

# ECG782: Multidimensional Digital Signal Processing

Spring 2014

TTh 14:30-15:45 CBC C313

Lecture 02

Image Basics

13/01/23

# Outline

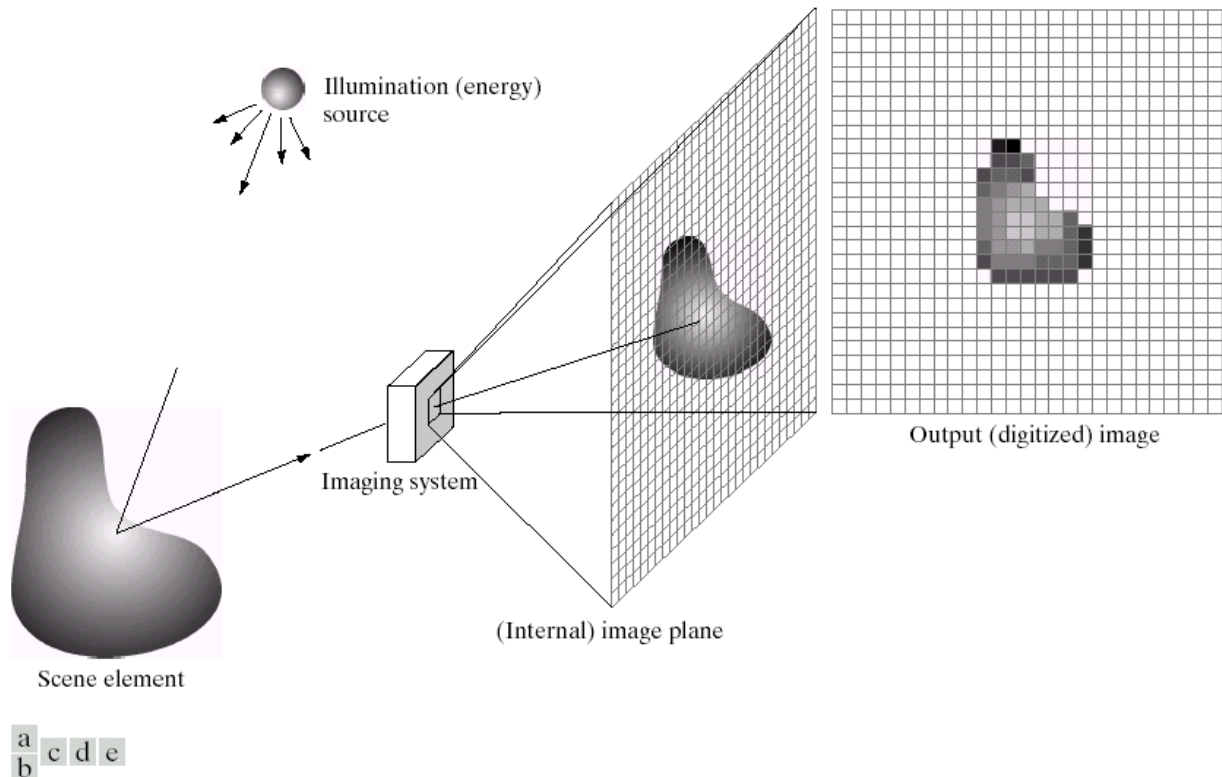
- Image Formation and Models
- Pixels
- Pixel Processing
- Color

# M-D Signals

- Use mathematical models to describe signals
  - A function depending on some variable with a physical meaning
- 1D signal
  - E.g. speech, audio, voltage, current
    - Dependent on “time”
- 2D signal
  - E.g. image
    - Dependent on spatial coordinates in a plane
- 3D signal
  - E.g. volume in space, video
- M-D signal
  - E.g. ???

# Image Formation

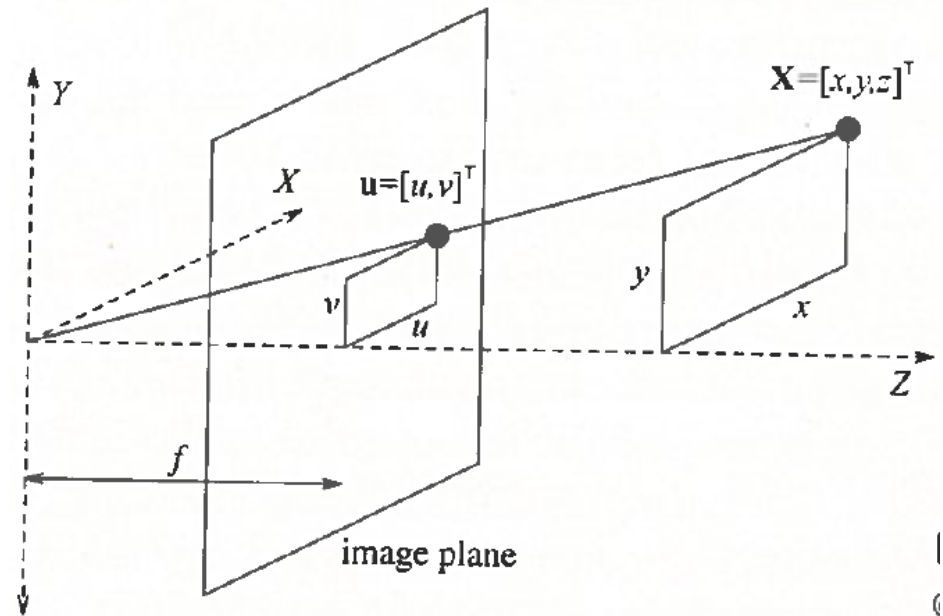
- Incoming light energy is focused and collected onto an image plane



**FIGURE 2.15** An example of the digital image acquisition process. (a) Energy (“illumination”) source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

# Image Formation Model

- Imaging takes the 3D world and projects it onto a 2D image
- Simple model for the process is called the pinhole camera
- $X = [x, y, z]^T$  - represents point in world 3D space
- $u = [u, v]^T$  - represents a 2D point on image plane
- $f$  – focal length of camera
- World-image relationship
  - $u = \frac{xf}{z}$                        $v = \frac{yf}{z}$

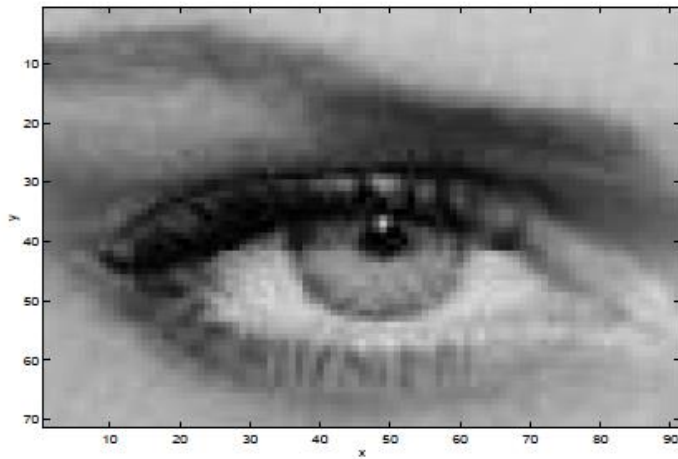


# Perspective Projection

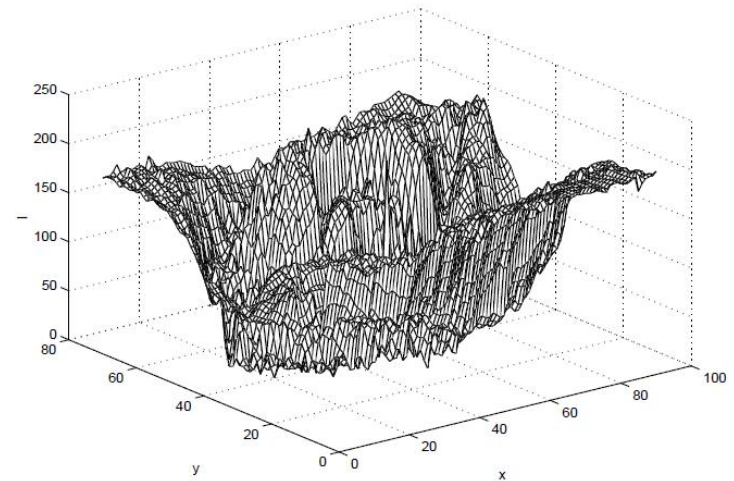
- Pinhole camera causes perspective distortion
  - Loss of information from perspective projection
  - The transform is not one-to-one
    - A line in space gets mapped to the same point
    - Need depth information to resolve ambiguity
- Orthographic (parallel) projection
  - Linear approximation with  $f \rightarrow \infty$
  - This is how far away objects  $z \rightarrow \infty$  are mapped onto image plane

# Image Representation

- Multiple equivalent representations
- Image



- Surface



- Matrix

```

188 186 188 187 168 130 101 99 110 113 112 107 117 140 153 153 156 158 156 153
189 189 188 181 163 135 109 104 113 113 110 109 117 134 147 152 156 163 160 156
190 190 188 176 159 139 115 106 114 123 114 111 119 130 141 154 165 160 156 151
190 188 188 175 158 139 114 103 113 126 112 113 127 133 137 151 165 156 152 145
191 185 189 177 158 138 110 99 112 119 107 115 137 140 135 144 157 163 158 150
193 183 178 164 148 134 118 112 119 117 118 106 122 139 140 152 154 160 155 147
185 181 178 165 149 135 121 116 124 120 122 109 123 139 141 154 156 159 154 147
175 176 176 163 145 131 120 118 125 123 125 112 124 139 142 155 158 158 155 148
170 170 172 159 137 123 116 114 119 122 126 113 123 137 141 156 158 159 157 150
171 171 173 157 131 119 116 113 114 118 125 113 122 135 140 155 156 160 160 152
174 175 176 156 128 120 121 118 113 112 123 114 122 135 141 155 155 158 159 152
176 174 174 151 123 119 126 121 112 108 122 115 123 137 143 156 155 152 155 150
175 169 168 144 117 117 127 122 109 106 122 116 125 139 145 158 156 147 152 148
179 179 180 155 127 121 118 109 107 113 125 133 130 129 139 153 161 148 155 157
176 183 181 153 122 115 113 106 105 109 123 132 131 131 140 151 157 149 156 159
180 181 177 147 115 110 111 107 107 105 120 132 133 133 141 150 154 148 155 157
181 174 170 141 113 111 115 112 113 105 119 130 132 134 144 153 156 148 152 151
180 172 168 140 114 114 118 113 112 107 119 128 130 134 146 157 162 153 153 148
186 176 171 142 114 114 116 110 108 104 116 125 128 134 148 161 165 159 157 149
185 178 171 138 109 110 114 110 109 97 110 121 127 136 150 160 163 158 156 150

```

# Image Representation

- Image  $f(x, y)$  is a 2D function
  - $f$  – amplitude, gray level, or brightness
  - $(x, y)$  – spatial coordinates
  - Conceptually,  $(x, y)$  are continuous but are discrete in practice
- In general, the function can be vector-valued
  - E.g. color images represented by (red, green, blue)
  - $f(x, y) = [r, g, b]^T$
- The image function can be M-dimensional
  - E.g. computed tomography (CT) images are 3D
    - $f(x, y, z)$  represents x-ray absorption at point  $(x, y, z)$

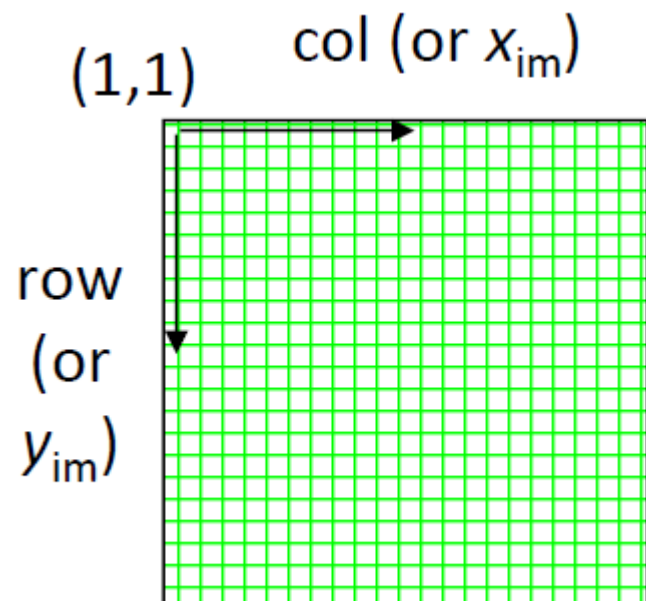


# Image as Function

- Think of an image as a function,  $f$ , that maps from  $R^2$  to  $R$ 
  - $0 < f(x, y) < \infty$  is the intensity at a point  $(x, y)$
- In reality, an image is defined over a rectangle with a finite range of values
  - $f: [a, b] \times [c, d] \rightarrow [0, 1]$
- Computationally,  $[0, 1]$  range is convenient but usually we have an 8-bit quantized representation
  - $0 < f(x, y) < 255$
- Color image is just three separate functions pasted together
  - $f(x, y) = [r(x, y); g(x, y); b(x, y)]$

# Image as Matrix

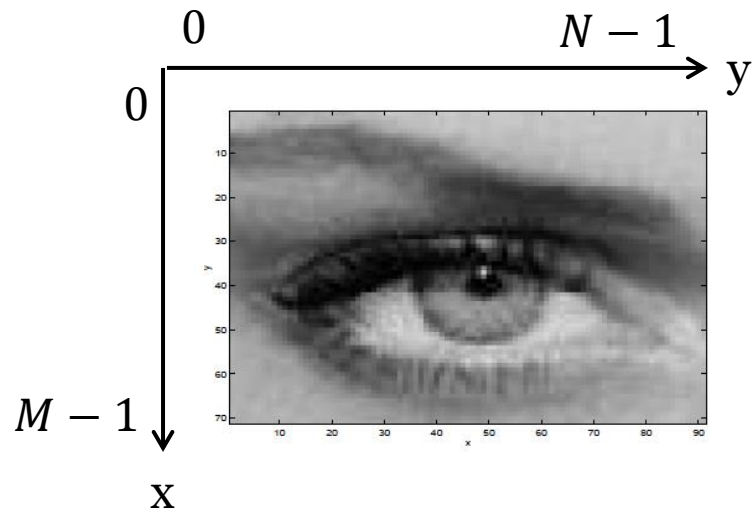
- Images are usually represented by matrices
  - $M \times N$  dimension
- Be aware that images can have different origin definitions
  - Bottom left - typical Cartesian coordinates
  - Upper left – typical image definition (matrix or table notation)
  - Matlab uses (1,1) for origin not (0,0)



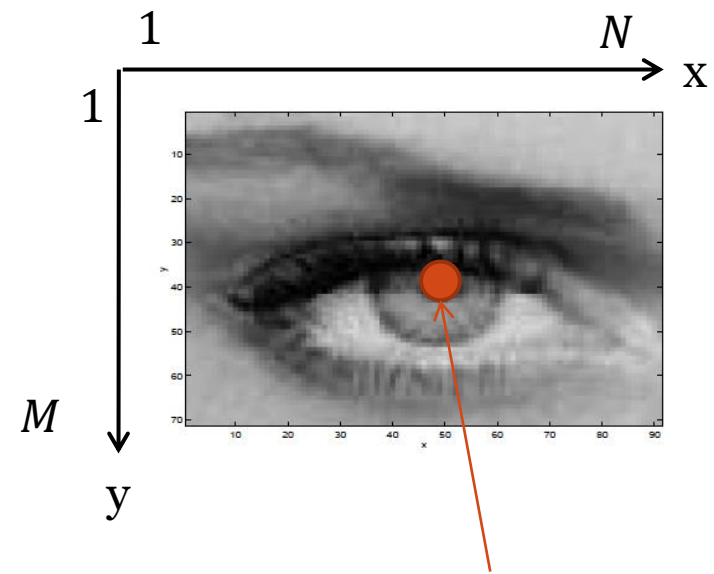
- Index an element either by
  - $(x, y)$
  - $(row, col)$

# Matrix Notation

- Mathematical
- Notation starts with  $f(0,0)$



- Matlab
- Notation starts with  $I(1,1)$ 
  - No zero indexing



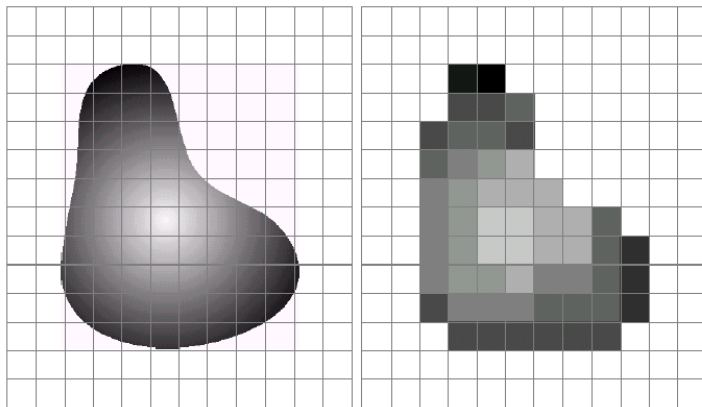
$(3,4) \rightarrow I(4,3)$

# Image Sampling

- A continuous image is sampled and ordered into a image grid
- Each grid element is known as a pixel
  - Voxel for volume element
- Consider the pixel as the smallest unit in an image
  - This is not quite a delta because it has a finite size on the CMOS sensor
  - It is possible to do sub-pixel processing (e.g. corner detection)

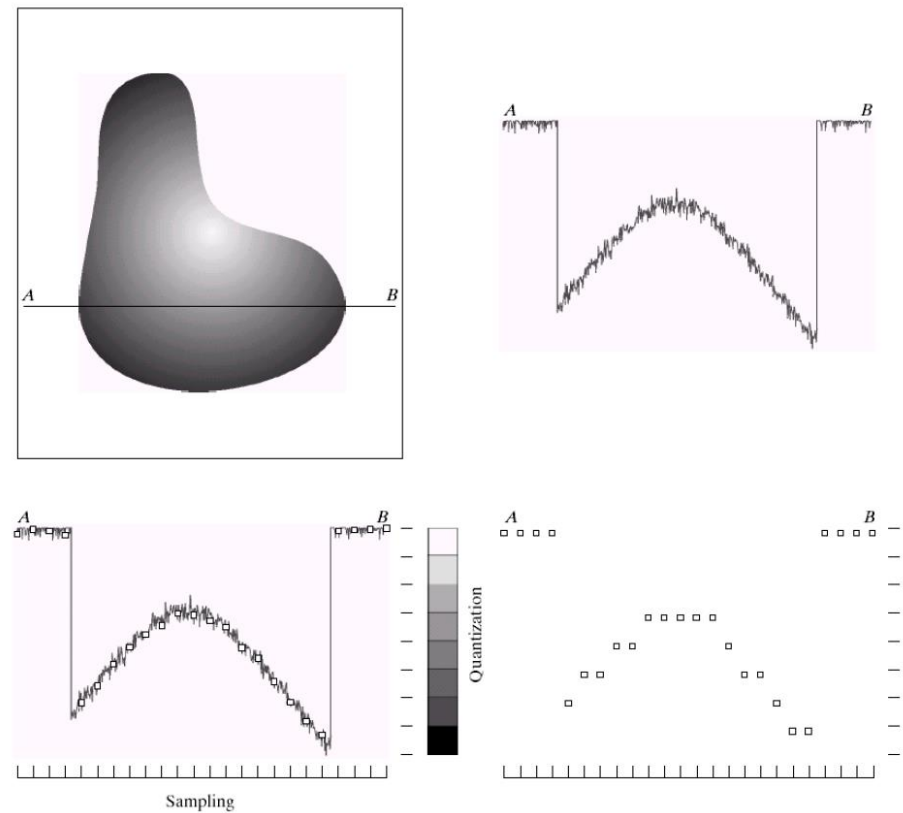
# Sampling and Quantization

- Sampling gives fixed grid of image
- Quantization gives the number of output levels  $L$



a b

**FIGURE 2.17** (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.



a b  
c d

**FIGURE 2.16** Generating a digital image. (a) Continuous image. (b) A scan line from  $A$  to  $B$  in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

# Quantization Levels

- $L$  = number of output levels
- $k$  = number of bits per pixel
- Output range of image
  - $[0, L - 1] = [0, 2^k]$
- Image storage size
  - $b = M \times N \times k$
  - Number of bits to store image with dimensions  $M \times N$
- 8-bits per channel is typical
  - Provide enough resolution to provide quality visual reproduction

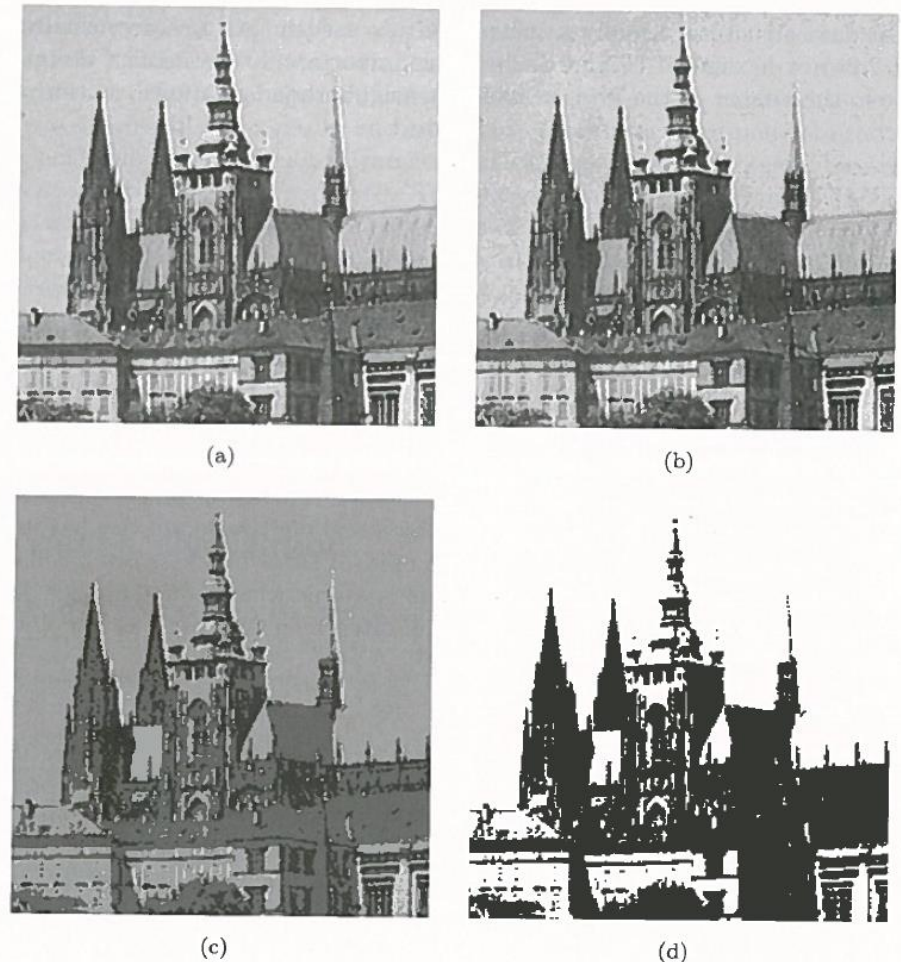
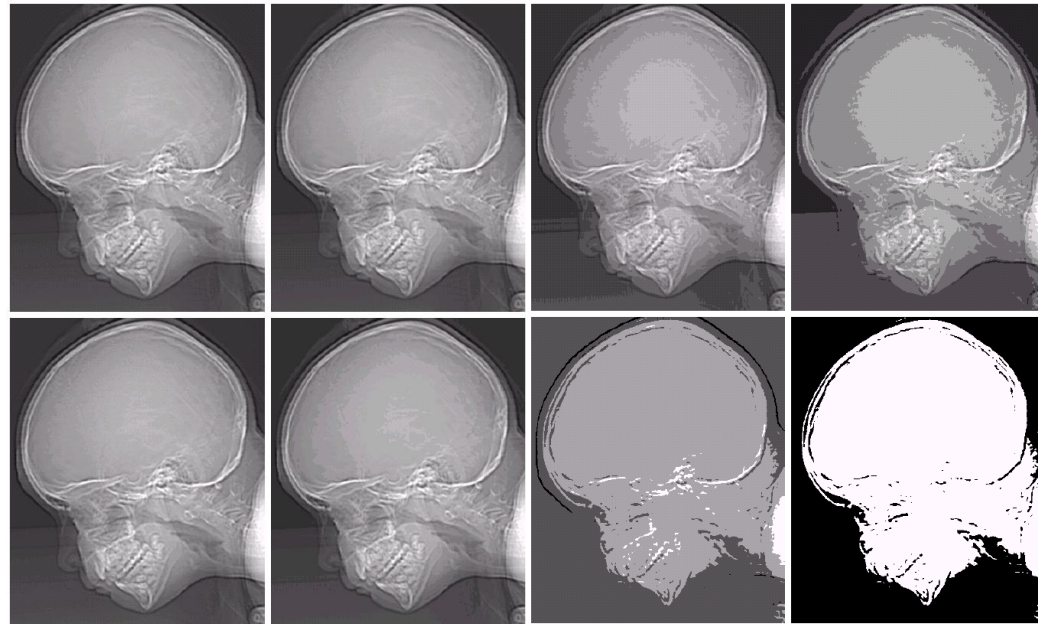


Figure 2.3: Brightness levels. (a) 64. (b) 16. (c) 4. (d) 2. © Cengage Learning 2015.

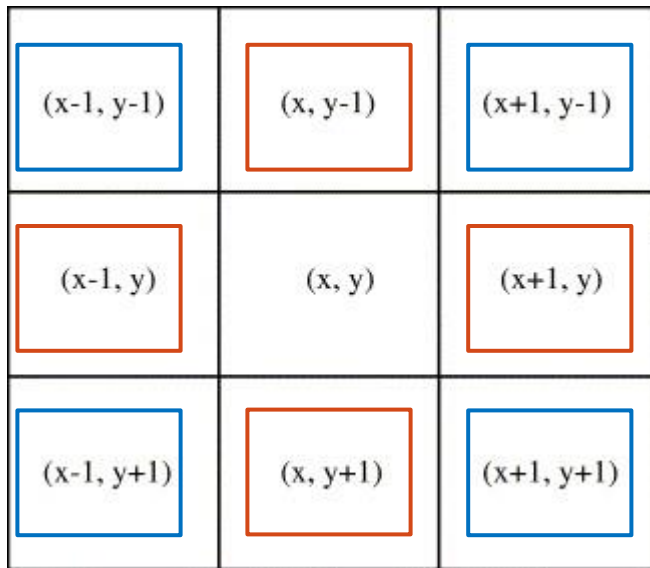
# Resolution

- Spatial resolution is the smallest discernible detail in an image
  - This is controlled by the sampling factor (the size  $M \times N$  of the CMOS sensor)
- Gray-level resolution is the smallest discernible change in gray level
  - Based on number of bits for representation



# Pixel Neighborhood

- The pixel neighborhood corresponds to nearby pixels

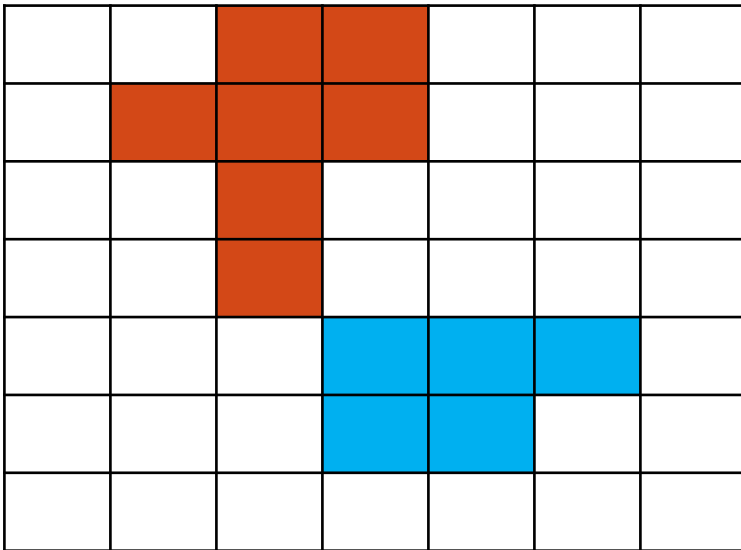


- 4-neighbors
  - Horizontal and vertical neighbors
- 8-neighbors
  - Include 4-neighbors and the diagonal pixels

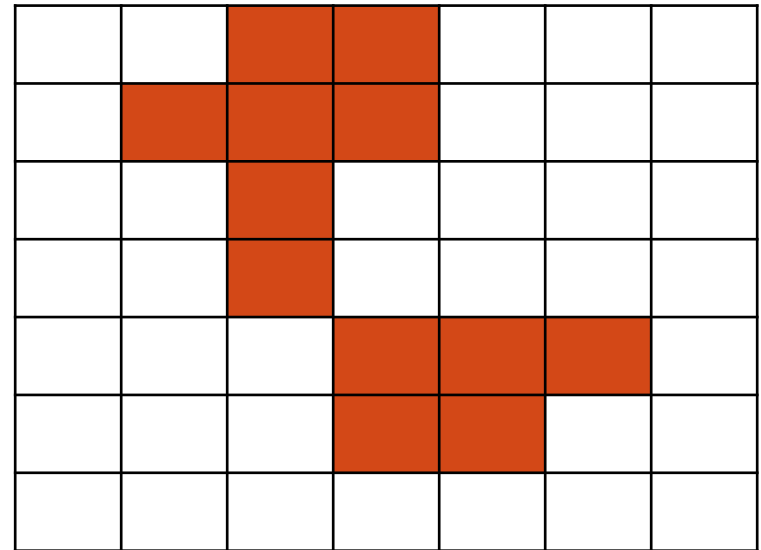


# Connectivity

- Path exists between pixels
- 4-connected



- 8-connected

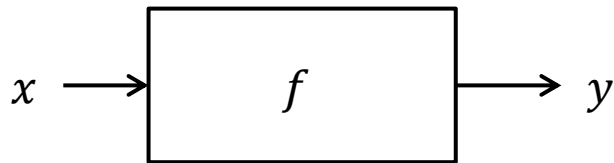


# Image Processing

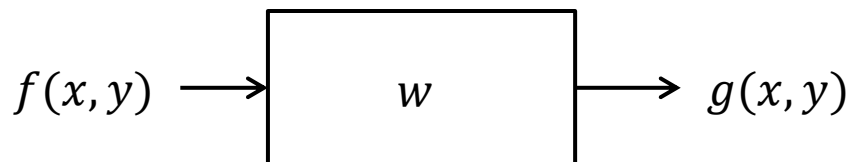
- Usually the first stage of computer vision applications
  - Pre-process an image to ensure it is in a suitable form for further analysis
- Typical operations include:
  - Exposure correction, color balancing, reduction in image noise, increasing sharpness, rotation of an image to straighten
- Digital Image Processing by Gonzalez and Woods is a great book to learn more

# 2D Signal Processing

- Image processing is an extension of signal processing to two independent variables
  - Input signal, output signal
- General system



- Image processing



# Point Operators/Processes

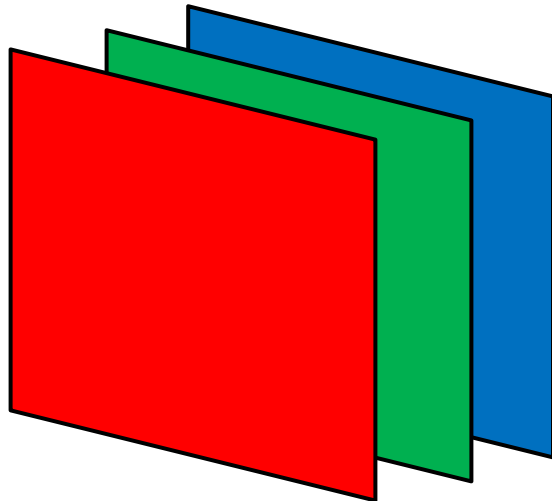
- Output pixel value only depends on the corresponding input pixel value
- Often times we will see operations like dividing one image by another
  - Matrix division is not defined
  - The operation is carried out between corresponding pixels in the two image
  - Element-by-element dot operation in Matlab
    - `>> I3 = I1 ./ I2`
    - Where I1 and I2 are the same size

# Pixel Transforms

- Gain and bias (Multiplication and addition of constant)
  - $g(x, y) = a(x, y)f(x, y) + b(x, y)$
  - $a$  (gain) controls contrast
  - $b$  (bias) controls brightness
    - Notice parameters can vary spatially (think gradients)
- Linear blend
  - $g(x) = (1 - \alpha)f_0(x) + \alpha f_1(x)$
  - We will see this used later for motion detection in video processing

# Color Transforms

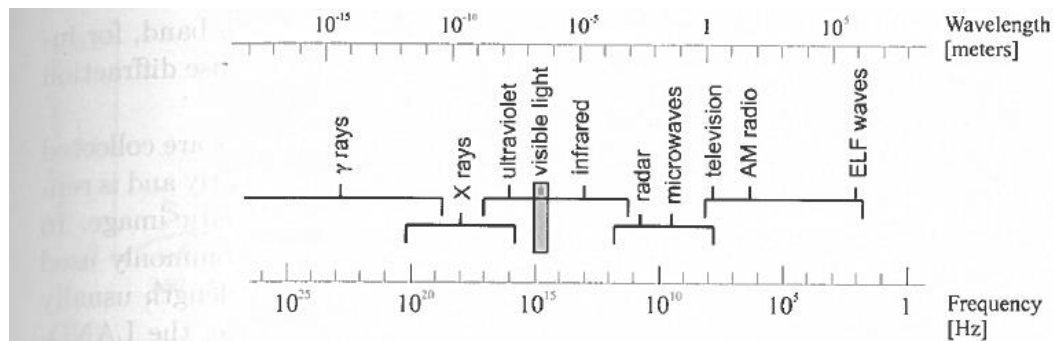
- Usually we think of a color image as three images concatenated together
  - Have a red, green, blue slice corresponding to the notion of primary colors



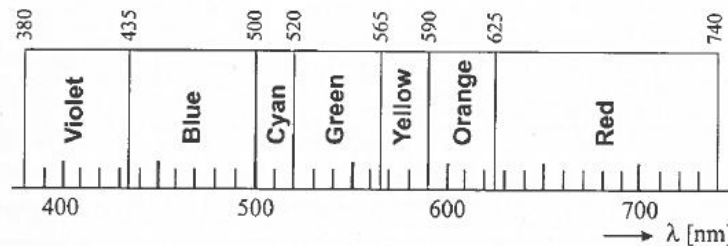
- Manipulations of these color channels may not correspond directly with desired perceptual response
  - Adding bias to all channels may actually change the apparent color instead of increasing brightness
- Need other representations of color for mathematical manipulation

# Color Images

- Color comes from underlying physical properties



**Figure 2.23:** Division of the electromagnetic spectrum (ELF is Extremely Low Frequencies).  
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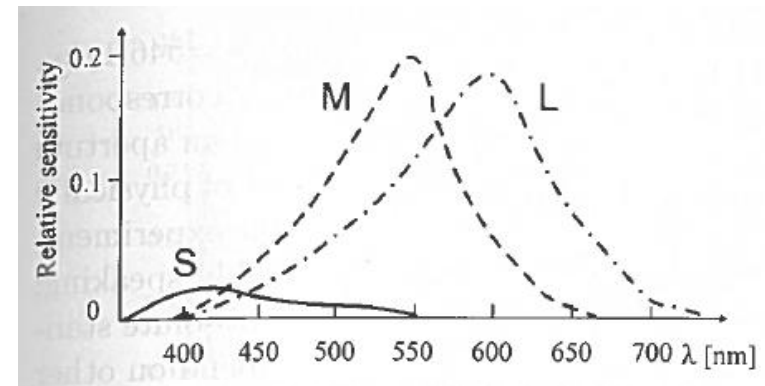


**Figure 2.24:** Wavelength  $\lambda$  of the spectrum visible to humans. © Cengage Learning 2015.

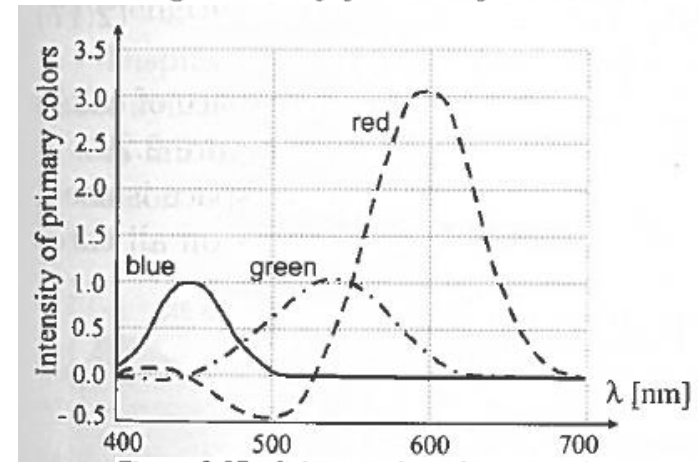
- However, humans do not perceive color in the same physical process
  - There is some subjectivity (e.g. color similarity)

# Human Color Perception

- Cones in human retina are sensitive to color
  - In the center of eye
  - 3 different types for different EM frequency sensitivity
- Rods are monochromatic
  - On outside of the eye and good for low lighting and motion sensing



**Figure 2.26:** Relative sensitivity of S, M, L cones of the human eye to wavelength. © Cengage Learning 2015.



**Figure 2.27:** Color matching functions obtained in the color matching experiment. Intensities of the selected primary colors which perceptually match spectral color of given wavelength  $\lambda$ . Based on [Wandell, 1995].



# Colorspaces

- Uniform method for defining colors
- Can transform from one to another
  - Want to take advantage of properties and color gamut
- XYZ
  - International absolute color standard
  - No negative mixing
- RGB
  - Additive color mixing for red, green, and blue
  - Widely used in computers
- CMYK
  - Cyan, magenta, yellow, black
  - Used for printers and based off of reflectivity
- HSV
  - Hue, saturation, and value = color, amount, brightness
  - Closer to human perception

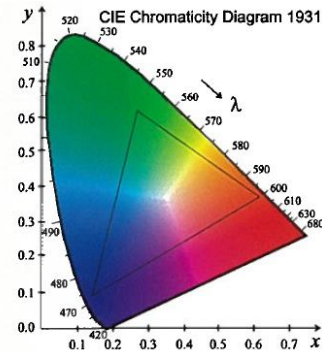


Plate 1: © Cengage Learning 2015. Page 35, Figure 2.30.

