EE292: Fundamentals of ECE

Fall 2012
TTh 10:00-11:15 SEB 1242

Lecture 4
120906

http://www.ee.unlv.edu/~b1morris/ee292/

Outline

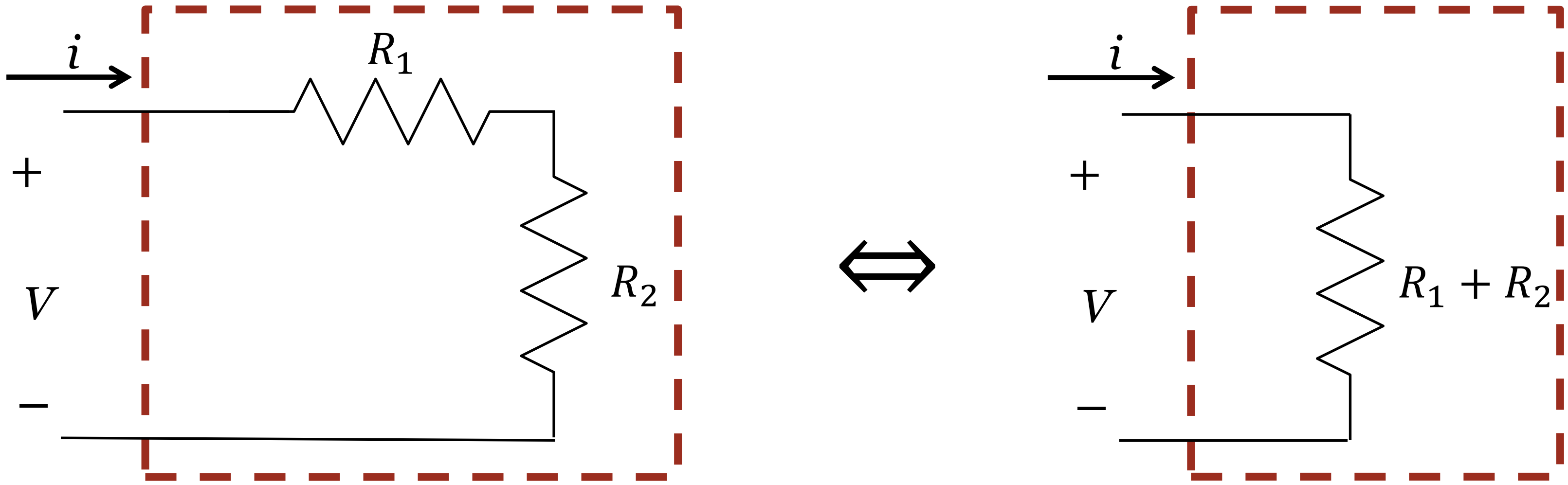
- Review
- Voltage Divider
- Current Divider
- Node-Voltage Analysis

Network Analysis

- Process of determining current, voltage, and power of every circuit (network) element
- Series and parallel equivalents help simplify circuits for network analysis
  - Repeated application of equivalents is usually required

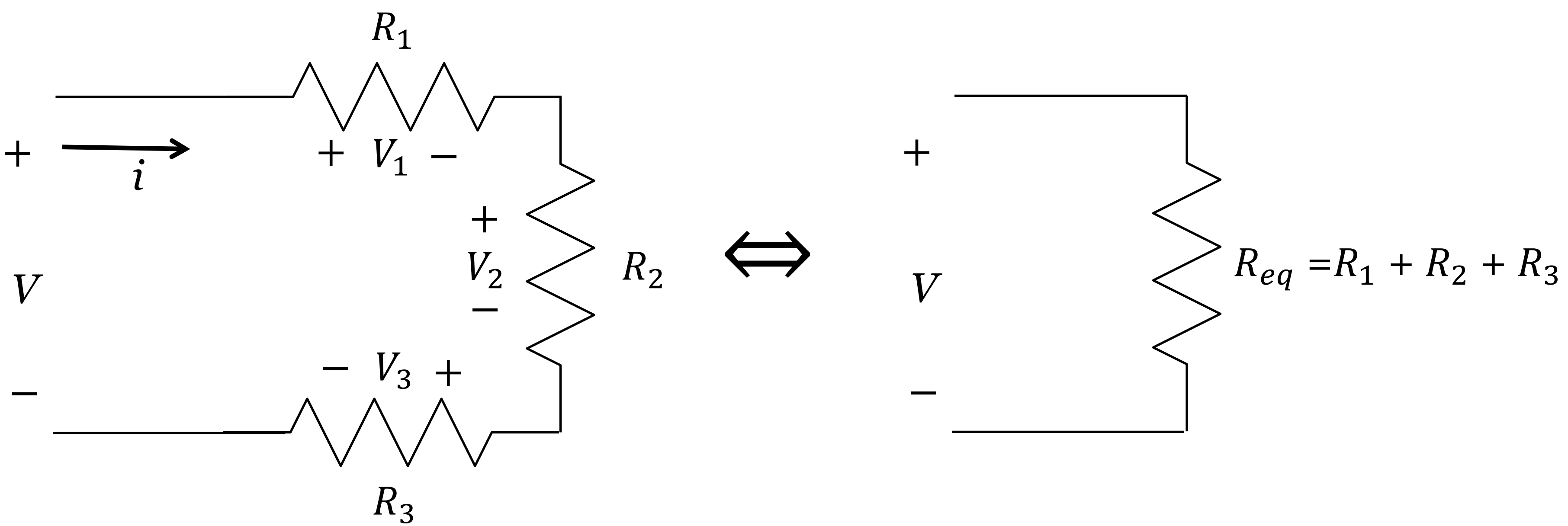
Equivalent Circuits

- A complicated circuit can be viewed more simply from its terminals as a
  1. single circuit element (best case)
  2. simple combination of elements (worst case)



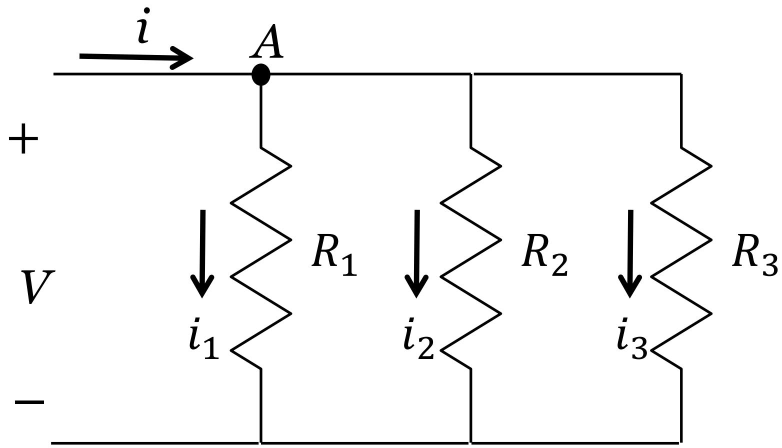
The internals are not important from the perspective of the terminals outside the black box

Series Resistance

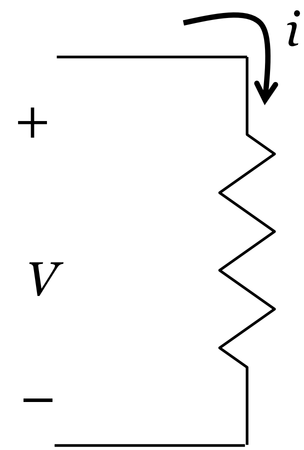


- A series circuit has resistances arranged in a chain with only one current path through all elements
  - Series resistance can be replaced by equivalent resistance
- $R_\Sigma = R_{eq} = \sum_i R_i$

Parallel Resistance



$\Leftrightarrow$

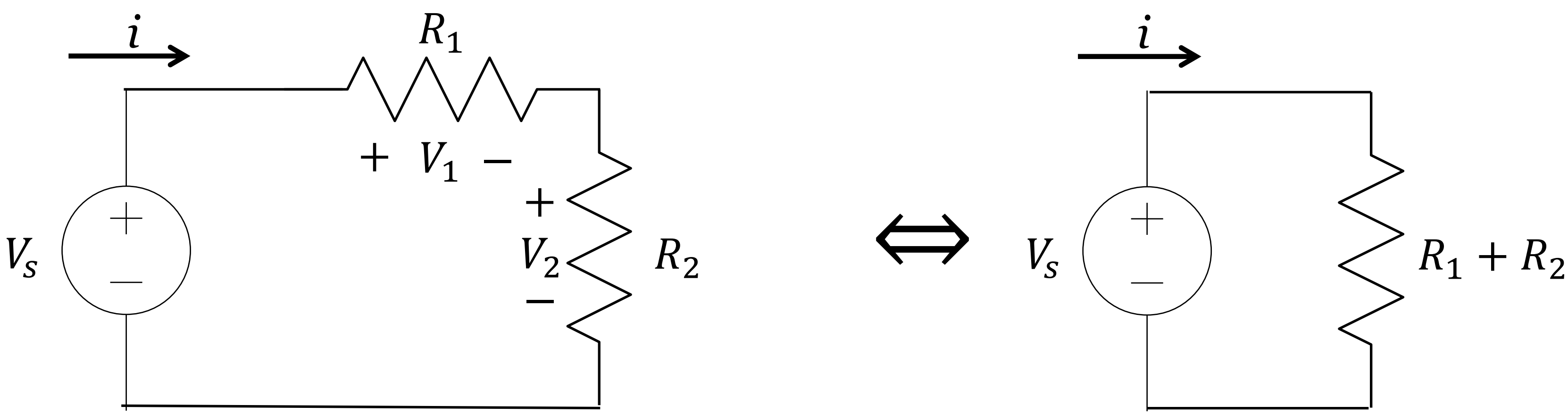


$$R_{eq} = 1/\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

- A parallel circuit has resistors arranged with their positive terminals all connected together and their negative terminals connected together
  - Parallel resistance can be replaced by equivalent resistance
  - $R_{eq} = 1/\sum_i \frac{1}{R_i}$
- Important special case for two resistors
  - $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

Voltage Divider

- For a voltage applied to a series combination, a fraction appears across each resistance

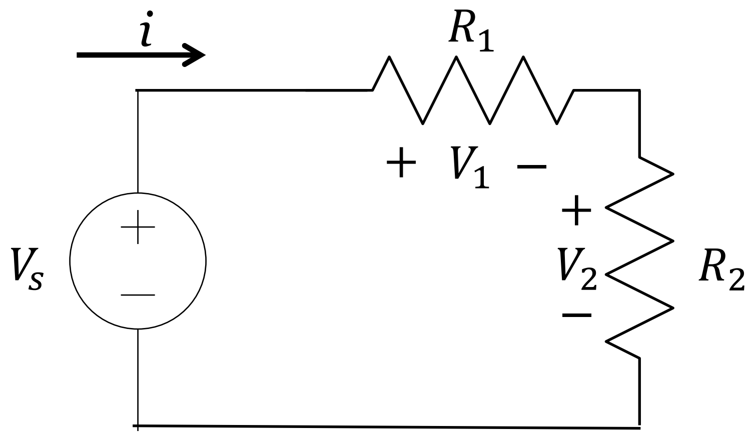


$$v_1 = iR_1 \qquad v_2 = iR_2 \qquad i = \frac{V_s}{R_1 + R_2}$$

$$= V_s\left(\frac{R_1}{R_1 + R_2}\right) \qquad = V_s\left(\frac{R_2}{R_1 + R_2}\right)$$

Voltage-Division Principle

- The fraction of voltage across a given resistance in a series connection is the ratio of the given resistance to the total series resistance



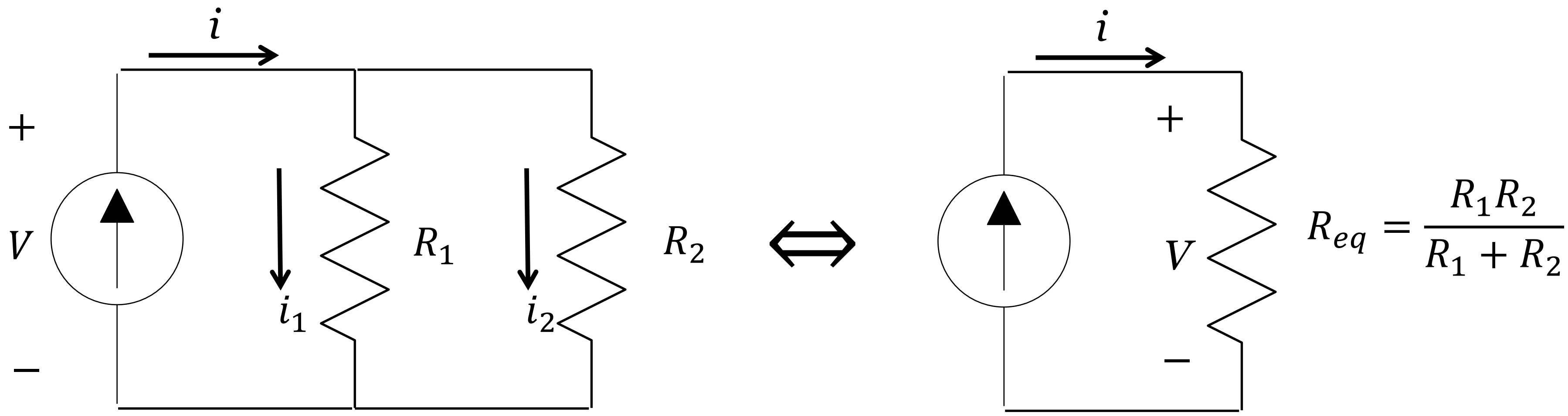
The larger resistor → more voltage drop across it

$$v_1 = iR_1 = V_s\left(\frac{R_1}{R_1 + R_2}\right)$$

$$v_2 = iR_2 = V_s\left(\frac{R_2}{R_1 + R_2}\right)$$

Current Divider

- For a current applied to a parallel combination, a fraction of the current flows through each resistor



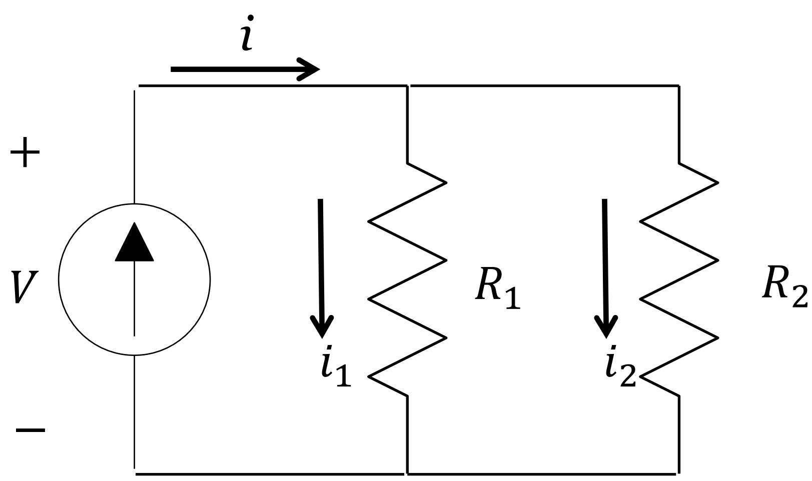
$$i_1 = \frac{v}{R_1} = \frac{i}{R_1}\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$
$$= \left(\frac{R_2}{R_1 + R_2}\right) i$$

$$i_2 = \frac{v}{R_2}$$
$$= \left(\frac{R_1}{R_1 + R_2}\right) i$$

$$v = i R_{eq}$$
$$= i\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

Current-Division Principle

- The fraction of current flowing in a given resistance is the ratio of the other resistance to the sum of the two resistances
  - Only applies for parallel pairs



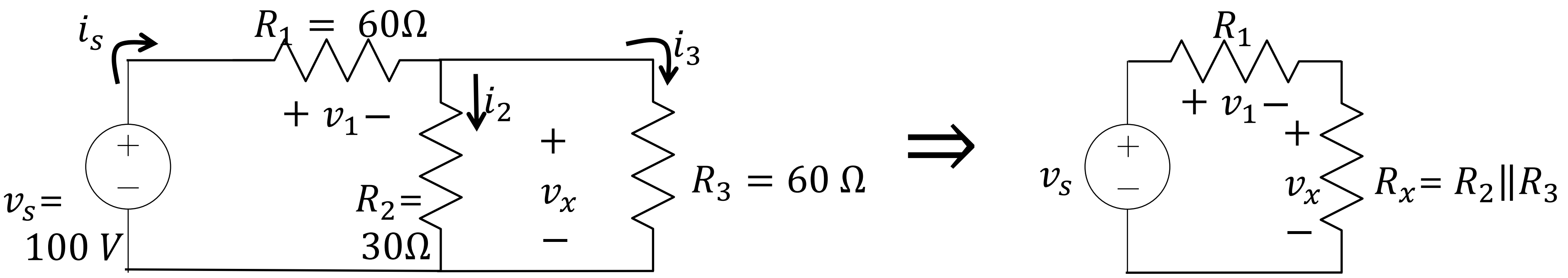
$$i_1 = \frac{v}{R_1} = \frac{i}{R_1}\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$
$$= \left(\frac{R_2}{R_1 + R_2}\right) i$$

$$i_2 = \frac{v}{R_2}$$
$$= \left(\frac{R_1}{R_1 + R_2}\right) i$$

Smaller the resistor → more current through parallel path

Example

- Find all voltages and currents

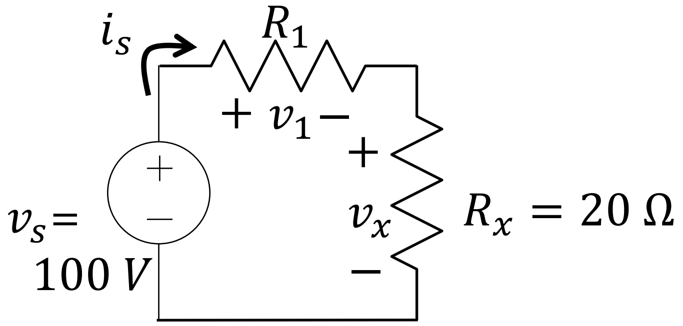


$$R_x = R_2 \| R_3 = \frac{R_2 R_3}{R_2 + R_3}$$

$$\frac{(60)(30)}{60 + 30} = 20\Omega$$

Example

- Using voltage divider



$$v_x = v_s \frac{R_x}{R_1 + R_x}$$

$$= 100\left(\frac{20}{20+60}\right) =$$

$$= 100\left(\frac{1}{4}\right) = 25\ V$$

$$v_1 = v_s \frac{R_1}{R_1 + R_x}$$

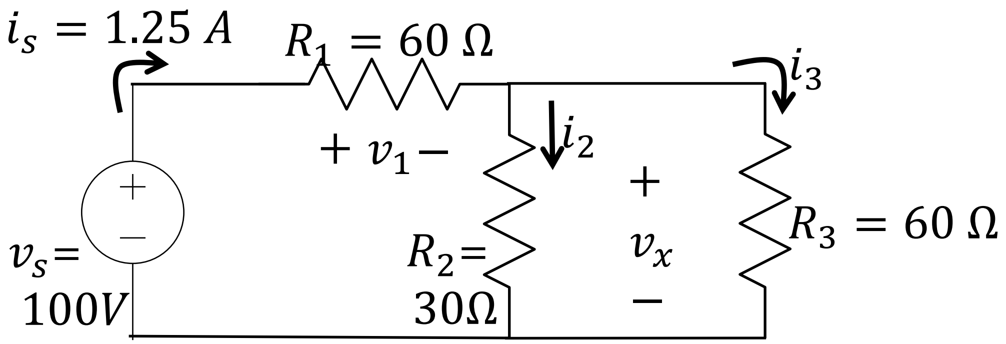
$$= 100\left(\frac{60}{20+60}\right) =$$

$$= 100\left(\frac{3}{4}\right) = 75\ V$$

$$i_s = \frac{v_x}{R_x} = \frac{25}{20} = 1.25A$$

Example

- Using current divider



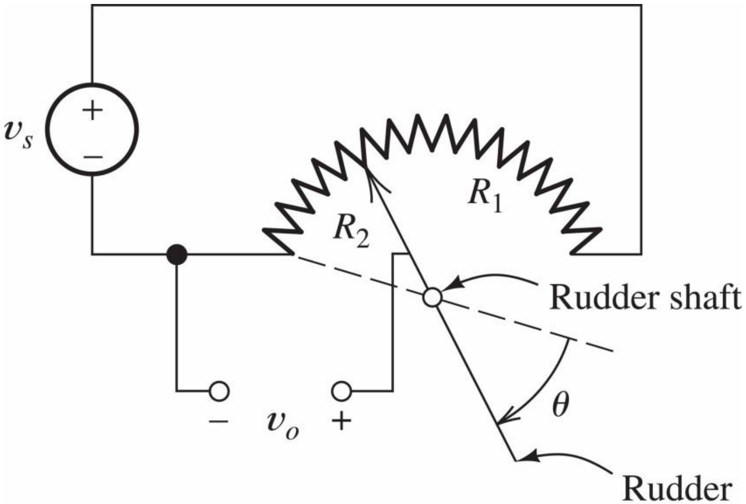
$$i_2 = i_s \frac{R_3}{R_2 + R_3}$$
$$= 1.25\left(\frac{60}{30+60}\right)$$
$$= 1.25\left(\frac{2}{3}\right)$$
$$= \frac{10}{12}\ A$$

$$i_3 = i_s\left(\frac{R_2}{R_2 + R_3}\right)$$
$$= 1.25\left(\frac{30}{30+60}\right)$$
$$= \frac{5}{4}\left(\frac{1}{3}\right)$$
$$= \frac{5}{12}\ A$$

Transducer Example

- Transducer – a device to convert from one form of energy to another
  - E.g. produce a voltage (current) proportional to a physical quantity (distance, pressure, temperature, etc.)

- Voltage-divider transducer

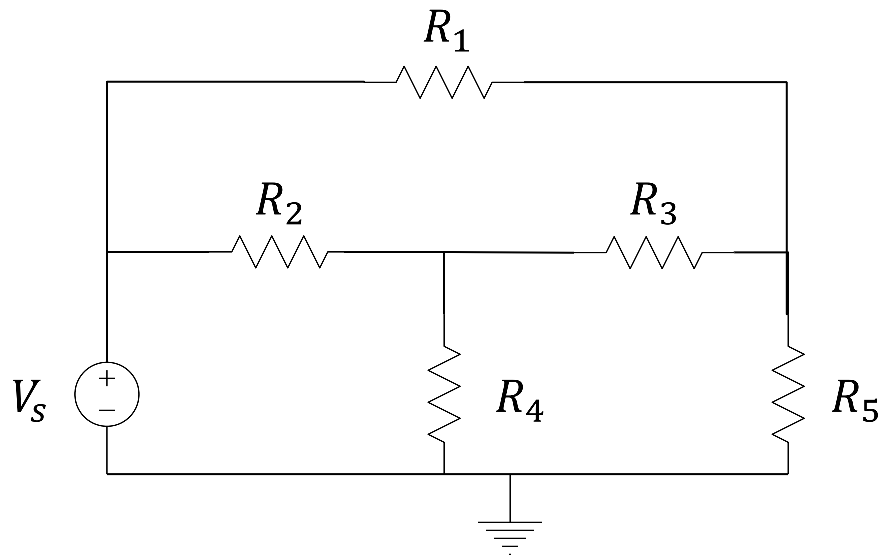


$$v_0 = v_s \frac{R_2}{R_1 + R_2} = \mathrm{K}\theta$$

Node-Voltage Analysis

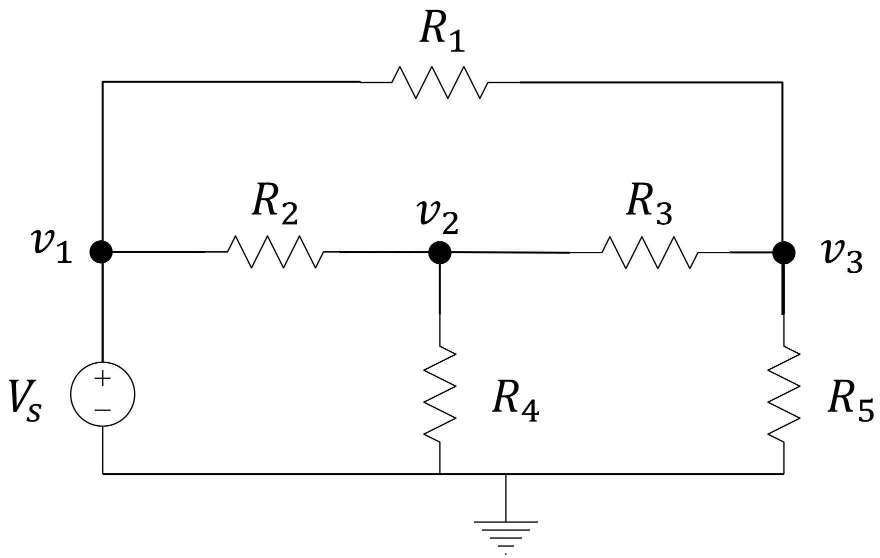
- A general technique that can be applied to any circuit to do analysis
  - Doesn't require series/parallel equivalents
- Define node voltages and use KCL to write equations in terms of these node voltages
- Node – a point where two or more circuit elements meet (are joined together)

Assigning Reference Node



1. Select a reference node
   - Typically select the bottom of a voltage source
   - Label this reference as ground (0 Volts)

Assigning Node Voltages



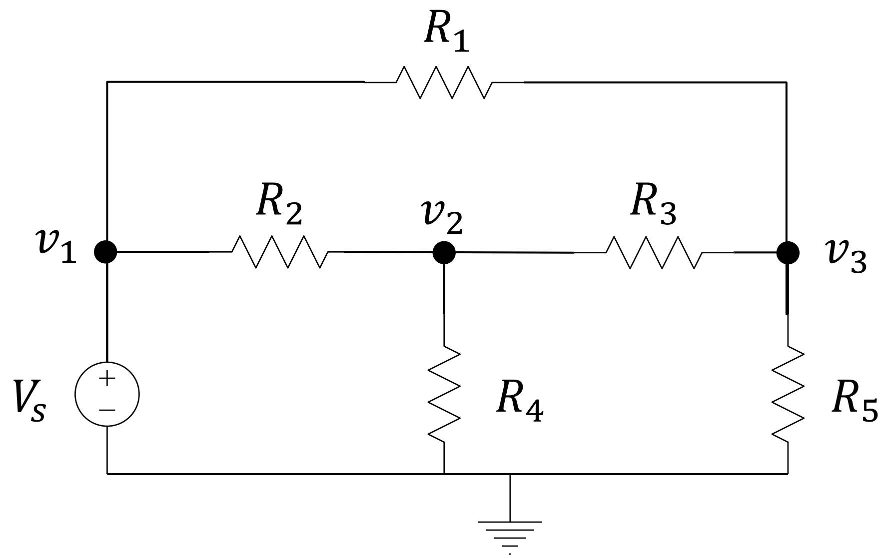
Notice:
$v_1 = V_s$

2. Label each additional node with a voltage
   - This indicates a positive polarity
   - Negative polarity is ground (reference node)
   - Voltage across an element is the difference in node voltages

$$v_{R3} = v_{23} = v_2 - v_3 \Rightarrow \text{current } i_{R3} \text{ from } 2 \longrightarrow 3$$

$$v_{R3} = v_{32} = v_3 - v_2 \Rightarrow \text{current } i_{R3} \text{ from } 3 \longrightarrow 2$$

KCL at Nodes

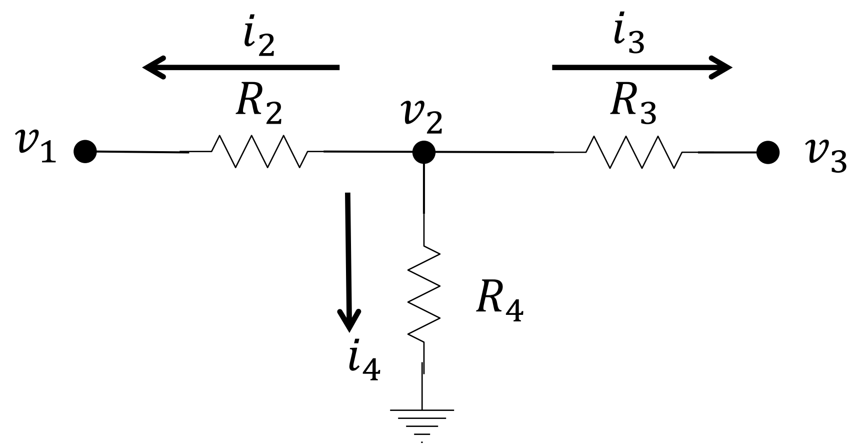


Notice:
$v_1 = V_s$

3. Write KCL equations at the nodes
   - Define all currents leaving nodes
     - Sum of currents leaving node = 0
   - When current sources are present, respect their direction
     - Current in = current out (current in only from a source)

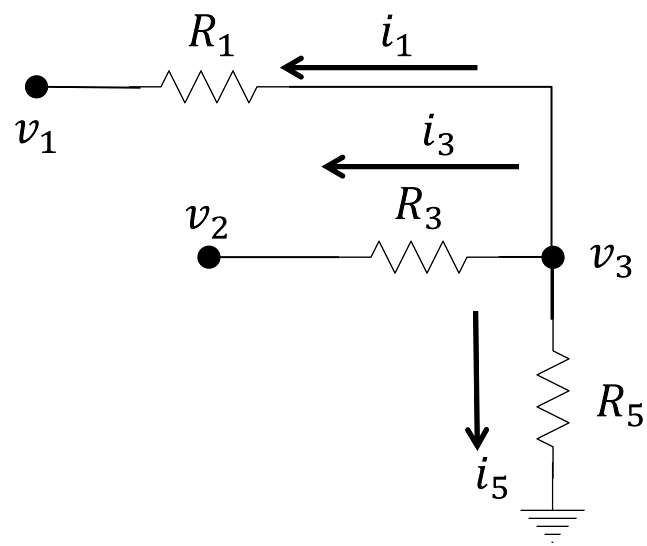
KCL Eqs.

- @ node 2



- $i_2 + i_3 + i_4 = 0$
- $\frac{v_2 - v_1}{R_2} + \frac{v_2 - v_3}{R_3} + \frac{v_2 - 0}{R_4} = 0$

- @ node 3



- $i_1 + i_3 + i_5 = 0$
- $\frac{v_3 - v_1}{R_1} + \frac{v_3 - v_2}{R_3} + \frac{v_3 - 0}{R_5} = 0$

Arrange KCL Equations

4. Arrange KCL equations in “standard” form to solve the system of equations
   - Use Ohm’s Law
   - $\frac{1}{R}v = i$ or $gv = i$
   - E.g. for two equations
   - $g_{11}v_1 + g_{12}v_2 = i_1$
   - $g_{21}v_1 + g_{22}v_2 = i_2$
   - In matrix form

$$\underbrace{\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}}_{G} \underbrace{\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}}_{V} = \underbrace{\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}}_{I}$$

Solve for Unknown Voltages

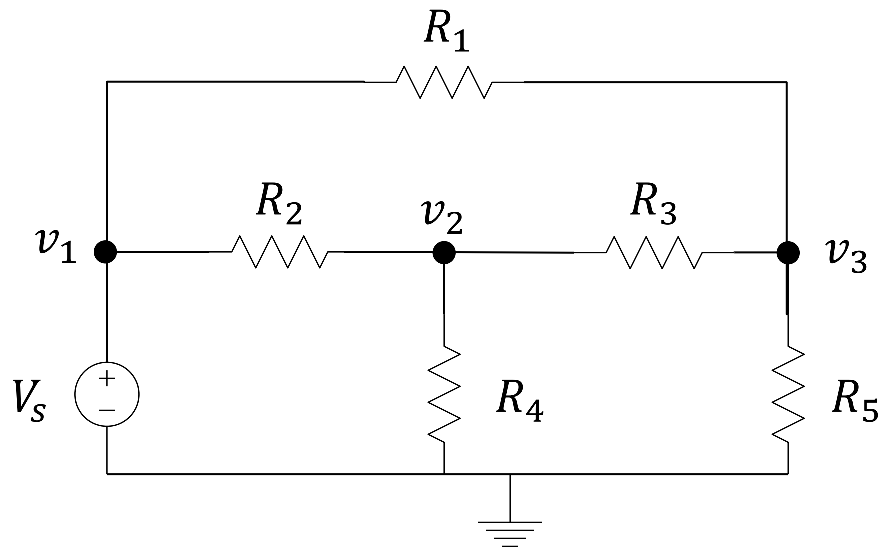
- KCL equations
- $\frac{v_2-v_1}{R_2}+\frac{v_2-v_3}{R_3}+\frac{v_2-0}{R_4}=0$
- $\frac{v_3-v_1}{R_1}+\frac{v_3-v_2}{R_3}+\frac{v_3-0}{R_5}=0$

- Standard form
- $\left(-\frac{1}{R_2}\right)v_1+\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)v_2+\left(-\frac{1}{R_3}\right)v_3=0$
- $\left(-\frac{1}{R_1}\right)v_1+\left(-\frac{1}{R_3}\right)v_2+\left(\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_5}\right)v_3=0$

- Notice 2 equations with 3 unknowns
  - Remember $v_1 = V_s$
- $\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)v_2+\left(-\frac{1}{R_3}\right)v_3=\left(\frac{1}{R_2}\right)v_s$
- $\left(-\frac{1}{R_3}\right)v_2+\left(\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_5}\right)v_3=\left(\frac{1}{R_1}\right)v_s$

$$\underbrace{\begin{bmatrix}\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right) & \left(-\frac{1}{R_3}\right)\\ \left(-\frac{1}{R_3}\right) & \left(\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_5}\right)\end{bmatrix}}_{G}\underbrace{\begin{bmatrix}v_2\\ v_3\end{bmatrix}}_{V}=\underbrace{\begin{bmatrix}\left(\frac{v_s}{R_2}\right)\\ \left(\frac{v_s}{R_1}\right)\end{bmatrix}}_{I}$$

Network Analysis



Notice:
$v_1 = V_s$

- Using values $V_s = 9\ V$, all $R = 10\ \Omega$ and Matlab
- `>> V = pinv(G) * I = [4.5; 4.5]`
- $v_1 = 9\ V, v_2, v_3 = 4.5\ V$

- Given node voltages it is possible to solve for all currents and power

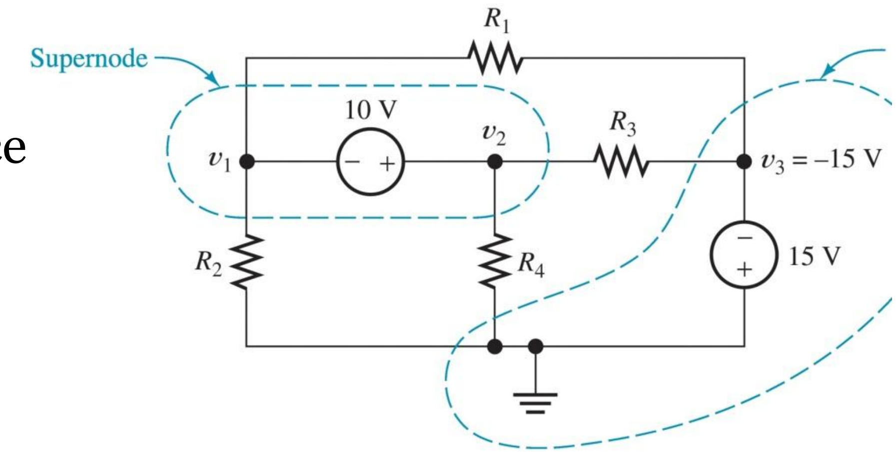
Supernodes

- A combination of circuit nodes useful for node-voltage analysis with non-grounded voltage sources
  - Adjust KCL so all current into a supernode is equal to the current flow out of supernode

- Acts like a node
  - Think of water pipes

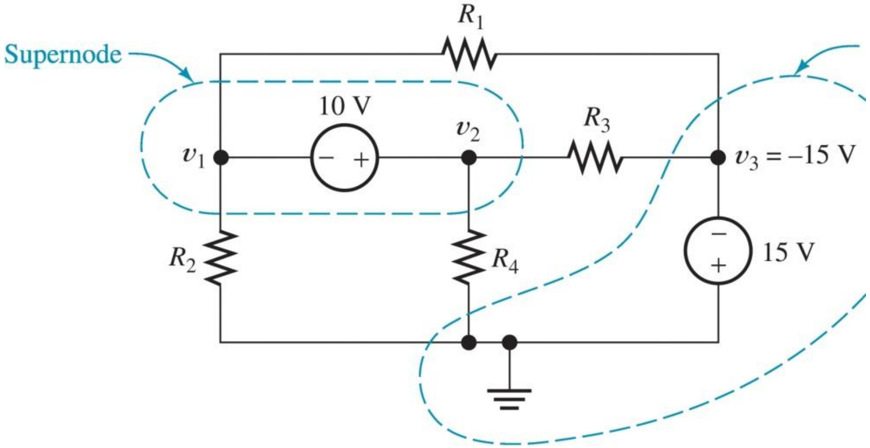
Supernode Example

- v_3 is not an unknown because it is connected to a voltage source
  - $v_3 = -15\ V$
- v_1, v_2 are unknown
  - Connected though voltage source
  - $v_2 - v_1 = 10\ V$
- KCL @ node 1
  - All currents leave node
  - $i_1 + i_2 + i_s = 0$
  - The voltage source introduces a new variable i_s
    - This is not desirable



Supernode Example

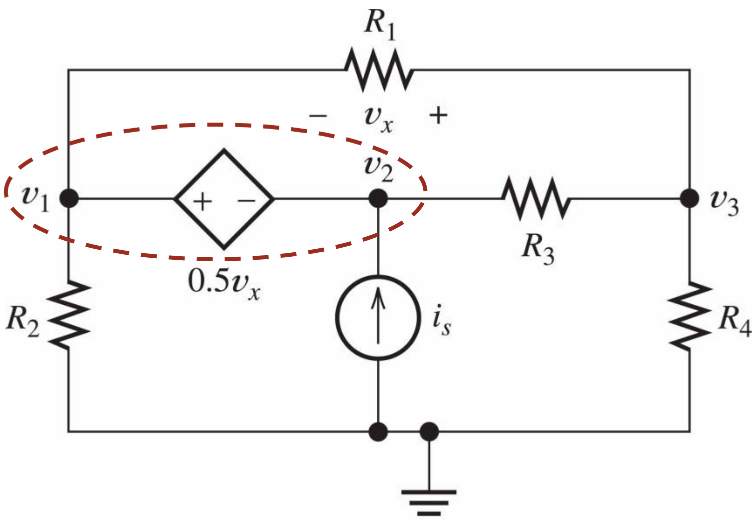
- Define a supernode to enclose the voltage source
- KCL @ supernode
  - All currents leave node
  - $\frac{v_1}{R_2} + \frac{v_1-(-15)}{R_1} + \frac{v_2}{R_4} + \frac{v_2-(-15)}{R_3} = 0$
- One equation with 2 unknowns
- KVL including supernode for additional constraints
  - $v_1 + 10 - v_2 = 0$
- Now two equations with 2 unknowns



Circuits with Controlled Sources

- Same technique as described but must account for dependent variable
  - Express dependent term as a function of node voltages and substitute back into node-voltage equations.

Node-Voltage with Controlled Source



1. Create supernode over node 1, 2
   - KVL with supernode

     a) $v_1 - 0.5v_x - v_2 = 0$

   - KCL @ supernode

     b) $\frac{v_1}{R_2} + \frac{v_1 - v_3}{R_1} + \frac{v_2 - v_3}{R_3} = i_s$

2. KCL @ node 3

   c) $\frac{v_3 - v_2}{R_3} + \frac{v_3 - v_1}{R_1} + \frac{v_3}{R_4} = 0$

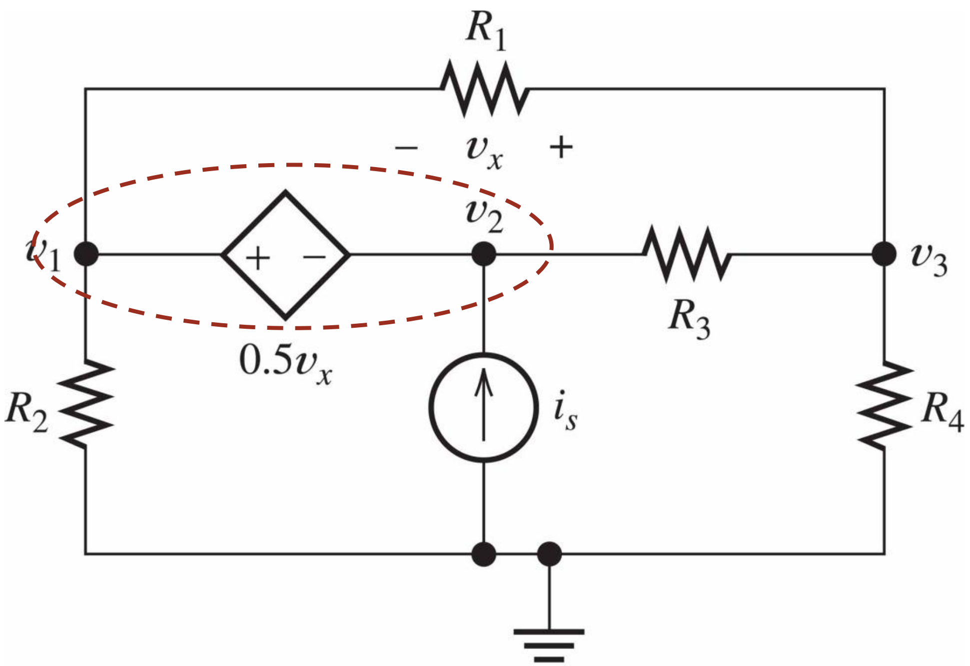
3 equations

4 unknown voltages

Node-Voltage with Controlled Source

3. Replace dependent variable with node voltages

- $v_x = v_3 - v_1 \quad \Rightarrow \quad v_1 - 0.5\,(v_3 - v_1) - v_2 = 0$



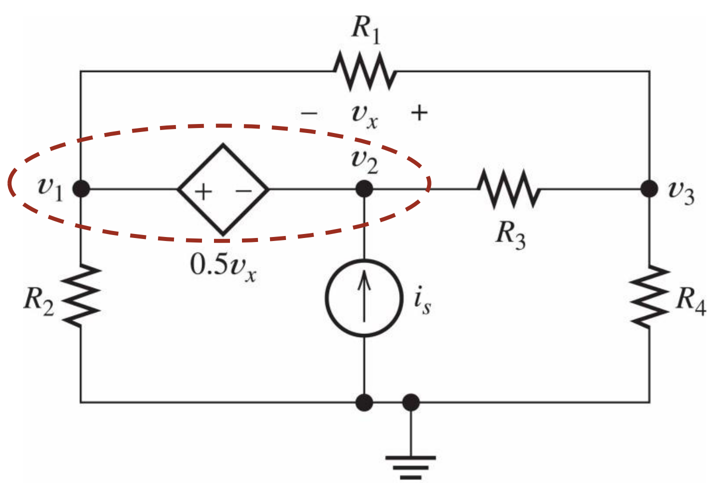
Node-Voltage with Controlled Source

4. Place in standard form

1. $v_1 - 0.5\,(v_3 - v_1) - v_2 = 0$ $\quad$ $(1.5)\,v_1 + (-1)\,v_2 + (-0.5)\,v_3 = 0$

2. $\frac{v_1}{R_2} + \frac{v_1 - v_3}{R_1} + \frac{v_2 - v_3}{R_3} = i_s \quad \Rightarrow \quad \left(\frac{1}{R_2} + \frac{1}{R_1}\right) v_1 + \left(\frac{1}{R_3}\right) v_2 + \left(-\frac{1}{R_1} - \frac{1}{R_3}\right) v_3 = i_s$

3. $\frac{v_3 - v_2}{R_3} + \frac{v_3 - v_1}{R_1} + \frac{v_3}{R_4} = 0$ $\quad$ $\left(-\frac{1}{R_1}\right) v_1 + \left(-\frac{1}{R_3}\right) v_2 + \left(\frac{1}{R_3} + \frac{1}{R_1} + \frac{1}{R_4}\right) v_3 = 0$



Node-Voltage Steps

1. Select a reference node and assign variables for the unknown node voltages. If the reference node is chosen at one end of an independent voltage source, one node voltage is known at the start, and fewer need to be computed
2. Write network equations. First use KCL to write current equations for nodes and supernodes. Write as many as you can without using all of the nodes. Then if you do not have enough equations because of voltage sources connected between nodes, use KVL to write additional equations
3. If the circuit contains dependent sources, find expressions for the controlling variables in terms of the node voltages. Substitute into the network equations, and obtain equations having only the node voltages as unknowns.
4. Put the equations into standard form and solve for the node voltages.