EE292: Fundamentals of ECE

Fall 2012
TTh 10:00-11:15 SEB 1242

Lecture 3
120904

http://www.ee.unlv.edu/~b1morris/ee292/

Outline

- Review
- Basic Circuit Examples
- Series Resistance
- Parallel Resistance
- Network Analysis
- Voltage Divider
- Current Divider

Current and Voltage

- Current – the flow of change
  - $i(t) = \frac{dq(t)}{dt}$
  - Must define a reference direction
    - The direction positive charge flows

- Voltage – the potential difference between 2 circuit nodes
  - The polarity defines the reference

Power and Energy

- Power – rate of energy transfer
  - $p(t) = v(t)i(t)$
  - Defined for passive reference configuration
    - Current flows into positive polarity terminal

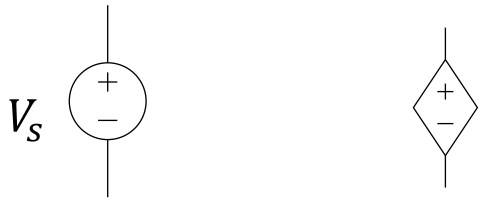
- Energy – amount of power delivered in time interval
  - $$w = \int_{t_1}^{t_2} p(t)dt$$

- $p, w > 0$ → energy absorbed by element
- $p, w < 0$ → energy supplied by element

Circuit Elements

- Conductors - charge carrying material
  - Ideal → wire

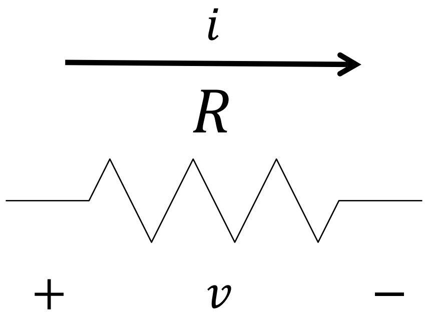
- Voltage sources – output specific voltage



- Current sources – output specific current

Ohm's Law and Resistance

- Ohm's Law
  - $v = iR$
- Resistance
  - $R = \frac{v}{i}$
  - Units of Ohms Ω



$$\begin{aligned} p &= vi \\ &= i^2R \\ &= \frac{V^2}{R} = GV^2 \end{aligned}$$

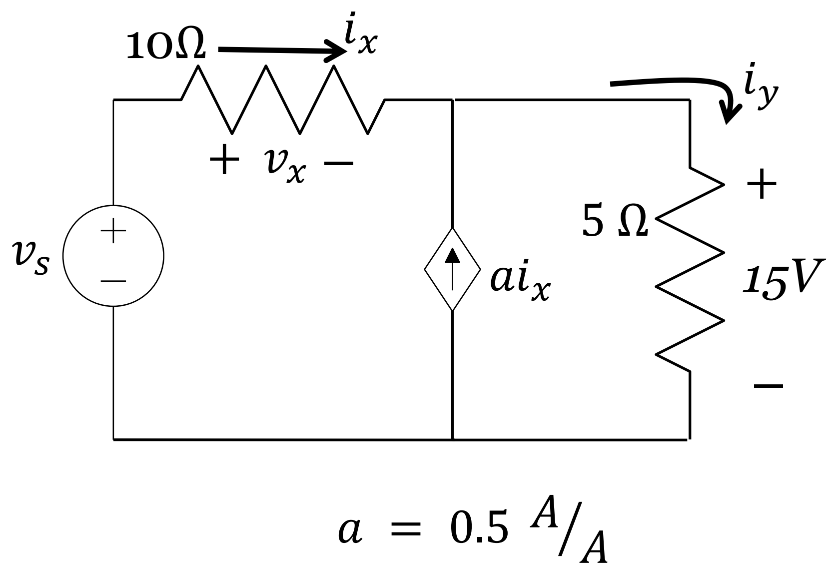
$$v = iR$$

Kirchhoff’s Laws

- KCL :
  - conservation of charge
  - sum of currents entering node is equal current leaving node
- KVL :
  - conservation of energy
  - Sum of voltages in circuit loop is zero

Basic circuit examples

- Example 1

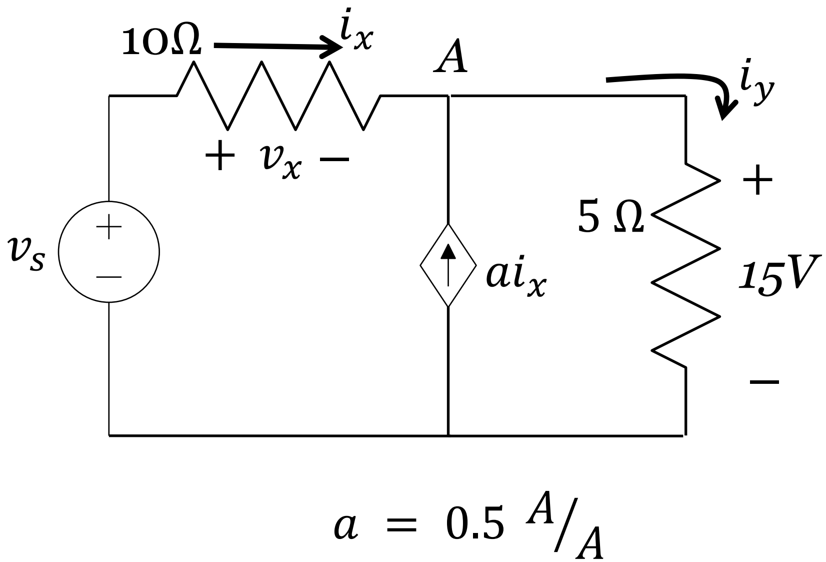


- Solve for v_s

a. Solve for i_y
   - Ohm's Law →$v = iR$
   - $i_y = \frac{v}{R} = \frac{15}{5} = 3 \text{ A}$

Basic circuit examples

- Example 1



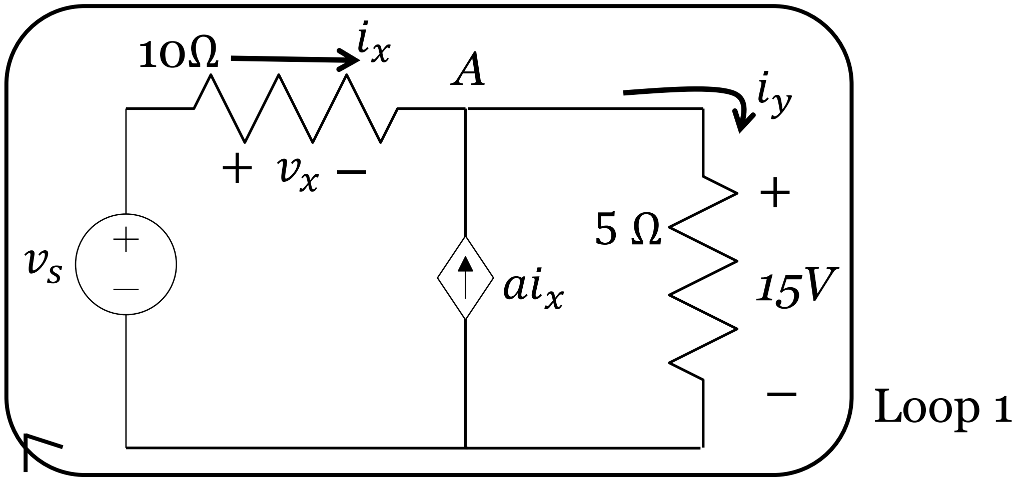
- Solve for v_s

b. KCL @ node A

- $i_x + ai_x = i_y$
- $i_x(1 + a) = i_y$
- $i_x = \frac{i_y}{1+a} = \frac{3}{1.5} = 2\ A$

Basic circuit examples

- Example 1



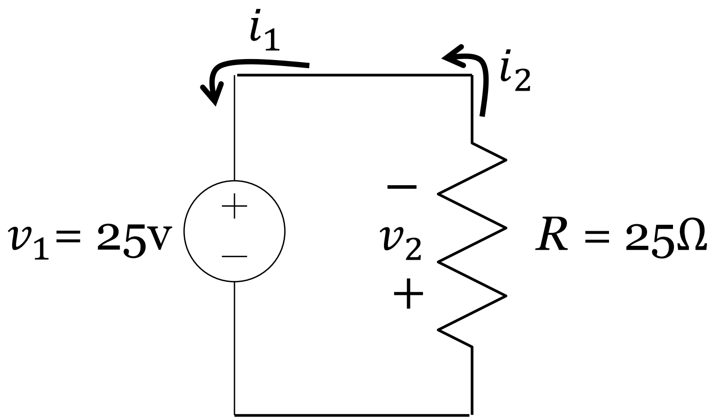
$a = 0.5\ {}^{A}/_{A}$

- Solve for v_s

c. KVL around Loop 1
- $V_s - V_x - 15 = 0$
- $V_s = V_x + 15 = i_x R + 15 = 2(10) + 15$
- $V_s = 35\ V$

Basic circuit examples

- Example 2

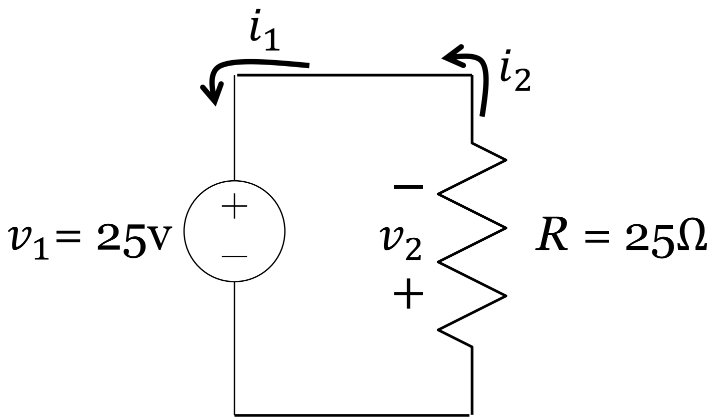


- Find i_R, v_R, and power for each element

a. By KCL (series connection)
   - $i_1 = i_2 = \frac{v_2}{R} = \frac{v_2}{25}$

Basic circuit examples

- Example 2

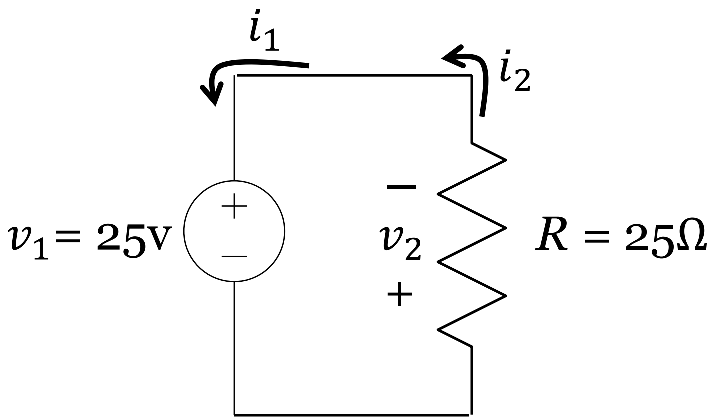


- Find i_R, v_R, and power for each element

b. By KVL (parallel connection)
  - $v_1 + v_2 = 0$
  - $v_2 = -v_1 = -25\ V$
  - $i_1 = i_2 = \frac{v_2}{R} = \frac{v_2}{25} = -\frac{25}{25} = -1\ A$

Basic circuit examples

- Example 2



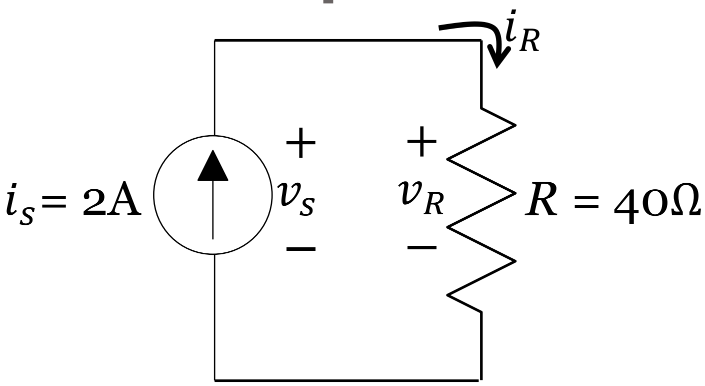
- Find i_R, v_R, and power for each element

c. Power of v_1
  - $p_1 = v_1 i_1 = 25(-1) = -25\ W$

d. Power of R
  - $p_R = v_2 i_2 = (-25)(-1) = 25\ W$

Basic circuit examples

- Example 3

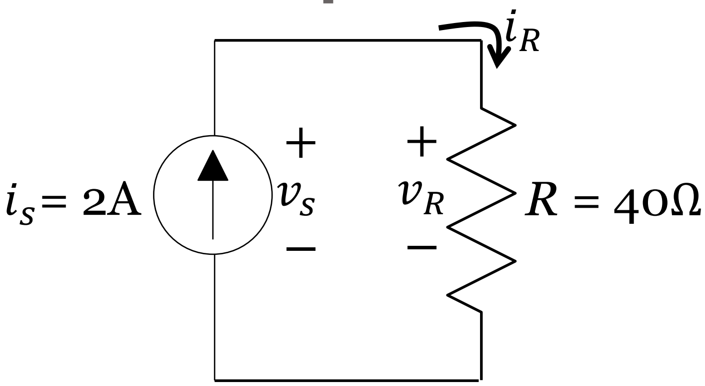


- Find i_R, v_R, v_s and power for each element

a. By KCL (series connection)
   - $i_s = i_R = 2\ A$

Basic circuit examples

- Example 3

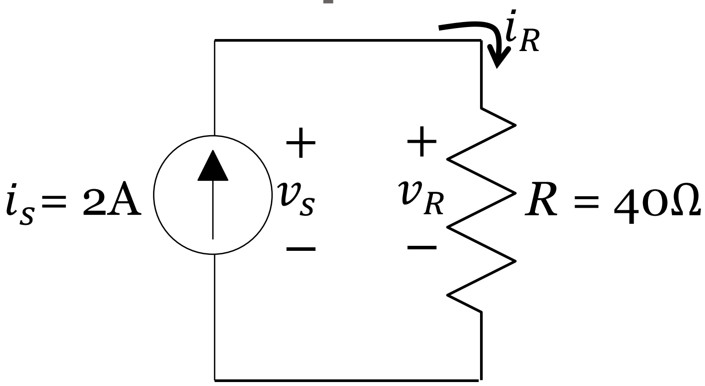


- Find i_R, v_R, v_s and power for each element

b. By KVL (parallel connection)
  - $v_s = v_R = i_R R = (2)(40) = 80\ V$

Basic circuit examples

- Example 3

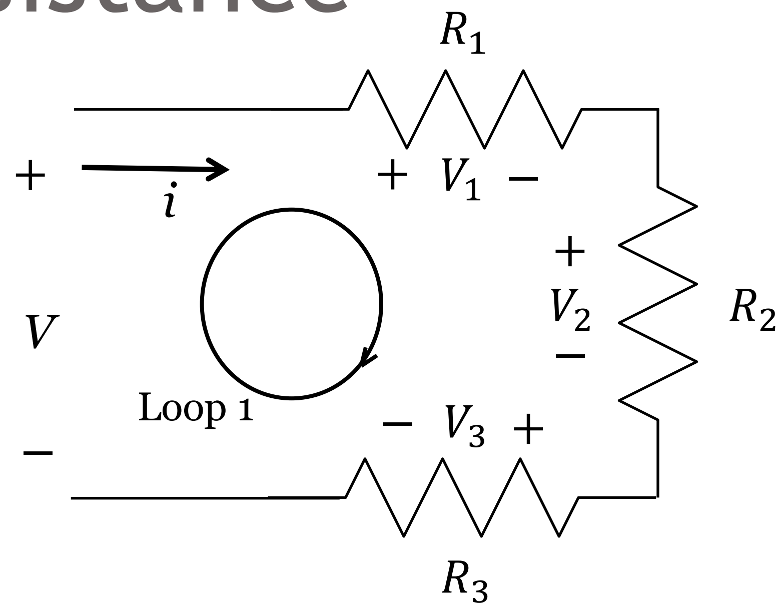


- Find i_R, v_R, v_s and power for each element

c. Power of i_s
   - $p_s = -v_s i_s = -(80)(2) = -160\ W$

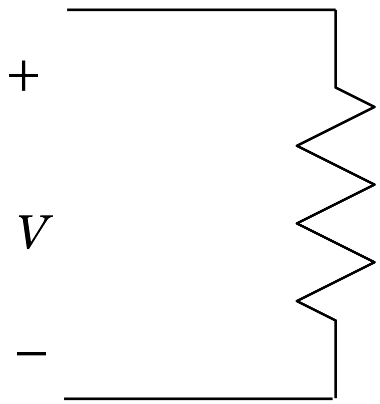
d. Power of R
   - $p_R = v_R i_R = (80)(2) = 160\ W$

Series resistance



- Using KCL, we know i goes through all R_1, R_2, R_3
- Using Ohm's Law :
  - $V_1 = iR_1$ $\quad$ $V_2 = iR_2$ $\quad$ $V_3 = iR_3$
- Using KVL around Loop 1
  - V - V_1 - V_2 - V_3 = 0
  - V = $V_1 + V_2 + V_3 = iR_1 + iR_2 + iR_3 = i(\underbrace{R_1 + R_2 + R_3}_{R_{eq}})$

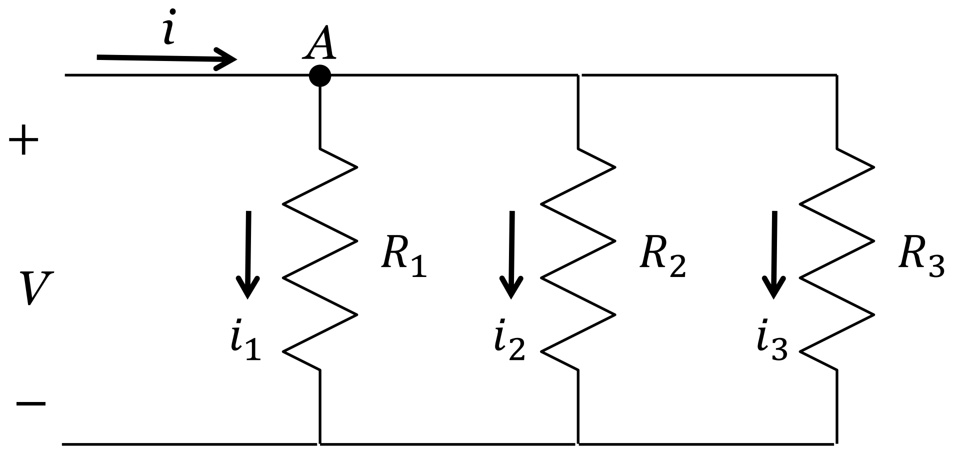
Series resistance



$R_{eq} = R_1 + R_2 + R_3$

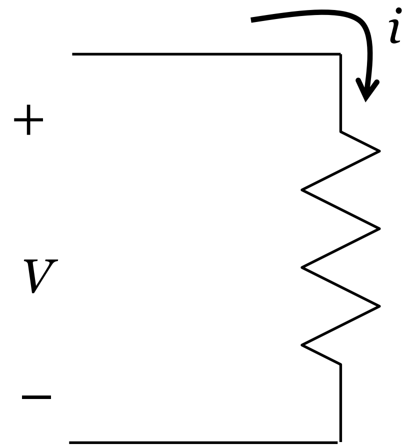
- Series resistance can be replaced by equivalent resistance
- $R_\Sigma = R_{eq} = \sum_i R_i$

Parallel Resistance



- Using KVL
  - $V_{R_1} = V_{R_2} = V_{R_3} = V$ *(parallel connection)*
- Using Ohm's Law
  - $i_1 = \frac{V}{R_1}$ $i_2 = \frac{V}{R_2}$ $i_3 = \frac{V}{R_3}$
- Using KCL @ A
  - $i = i_1 + i_2 + i_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\underbrace{(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3})}_{\frac{1}{R_{eq}}}$

Parallel Resistance



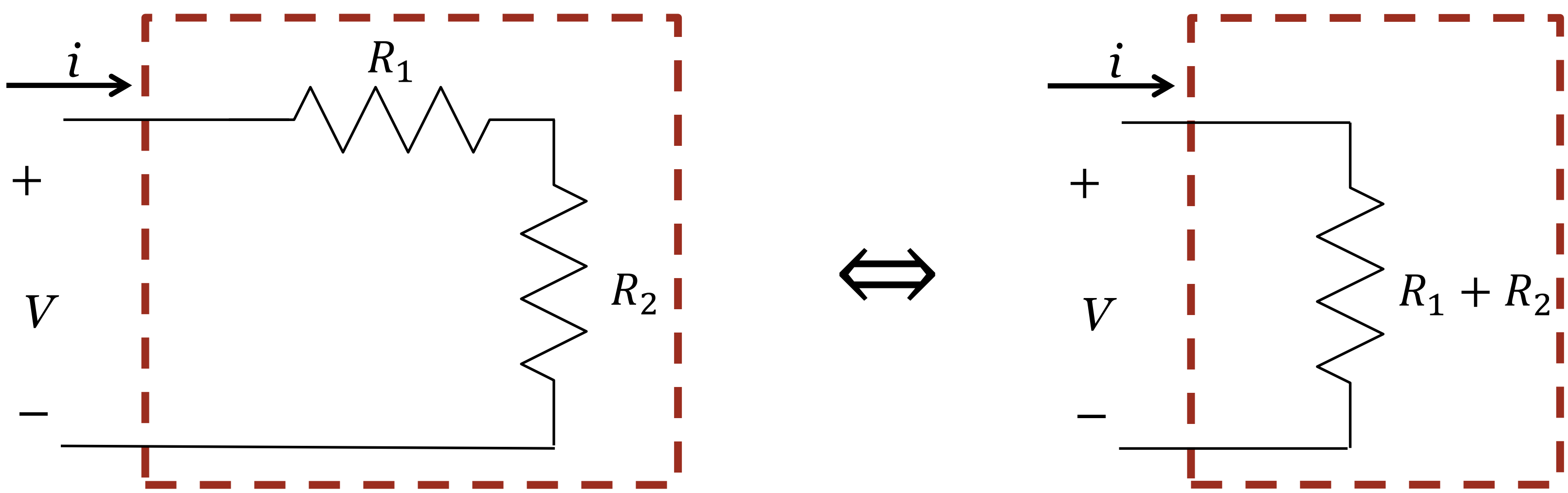
$$R_{eq} = 1/\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

- Parallel resistance can be replaced by equivalent resistance
  - $R_{eq} = {}^{1}\!\Big/_{\sum_i \frac{1}{R_i}}$
- Important special case
  - $\underbrace{R_1 \parallel R_2}_{\text{shorthand}} \Rightarrow R_{eq} = \dfrac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \dfrac{1}{\frac{R_1 + R_2}{R_1 R_2}} = \dfrac{R_1 R_2}{R_1 + R_2}$

Examples Combining Resistors

Equivalent Circuits

- A complicated circuit can be viewed more simply from its terminals as a
  1. single circuit element (best case)
  2. simple combination of elements (worst case)



The internals are not important from the perspective of the terminals outside the black box

Network Analysis

- Process of determining current, voltage, and power of every circuit (network) element
- Series and parallel equivalents help simplify circuits for network analysis
  - Repeated application of equivalents is usually required

Steps for Circuit Analysis with Series/Parallel Equivalents

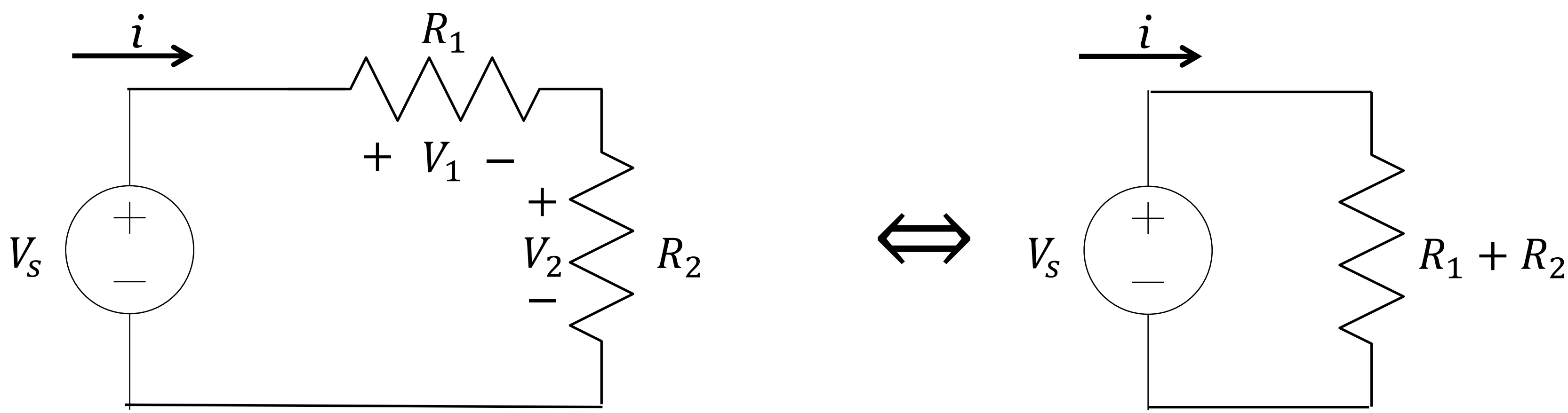
1. Begin by locating a combination of resistances in series/parallel
   - Often you should start furthest from the source
2. Redraw the circuit with the equivalent resistance for the combination in step 1
3. Repeat steps 1 and 2 until the circuit is reduced as far as possible
   - Often this ends up as a single source and single resistance
4. Solve for currents and voltages in the final equivalent circuit. Then, transfer results back one step and solve for additional unknowns. Again transfer results back one step and solve. Repeat until all voltages and currents are known in the original circuit.
5. Check your results to make sure that KCL is satisfied at each node, KVL is satisfied for each loop, and the powers add to zero.

Example

- Find current, voltage, and power of each circuit element

Voltage Divider

- For a voltage applied to a series combination, a fraction appears across each resistance



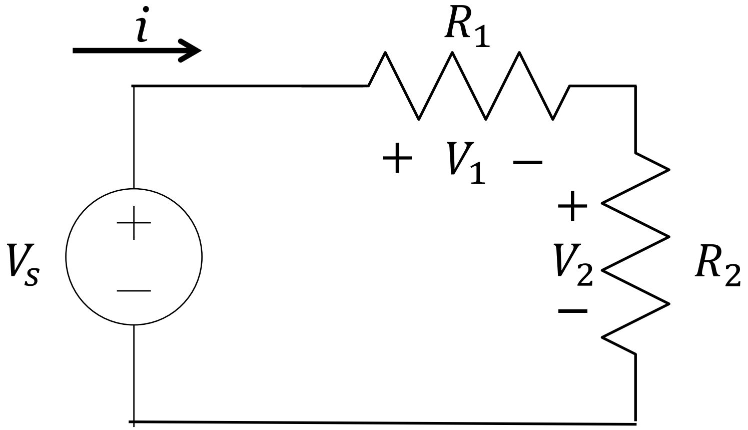
$$v_1 = iR_1 = V_s\left(\frac{R_1}{R_1+R_2}\right)$$

$$v_2 = iR_2 = V_s\left(\frac{R_2}{R_1+R_2}\right)$$

$$i = \frac{V_s}{R_1+R_2}$$

Voltage-Division Principle

- The fraction of voltage across a given resistance in a series connection is the ratio of the given resistance to the total series resistance



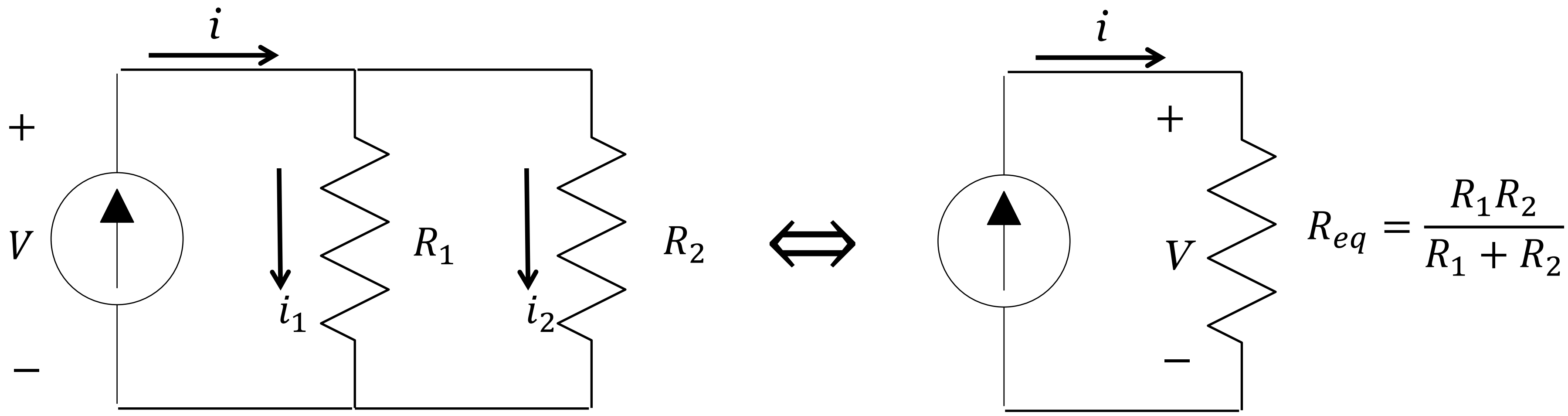
The larger resistor → more voltage drop across it

$$v_1 = iR_1 = V_s\left(\frac{R_1}{R_1 + R_2}\right)$$

$$v_2 = iR_2 = V_s\left(\frac{R_2}{R_1 + R_2}\right)$$

Current Divider

- For a current applied to a parallel combination, a fraction of the current flows through each resistor



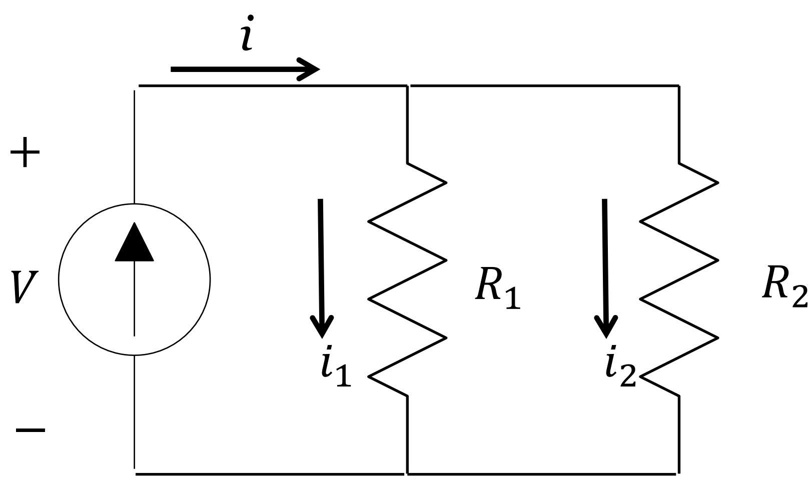
$$i_1 = \frac{v}{R_1} = \frac{i}{R_1}\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$
$$= \left(\frac{R_2}{R_1 + R_2}\right) i$$

$$i_2 = \frac{v}{R_2}$$
$$= \left(\frac{R_1}{R_1 + R_2}\right) i$$

$$v = iR_{eq}$$
$$= i\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

Current-Division Principle

- The fraction of current flowing in a given resistance is the ratio of the other resistance to the sum of the two resistances
  - Only applies for parallel pairs



$$i_1 = \frac{v}{R_1} = \frac{i}{R_1}\left(\frac{R_1 R_2}{R_1 + R_2}\right) = \left(\frac{R_2}{R_1 + R_2}\right) i$$

$$i_2 = \frac{v}{R_2} = \left(\frac{R_1}{R_1 + R_2}\right) i$$

Smaller the resistor → more current through parallel path

Example

- Find all voltages and currents