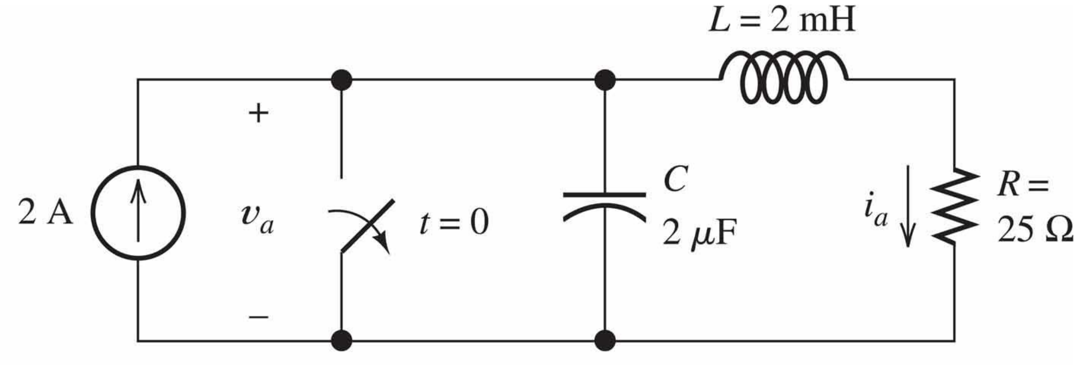
DC Steady-State Analysis

- Analysis of C, L circuits in DC operation
  - Steady-state – non-changing sources

- Capacitors $i = C\frac{dv}{dt}$
  - Voltage is constant → no current → open circuit
- Inductors $v = L\frac{di}{dt}$
  - Current is constant → no voltage → short circuit

Example

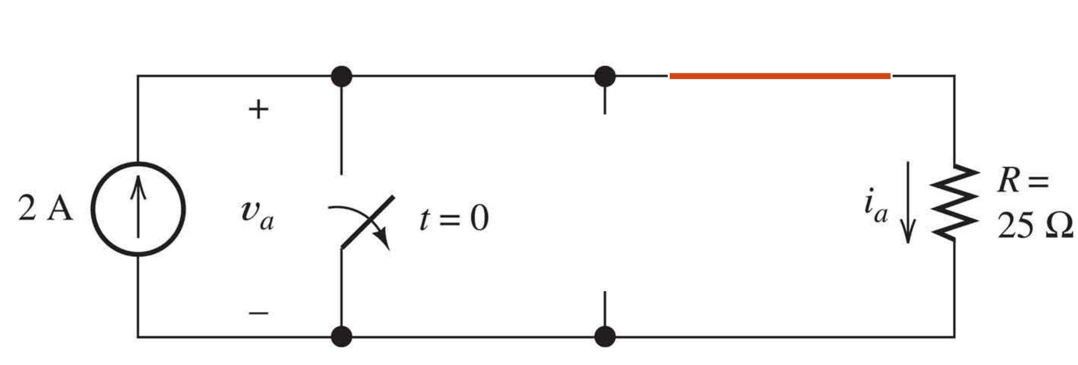
- Solve for the steady-state values



- Capacitors
  - open circuit
- Inductors
  - short circuit

Example

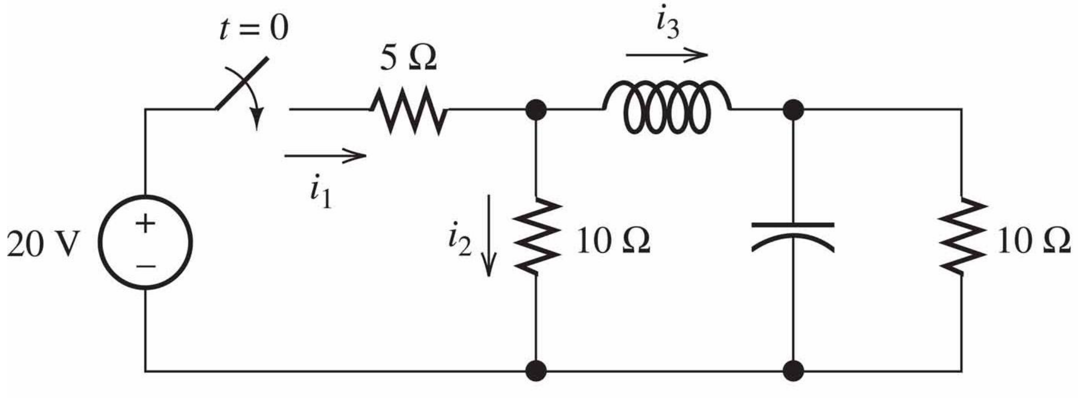
- Steady-state circuit



- $v_a = I_s R = 2 \cdot 25 = 50\ V$
- $i_a = I_s = 2\ A$

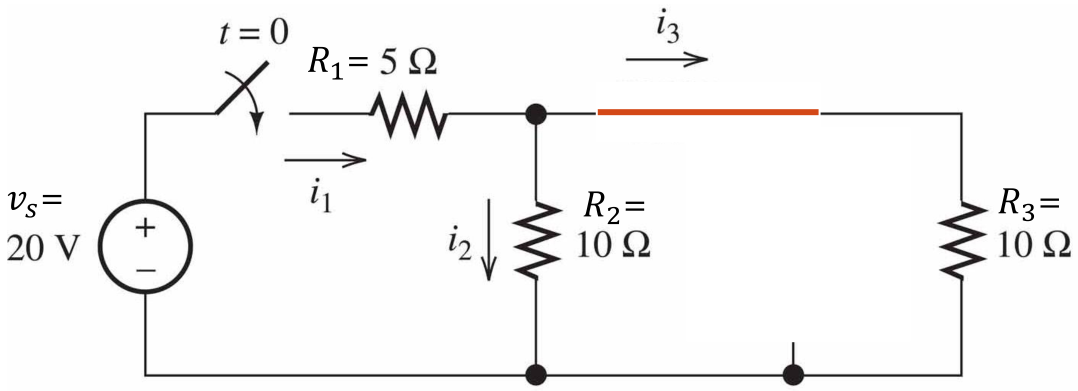
Example

- Solve for the steady-state values



Example

- Steady-state circuit



- Solve for i_1 by equivalent resistance
  - $i_1 = \frac{v_S}{R_1 + R_2 || R_3} = \frac{20}{5+5} = 2\ A$
- Solve for i_2, i_3 by current divider
  - $i_2 = i_3 = i_1 \frac{R_3}{R_2 + R_3} = 0.5 i_1 = 1\ A$

Transients

- The study of time-varying currents and voltages
  - Circuits contain sources, resistances, capacitances, inductances, and switches
- Studied using our basic analysis methods
  - KCL, KVL, node-voltage, mesh-current
  - But, more complex due to differential relationships between current and voltage with capacitors and inductors

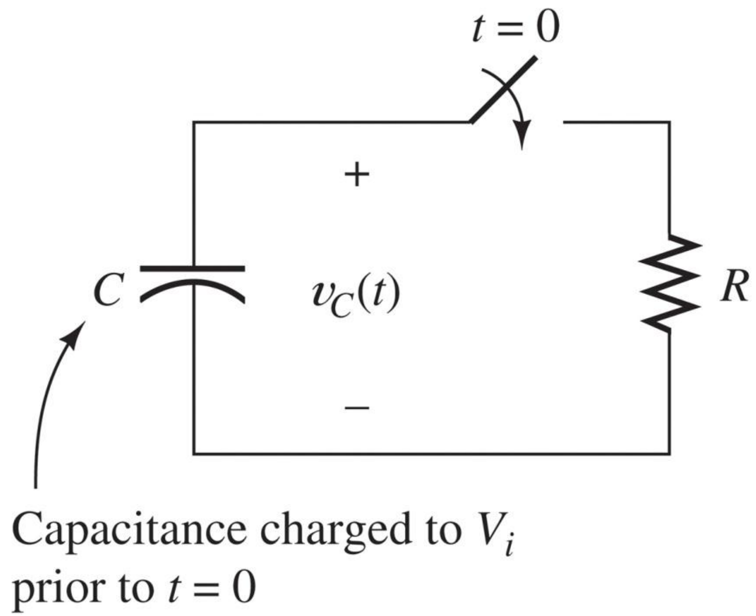
First-Order RC Circuit

- Contains a single capacitor (C) and resistor (R)
  - Denoted as first order because the differential equation will only contain a first derivative

Discharging a Capacitor

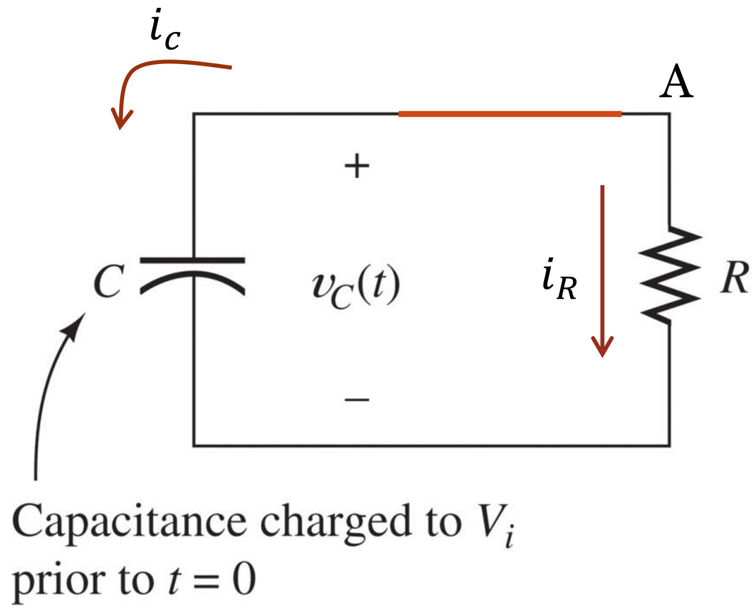
- What happens in this circuit?

- Switch closed at time $t = 0$
  - Current is able to flow
- Charge on capacitor will flow as current and be absorbed by the resistor
  - Discharge capacitor through resistor R



Discharging a Capacitor

- KCL @ A
- $i_c + i_R = 0$
- $C\frac{dv_c(t)}{dt} + \frac{v_c(t)}{R} = 0$
- $RC\frac{dv_c(t)}{dt} + v_c(t) = 0$



- Differential equation describes the voltage across the capacitor over time

Discharging a Capacitor

- $RC\frac{dv_c(t)}{dt} + v_c(t) = 0$

- Solution is of the exponential form
- $v_c(t) = Ke^{st}$
  - K is a gain constant to be found
  - s is the exponential time constant to be found

  - From your favorite differential equation class you know this as a homogeneous differential equation

Discharging a Capacitor

- $RC\frac{dv_c(t)}{dt} + v_c(t) = 0$

- Substitute $v_c(t) = Ke^{st}$ into differential equation
  - $\frac{dv_c(t)}{dt} = \frac{d}{dt}Ke^{st} = \mathrm{K}se^{st}$
- $RCKse^{st} + Ke^{st} = 0$

- Solve for s
- $(RCs + 1)Ke^{st} = 0$
  - $(RCs + 1) = 0$
  - $s = -\frac{1}{RC}$ → RC is known as the time constant

- $v_c(t) = Ke^{-t/(RC)}$

Discharging a Capacitor

- $v_c(t) = Ke^{-t/(RC)}$
- Solve for k
  - Capacitor voltage cannot change instantaneously when switched
    - $i = C\frac{dv}{dt}$ requires infinite current
  - Voltage before and after switch are the same
    - $v_c(0^-) = v_c(0^+)$
  - $v_c(0^+) = V_i = Ke^0 = K$
    - V_i is initial charge on capacitor
    - $K = V_i$

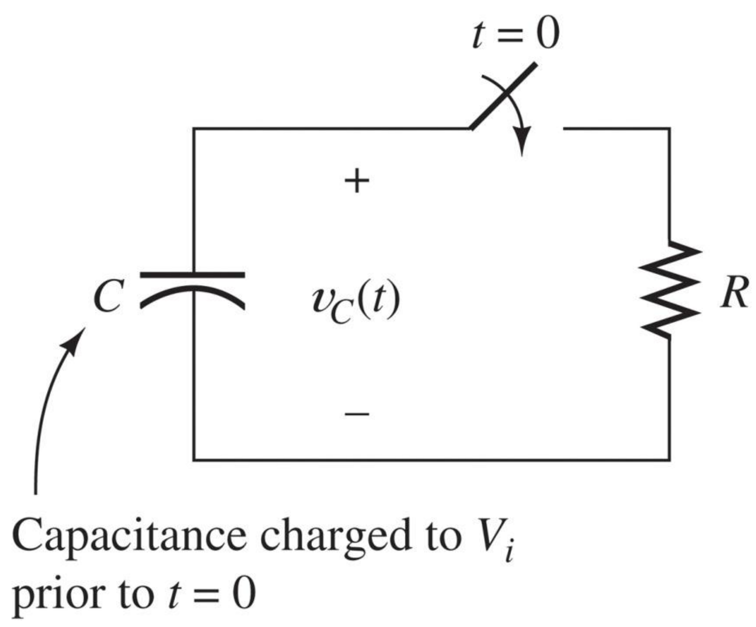
- $v_c(t) = V_i e^{-t/(RC)}$

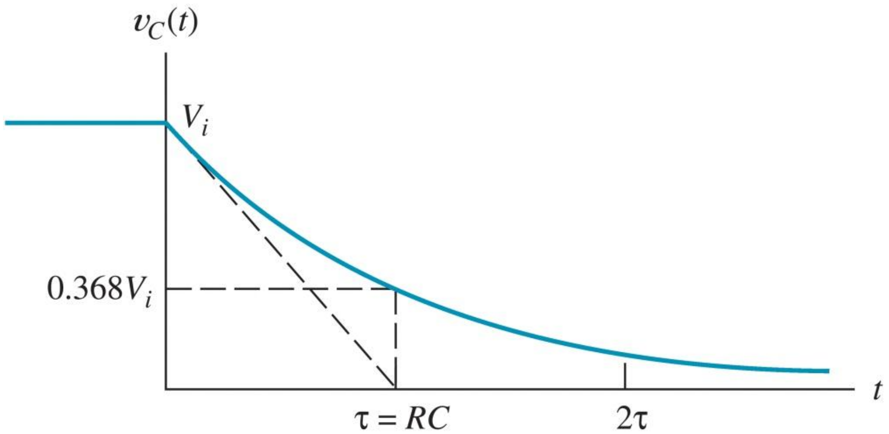
Voltage/Time Characteristics

- $v_c(t) = V_i e^{-t/(RC)}$

- $\tau = RC$
- Time constant of the circuit
- The amount of time for voltage to decay by a factor of $e^{-1} = 0.368$

- Decays to 0 in about five time constants (5τ)
- Large τ → longer decay time
  - Larger R → less current
  - Larger C → more charge

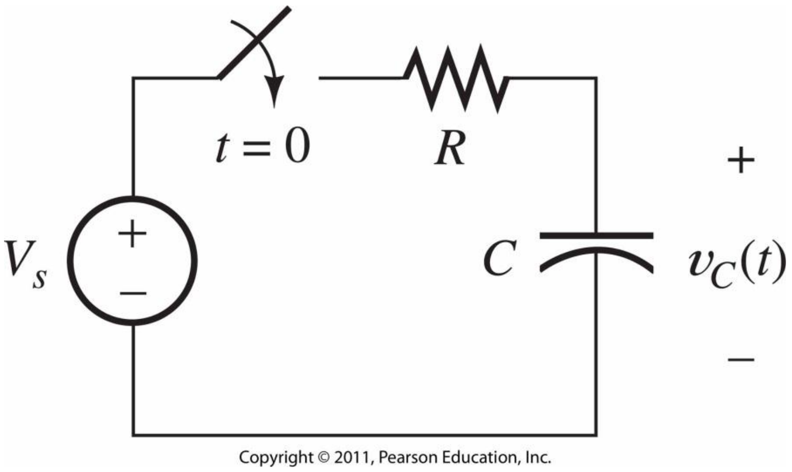




Charging a Capacitance

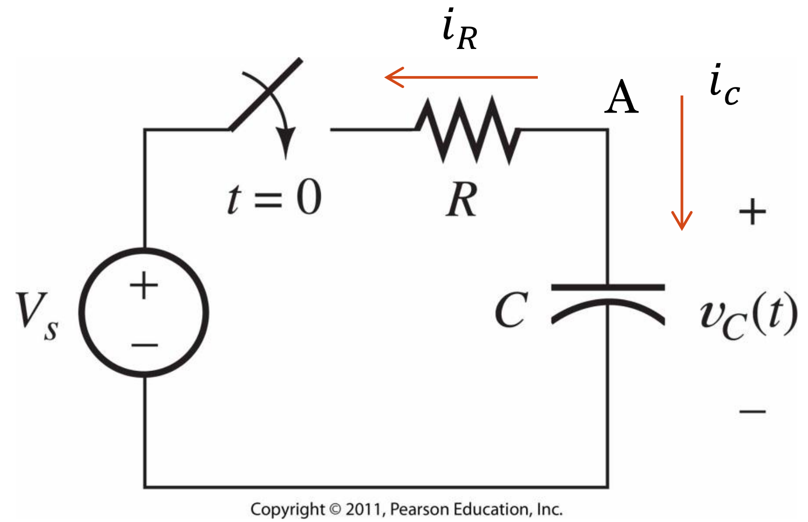
- What happens in this circuit?

- When switch is closed:
- Current flows through the resistor into the capacitor
- Capacitor is charged until fully charged
  - $v_c(t) = V_s$



Charging a Capacitance

- Assume capacitor is fully discharged → no voltage across capacitor
  - $v_c(0^-) = 0$
- KCL @ A
- $i_c + i_R = 0$
- $C\frac{dv_c(t)}{dt} + \frac{v_c(t)-V_s}{R} = 0$
- $RC\frac{dv_c(t)}{dt} + v_c(t) = V_s$



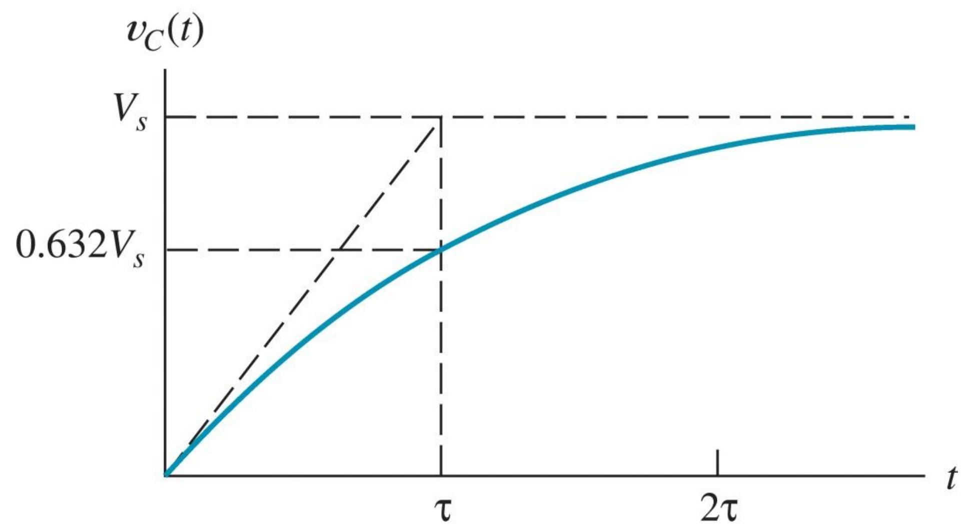
- Assume solution of the form
  - $v_c(t) = K_1 + K_2e^{st}$

- Solve for s, K_1
- $RCK_2se^{st} + K_1 + K_2e^{st} = V_s$
- $(1 + RCs)K_2e^{st} + K_1 = V_s$
  - $s = -\frac{1}{RC}$
  - $K_1 = V_s$

Charging a Capacitance

- $v_c(t) = V_s + K_2 e^{-t/(RC)}$
- Solve for K_2
- $v_c(0^+) = 0 = V_s + K_2 e^0$
  - $K_2 = -V_s$

- Final solution
- $v_c(t) = V_s - V_s e^{-t/(RC)}$



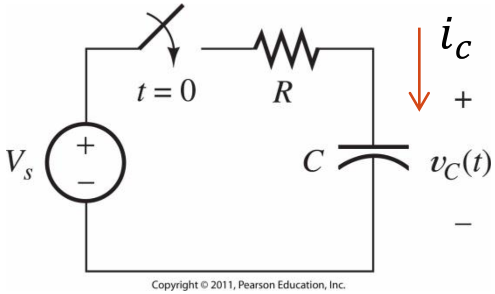
Transient response – eventually decays to a negligible value

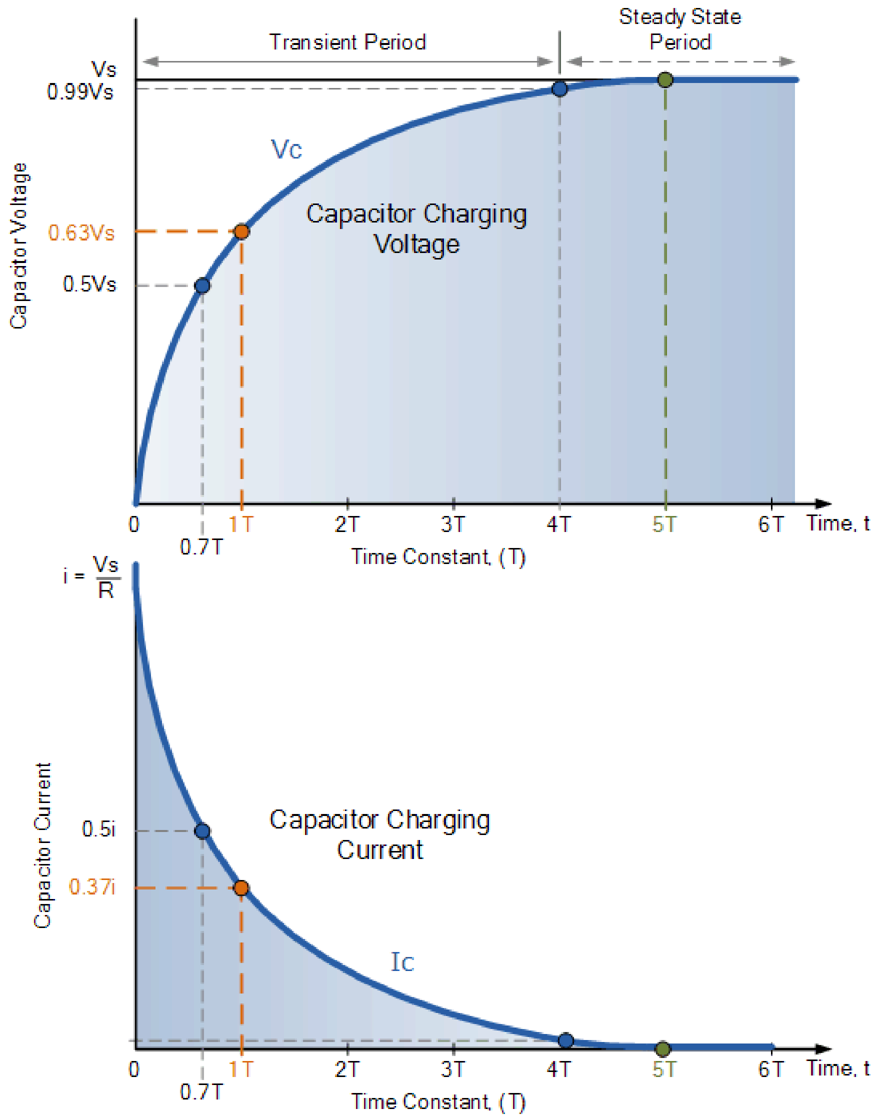
Steady-state response or forced response

RC Current

- The previous examples examined voltage but current could also be examined

- Voltage
  - $v_c(t) = V_s - V_s e^{-t/(RC)}$
- Current
  - $i_c = \frac{V_s - v_c(t)}{R} = C\frac{dv_c(t)}{dt}$
  - $i_c = C\left(\frac{V_s}{RC}e^{-t/(RC)}\right)$
  - $i_c = \frac{V_s}{R}e^{-t/(RC)}$





http://www.electronics-tutorials.ws/rc/rc_1.html

General RC Solution

- Notice both the current and voltage in an RC circuit has an exponential form

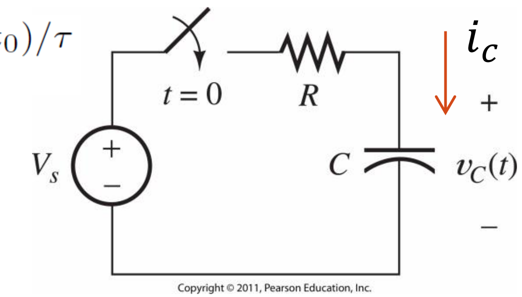
- The general solution for current/voltage is:

$$x(t) = x_f + \left[x(t_0^+) - x_f\right] e^{-(t-t_0)/\tau}$$

  - x – represents current or voltage
  - t_0 – represents time when source switches
  - x_f - final (asymptotic) value of current/voltage
  - τ – time constant (RC)
- Find values and plug into general solution

Example

$$x(t) = x_f + \left[x(t_0^+) - x_f\right] e^{-(t-t_0)/\tau}$$



- Solve for $v_c(t)$
  - $v_f = V_s$ steady-state analysis
  - $v_c(0^+) = 0$ no voltage when switch open
  - $\tau = RC$ equivalent resistance/capacitance
- $v_c(t) = V_s + [0 - V_s]e^{-t/(RC)} = V_s - V_s e^{-t/(RC)}$
- Solve for $i_c(t)$
  - $i_f = 0$ fully charged cap → no current
  - $i_c(0^+) = \frac{V_S - v_C(0^+)}{R} = \frac{V_S - 0}{R} = \frac{V_S}{R}$
- $i_c(t) = 0 + \left[\frac{V_S}{R} - 0\right] e^{-t/(RC)} = \frac{V_S}{R} e^{-t/(RC)}$

First-Order RL Circuits

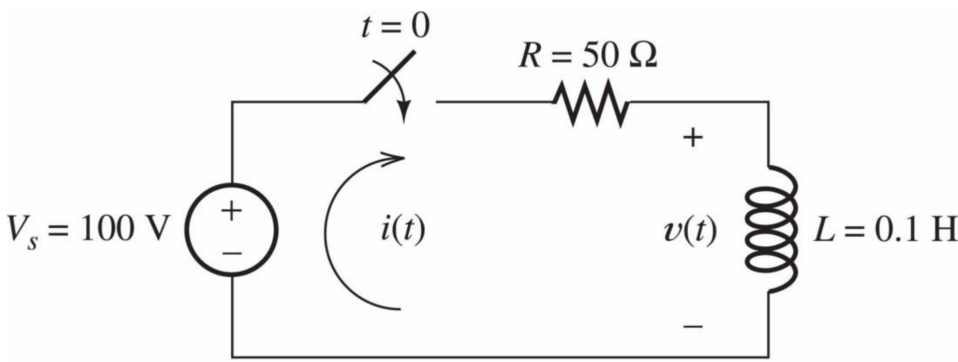
- Contains DC sources, resistors, and a single inductance

- Same technique to analyze as for RC circuits

1. Apply KCL and KVL to write circuit equations
2. If the equations contain integrals, differentiate each term in the equation to produce a pure differential equation
   - Use differential forms for I/V relationships for inductors and capacitors
3. Assume solution of the form $K_1 + K_2 e^{st}$
4. Substitute the solution into the differential equation to determine the values of K_1 and s
5. Use initial conditions to determine the value of K_2
6. Write the final solution

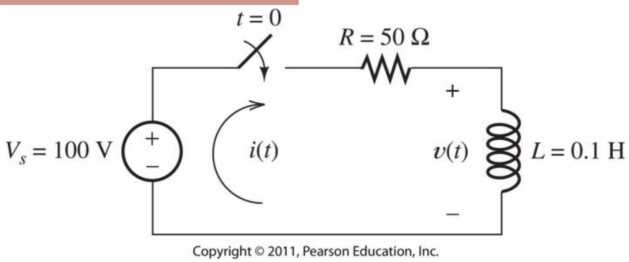
RL Example

- Before switch
  - $i(0^-) = 0$
- KVL around loop
  - $V_s - Ri(t) - L\frac{di(t)}{dt} = 0$
  - $i(t) + \frac{L}{R}\frac{di(t)}{dt} = \frac{V_s}{R}$
    - Notice this is the same equation form as the charging capacitor example
- Solution of the form
  - $i(t) = K_1 + K_2 e^{st}$
- Solving for K_1, s
  - $K_1 + K_2 e^{st} + \frac{L}{R} K_2 s e^{st} = \frac{V_s}{R}$
    - $K_1 = \frac{V_s}{R}$
    - $\left(1 + \frac{L}{R}s\right) \rightarrow S = -\frac{R}{L}$



- Solving for K_2
  - $i(0^+) = 0 = \frac{V_s}{R} + K_2 e^{-tR/L}$
  - $0 = \frac{V_s}{R} + K_2 e^0$
  - $K_2 = -\frac{V_s}{R}$
- Final Solution
  - $i(t) = \frac{V_s}{R} - \frac{V_s}{R} e^{-tR/L}$
  - $i(t) = 2 - 2e^{-500t}$

RL Example

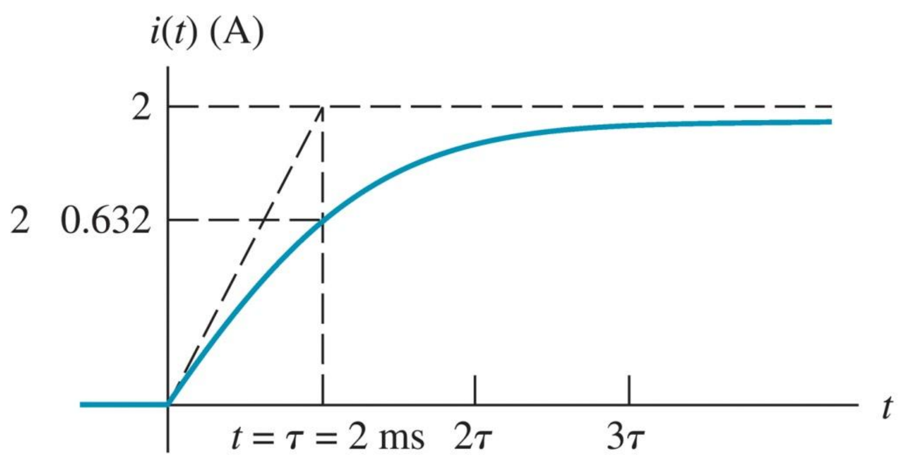


- $i(t) = 2 - 2e^{-500t}$
- Notice this is in the general form we used for RC circuits

$$x(t) = x_f + \left[x(t_0^+) - x_f\right] e^{-(t-t_0)/\tau}$$

  - $\tau = \frac{L}{R}$

- Find voltage $v(t)$
  - $v_f = 0$, steady-state short
  - $v(0^+) = 100$
    - No current immediately through R, $v = L\frac{di(t)}{dt}$
- $v(t) = 100e^{-t/\tau}$



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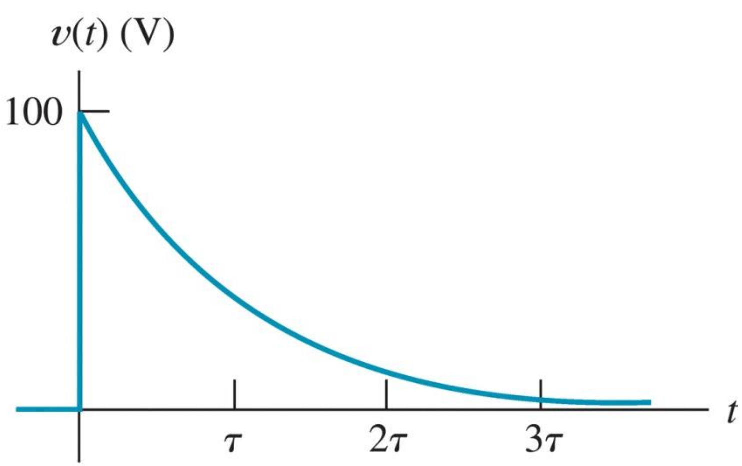

(b)