EE292: Fundamentals of ECE

Fall 2012
TTh 10:00-11:15 SEB 1242

Lecture 5
120911

http://www.ee.unlv.edu/~b1morris/ee292/

Outline

- Review Node-Voltage Analysis
- Mesh-Current Analysis
- Superposition
- Thevenin Equivalent Circuits
- Norton Equivalent Circuits

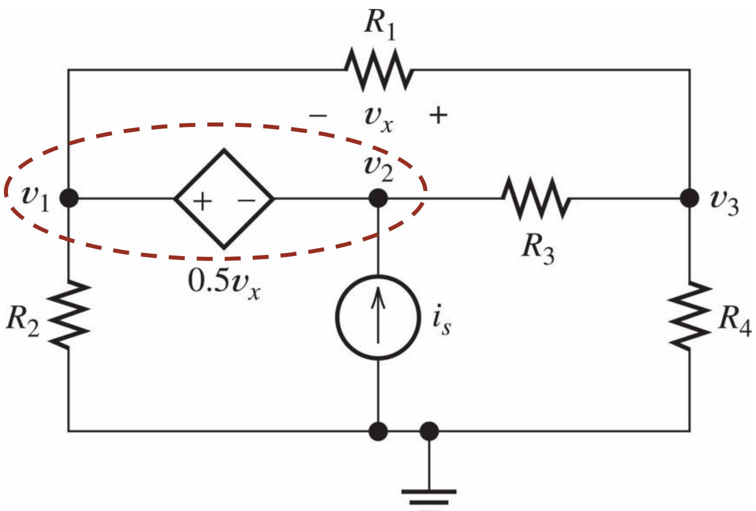
Node-Voltage Analysis

- Voltages at nodes are unknown
- Use KCL equations at nodes

- Steps:

1. Select a reference node
2. Label each additional by a node voltage
3. Write network equations
    - Use KCL at nodes/supernodes, KVL for any additional equations
    - Dependent source equations should be re-written in terms of node voltages
4. Put the equations into standard form and solve for the node voltages

Node-Voltage with Controlled Source



1. Create supernode over node 1, 2
   - KVL with supernode

     a) $v_1 - 0.5v_x - v_2 = 0$
   - KCL @ supernode

     b) $\frac{v_1}{R_2} + \frac{v_1 - v_3}{R_1} + \frac{v_2 - v_3}{R_3} = i_s$

2. KCL @ node 3

   c) $\frac{v_3 - v_2}{R_3} + \frac{v_3 - v_1}{R_1} + \frac{v_3}{R_4} = 0$

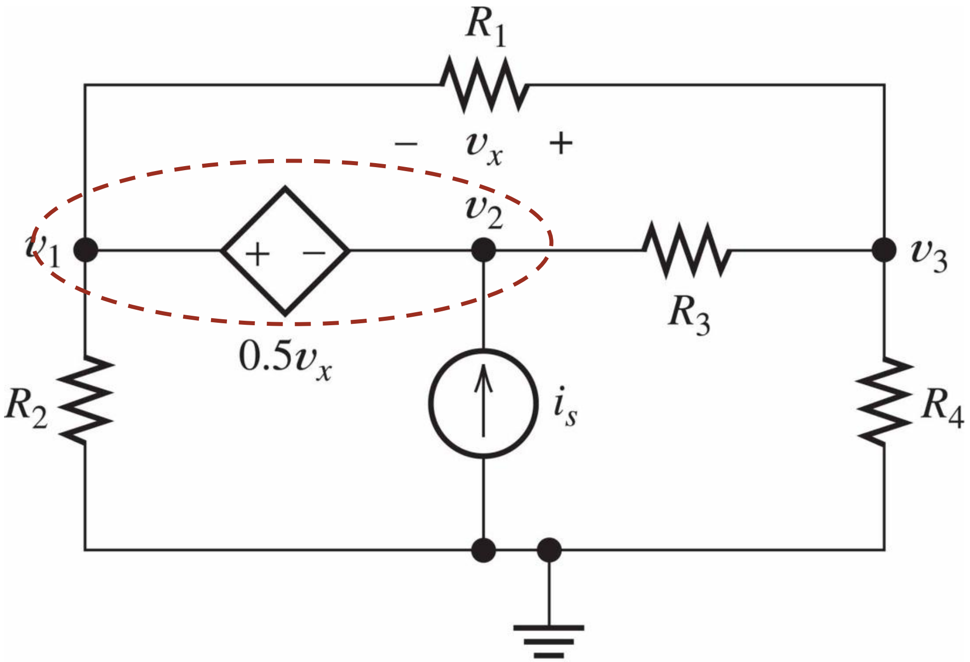
3 equations

4 unknown voltages

Node-Voltage with Controlled Source

3. Replace dependent variable with node voltages

- $v_x = v_3 - v_1 \quad \Rightarrow \quad v_1 - 0.5\,(v_3 - v_1) - v_2 = 0$



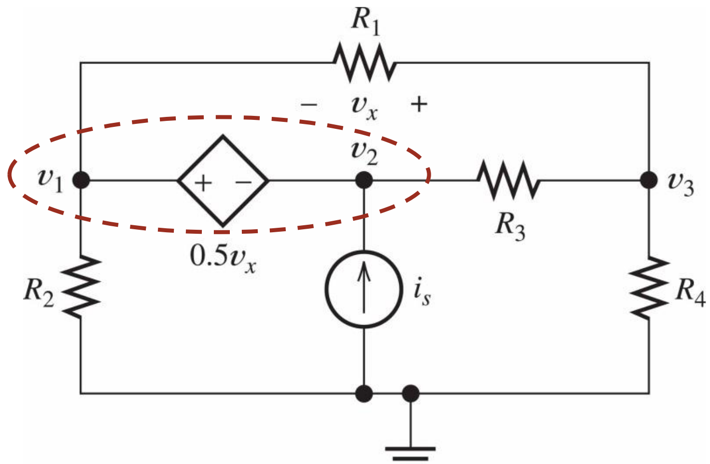
Node-Voltage with Controlled Source

4. Place in standard form

1. $v_1 - 0.5\,(v_3 - v_1) - v_2 = 0$ $\qquad (1.5)\,v_1 + (-1)\,v_2 + (-0.5)\,v_3 = 0$

2. $\frac{v_1}{R_2} + \frac{v_1 - v_3}{R_1} + \frac{v_2 - v_3}{R_3} = i_s \quad \Rightarrow \quad \left(\frac{1}{R_2} + \frac{1}{R_1}\right)v_1 + \left(\frac{1}{R_3}\right)v_2 + \left(-\frac{1}{R_1} - \frac{1}{R_3}\right)v_3 = i_s$

3. $\frac{v_3 - v_2}{R_3} + \frac{v_3 - v_1}{R_1} + \frac{v_3}{R_4} = 0 \qquad \left(-\frac{1}{R_1}\right)v_1 + \left(-\frac{1}{R_3}\right)v_2 + \left(\frac{1}{R_3} + \frac{1}{R_1} + \frac{1}{R_4}\right)v_3 = 0$



Mesh-Current Analysis

- Another general circuit analysis technique for planar networks
  - Planar – non-crossing elements

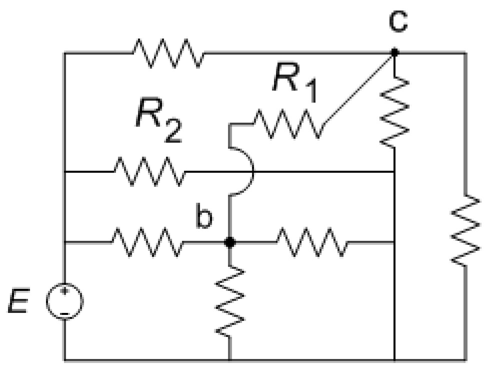


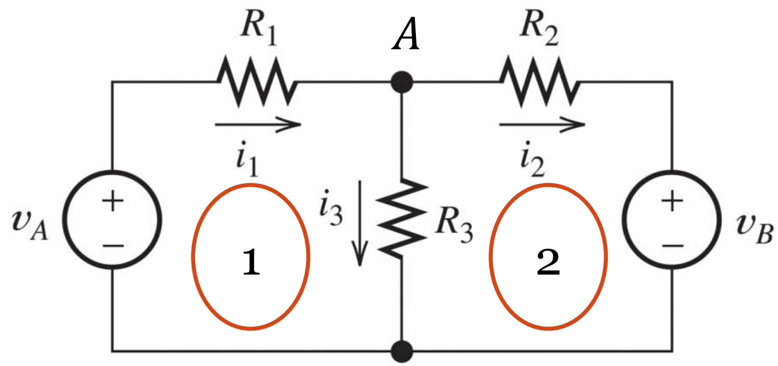
Fig. 5: A Non-Planar Circuit

- Define unknown mesh currents and use KVL

- Mesh – loop of a circuit that contains no other loops

Example Branch Currents

- KVL around loop 1
  - $v_A - i_1 R_1 - i_3 R_3 = 0$
- KVL around loop 2
  - $i_3 R_3 - i_2 R_2 - v_B = 0$



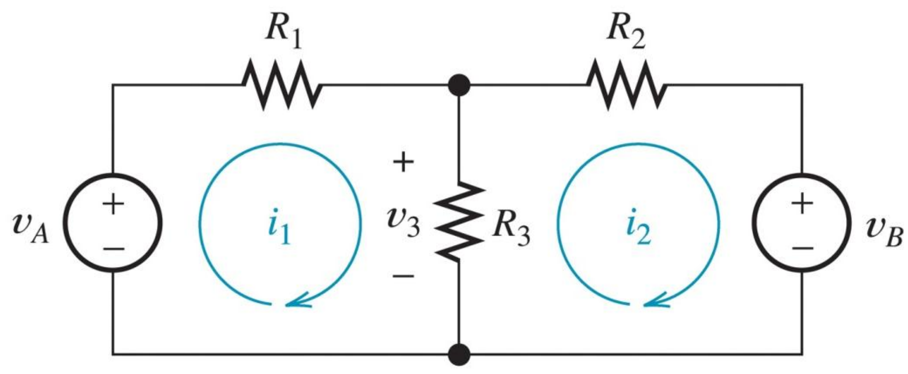
(a) Circuit with branch currents

- Notice: only 2 equations but three unknowns
- KCL @ node *A*
  - $i_1 = i_2 + i_3 \Rightarrow i_3 = i_1 - i_2$
- Substituting for i_3
  - $v_A - i_1 R_1 - (i_1 - i_2) R_3 = 0$
  - $(i_1 - i_2) R_3 - i_2 R_2 - v_B = 0$

2 equations
2 unknown currents

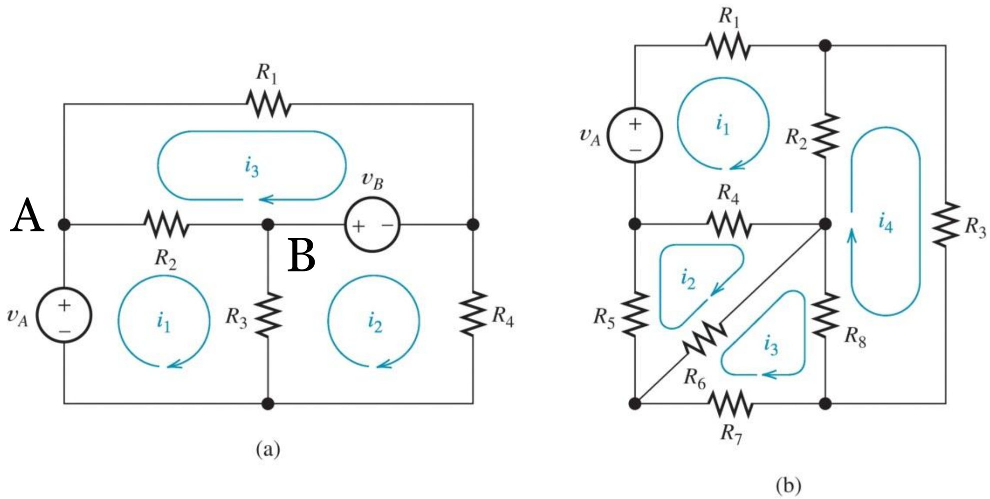
Example Mesh Currents

- Use loops as mesh currents
- What is current through R_3?
    - $v_3 = R_3(i_3) = R_3(i_1 - i_2)$
- KVL with mesh i_1
    - $v_A - i_1R_1 - (i_1 - i_2)R_3 = 0$
- KVL with mesh i_2
    - $(i_1 - i_2)R_3 - i_2R_2 - v_B = 0$
- Same equations as branch currents without KCL substitution



(b) Circuit with mesh currents

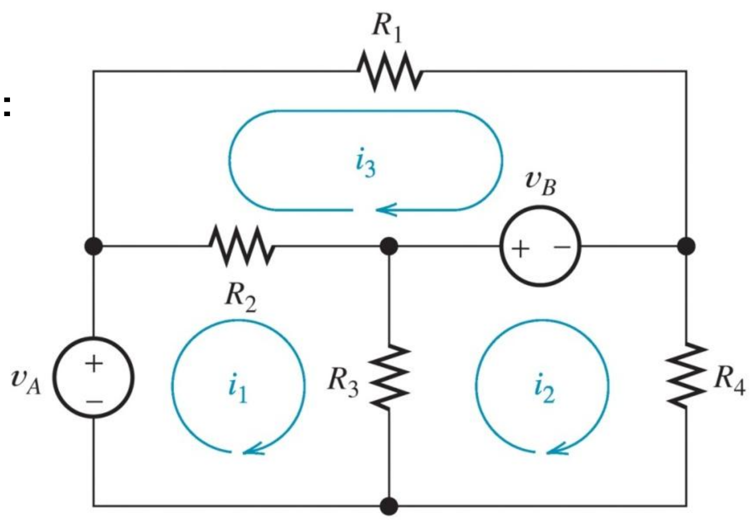
Choosing Mesh Current



- Define a mesh clockwise through a minimum loop
  - Minimum loop – a loop that contains no other loops
- Current through an element is the sum of mesh currents
  - Respect the mesh current directions
  - $i_{R2} = i_1 - i_3$ for left to right current (A→B)
  - $i_{R2} = i_3 - i_1$ for right to left current (B→A)

Write KVL Equations

- KVL with mesh i_1
  - $v_A - (i_1 - i_3)R_2 - \ (i_1 - i_2)R_3$
- KVL with mesh i_2
  - $v_B - \ (i_1 - i_2)R_3 + i_2 R_4 = 0$
- KVL with mesh i_3
  - $-i_3 R_1 + v_B - \ (i_3 - i_1)R_2 = 0$



Arrange Eqs. in Standard Form

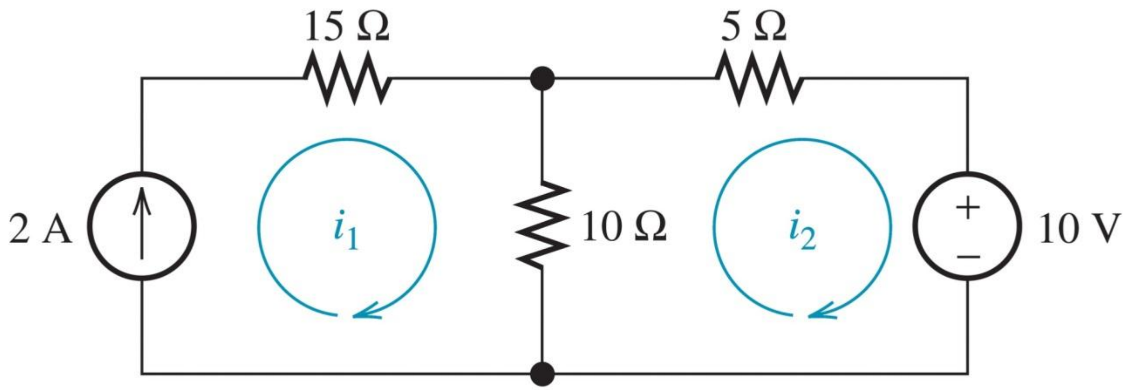
- Standard form => RI=V

$$
\begin{aligned}
v_A - (i_1 - i_3)R_2 - (i_1 - i_2)R_3 &= 0 \\
v_B - (i_1 - i_2)R_3 + i_2 R_4 &= 0 \\
-i_3 R_1 + v_B - (i_3 - i_1)R_2 &= 0
\end{aligned}
\qquad
\begin{aligned}
(-R_2 - R_1)i_1 + (R_3)i_2 + (R_2)i_3 &= -v_A \\
(-R_3)i_1 + (R_3 + R_4)i_2 + (0)i_3 &= -v_B \\
(R_2)i_1 + (0)i_2 + (-R_1 - R_2)i_3 &= -v_B
\end{aligned}
$$

- In matrix form that can be solved by Matlab

$$
\underbrace{\begin{bmatrix} -(R_2 + R_1) & R_3 & R_2 \\ -R_3 & (R_3 + R_4) & 0 \\ R_2 & 0 & -(R_1 + R_2) \end{bmatrix}}_{R}
\underbrace{\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}}_{I}
=
\underbrace{\begin{bmatrix} -v_A \\ -v_B \\ -v_B \end{bmatrix}}_{V}
$$

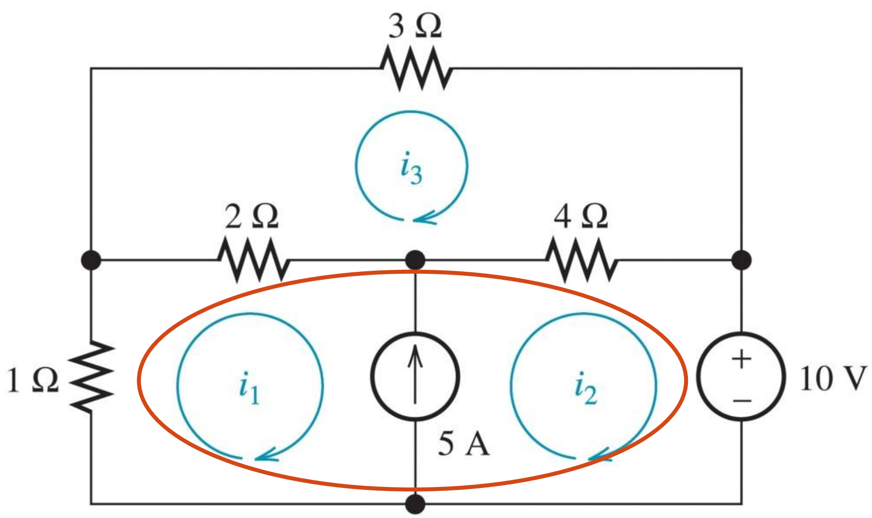
Mesh Currents with Current Sources



- Voltage across a current source is unknown
  - KVL with i_1 will have a new unknown voltage
- But, by definition
  - $i_1 = 2$ A because of the current source
  - Results in a single unknown mesh current i_2
- KVL with mesh i_1
  - $10 + i_2 5 - (i_1 - i_2)10 = 0$
  - $5i_2 - (2 - i_2)10 = -10$
  - $15i_2 = 10$
  - $i_2 = 2/3$ A

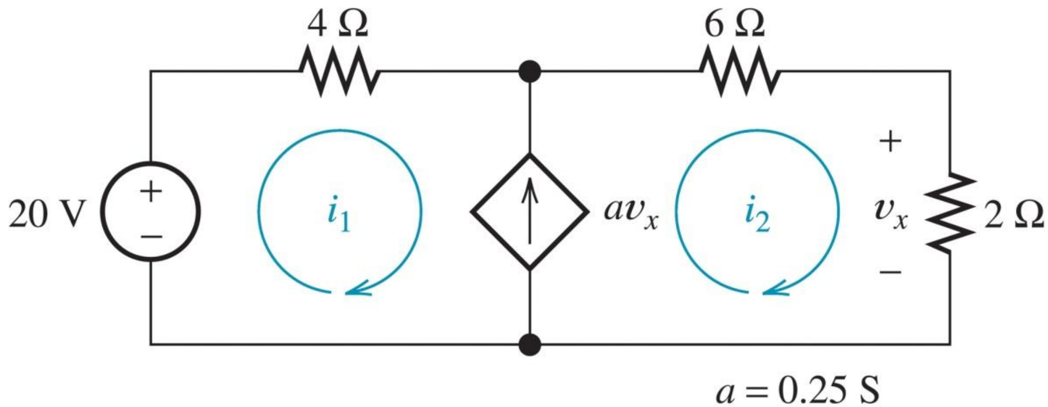
Supermeshes

- A combination of meshes useful for mesh-current analysis with shared current sources
  - Adjust KVL to get an equation around the supermesh
- Same idea as the supernode in node-voltage analysis

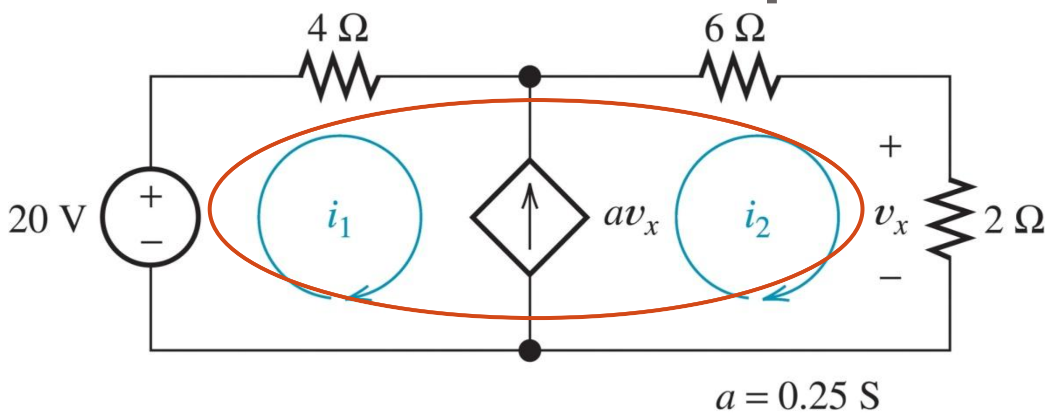


Meshes and Controlled Sources

- Same technique as previously described
  - Must express controlling variable in terms of the mesh currents



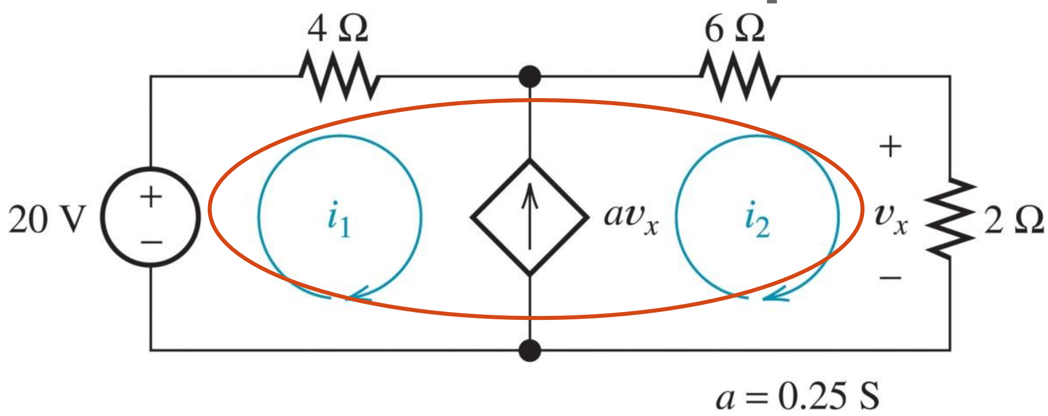
Example Controlled Supermesh



- 2 unknown mesh currents → need 2 equations

1. Create supermesh because of shared current source
2. KVL around supermesh
   - $20 - 4i_1 - 6i_2 - 2i_2 = 0$
3. Replace controlling variable v_x with mesh currents
   - $v_x = 2i_2$

Example Controlled Supermesh



- 2 unknown mesh currents → need 2 equations

4. Write (mesh) expression for the current source
   - $av_x = a(2i_2) = i_2 - i_1$
   - $(2a - 1)i_2 = -i_1$
   - $i_1 = 0.5i_2$
5. Plug back into KVL equation
   - $20 - 4i_1 - 6i_2 - 2i_2 = 0$
   - $20 - 4i_2/2 - 6i_2 - 2i_2 = 0$
   - $10i_2 = 20$
   - $i_2 = 2$ A and $i_1 = 1$ A

Steps for Mesh-Current Analysis

1. If necessary, redraw the network without crossing conductors or elements. Then define the mesh currents flowing around each of the open areas defined by the network. For consistency, we usually select a clockwise direction for each of the mesh currents
2. Write network equations, stopping after the number of equations is equal to the number of mesh currents. First, use KVL to write voltage equations for meshes that do not contain sources. Next, if any current sources are present, write expressions for their currents in terms of the mesh currents. Finally, if a current source is common to two meshes, write a KVL equation for the supermesh

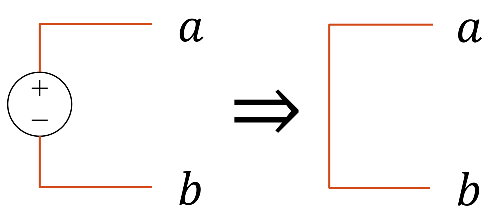
Steps for Mesh-Current Analysis

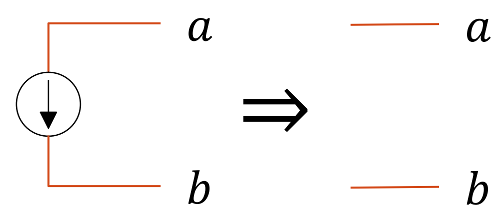
3. If the circuit contains dependent sources, find expressions for the controlling variable in terms of the mesh currents. Substitute into the network equations and obtain equations having only the mesh currents as unknowns
4. Put the equations into standard form. Solve for the mesh currents by use of determinants or other means
5. Use the values found for the mesh currents to calculate any other currents or voltages of interest.

Superposition Principle

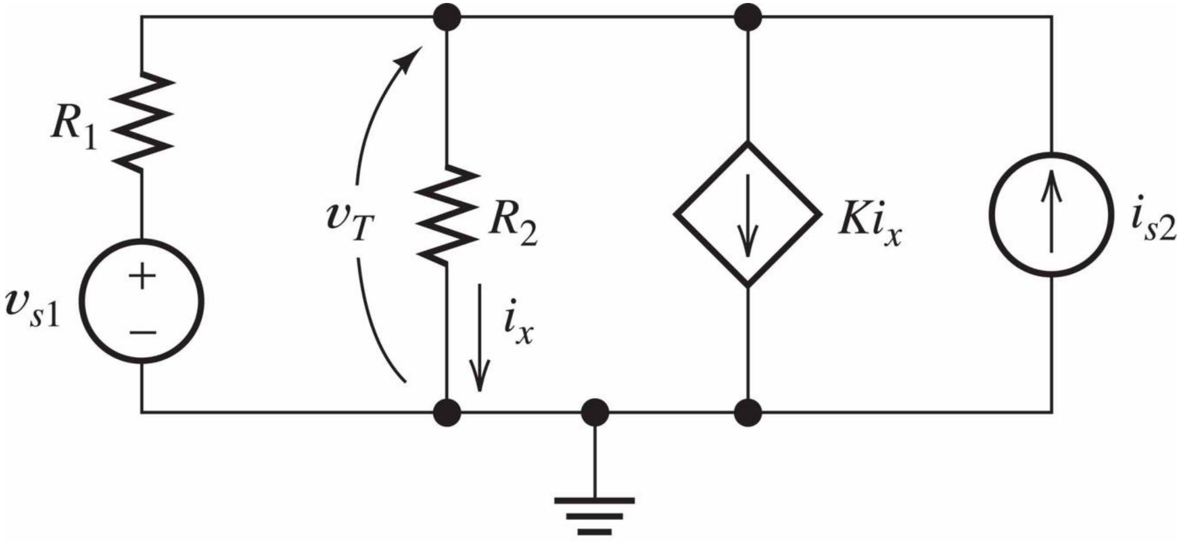
- Given a circuit with multiple independent sources, the total response is the sum of the responses to each individual source
  - Requires linear dependent sources

- Analyze each independent source individually
  - Must zero out independent sources, but keep dependent sources
    - A voltage source becomes a short circuit
    - A current source becomes an open circuit





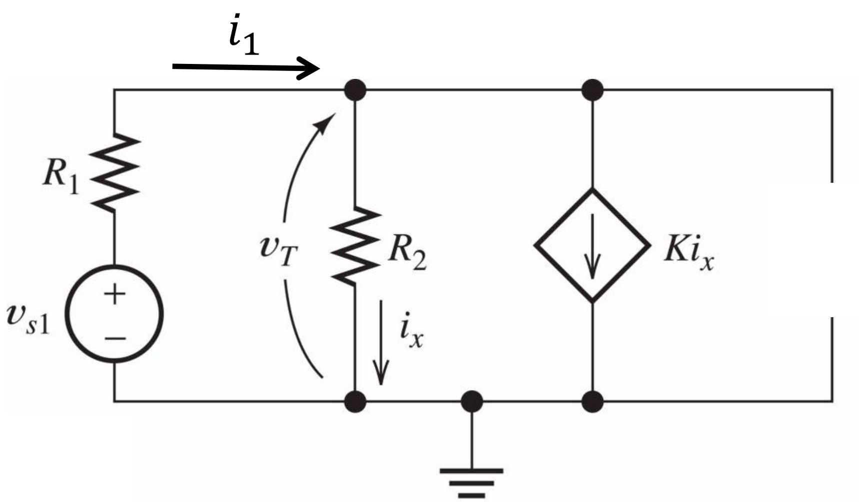
Superposition Example



- 2 independent sources
  - Response is sum of each source response
  - $v_T = v_{T1} + v_{T2}$

Voltage Source $s1$

- Zero sources – open current source
- KCL @ T1



$$i_1 = i_x + Ki_x = (1+K)i_x$$

$$\frac{v_{s1} - v_{T1}}{R_1} = (1+K)\frac{v_{T1}}{R_2}$$

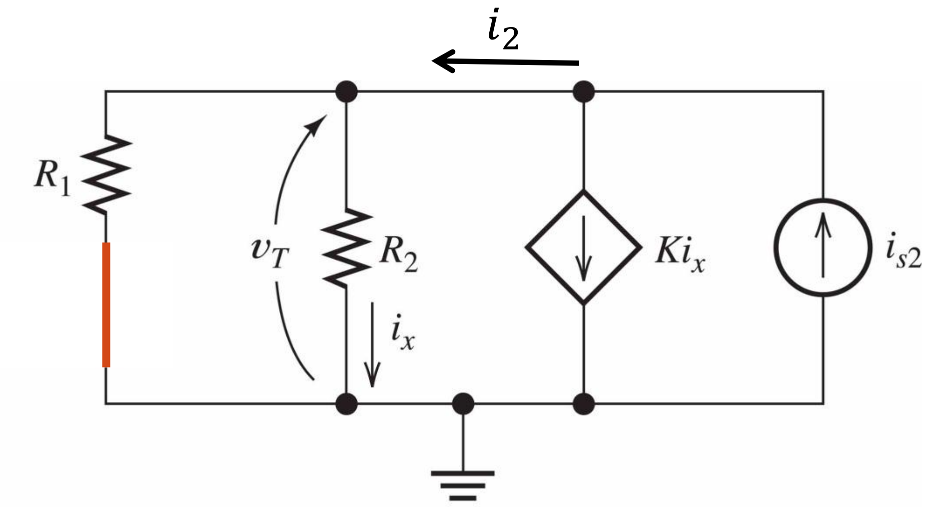
$$\left(\frac{1+K}{R_2} + \frac{1}{R_1}\right) v_{T1} = \frac{v_{s1}}{R_1}$$

$$\left[\frac{R_1(1+K) + R_2}{R_1 R_2}\right] v_{T1} = \frac{v_{s1}}{R_1}$$

$$v_{T1} = \left(\frac{R_2}{R_1(1+K) + R_2}\right) v_{s1}$$

Current Source $s2$

- Zero sources – short voltage source
- KCL @ T2



$$i_x = i_2\left(\frac{R_1}{R_1+R_2}\right)$$

$$i_2 = \frac{v_{T2}}{R_1||R_2} = \left(\frac{R_1+R_2}{R_1R_2}\right)v_{T2}$$

$$\begin{aligned} i_{s2} &= i_2 + Ki_x \\ &= i_2 + Ki_2\left(\frac{R_1}{R_1+R_2}\right) \\ &= \left(1+\frac{KR_1}{R_1+R_2}\right)i_2 \\ &= \left(\frac{R_1+R_2+KR_1}{R_1+R_2}\right)\left(\frac{R_1+R_2}{R_1R_2}\right)v_{T2} \\ &= \left(\frac{(1+K)R_1+R_2}{R_1R_2}\right)v_{T2} \\ v_{T2} &= \left(\frac{R_1R_2}{(1+K)R_1+R_2}\right)i_{s2} \end{aligned}$$