ECG782: Multidimensional Digital Signal Processing

Spring 2014
TTh 14:30-15:45 CBC C313

Lecture 02
Image Basics
13/01/23

http://www.ee.unlv.edu/~b1morris/ecg782/
Outline

- Image Formation and Models
- Pixels
- Pixel Processing
- Color
M-D Signals

• Use mathematical models to describe signals
  ▫ A function depending on some variable with a physical meaning
• 1D signal
  ▫ E.g. speech, audio, voltage, current
    • Dependent on “time”
• 2D signal
  ▫ E.g. image
    • Dependent on spatial coordinates in a plane
• 3D signal
  ▫ E.g. volume in space, video
• M-D signal
  ▫ E.g. ???
Image Formation

- Incoming light energy is focused and collected onto an image plane.
Image Formation Model

- Imaging takes the 3D world and projects it onto a 2D image
- Simple model for the process is called the pinhole camera

\[ X = [x, y, z]^T \] - represents point in world 3D space
\[ u = [u, v]^T \] - represents a 2D point on image plane
\( f \) – focal length of camera

World-image relationship
- \[ u = \frac{x f}{z} \]
- \[ v = \frac{y f}{z} \]
Perspective Projection

- Pinhole camera causes perspective distortion
  - Loss of information from perspective projection
  - The transform is not one-to-one
    - A line in space gets mapped to the same point
    - Need depth information to resolve ambiguity
- Orthographic (parallel) projection
  - Linear approximation with $f \to \infty$
  - This is how far away objects $z \to \infty$ are mapped onto image plane
Image Representation

- Multiple equivalent representations
- Image
- Surface
- Matrix
Image Representation

- Image $f(x, y)$ is a 2D function
  - $f$ – amplitude, gray level, or brightness
  - $(x, y)$ – spatial coordinates
  - Conceptually, $(x, y)$ are continuous but are discrete in practice

- In general, the function can be vector-valued
  - E.g. color images represented by (red, green, blue)
  - $f(x, y) = [r, g, b]^T$

- The image function can be $M$-dimensional
  - E.g. computed tomography (CT) images are 3D
    - $f(x, y, z)$ represents x-ray absorption at point $(x, y, z)$
Image as Function

- Think of an image as a function, $f$, that maps from $\mathbb{R}^2$ to $\mathbb{R}$
  - $0 < f(x, y) < \infty$ is the intensity at a point $(x, y)$
- In reality, an image is defined over a rectangle with a finite range of values
  - $f: [a, b] \times [c, d] \rightarrow [0,1]$
- Computationally, [0,1] range is convenient but usually we have an 8-bit quantized representation
  - $0 < f(x, y) < 255$
- Color image is just three separate functions pasted together
  - $f(x, y) = [r(x, y); g(x, y); b(x, y)]$
Image as Matrix

- Images are usually represented by matrices
  - $M \times N$ dimension
- Be aware that images can have different origin definitions
  - Bottom left - typical Cartesian coordinates
  - Upper left – typical image definition (matrix or table notation)
  - Matlab uses (1,1) for origin not (0,0)
- Index an element either by
  - $(x, y)$
  - $(row, col)$
Matrix Notation

- Mathematical
  - Notation starts with $f(0,0)$

- Matlab
  - Notation starts with $I(1,1)$
    - No zero indexing

$\begin{align*}
\text{Mathematical} \\
\text{Notation starts with } f(0,0) \\
\text{Matlab} \\
\text{Notation starts with } I(1,1) \\
\text{No zero indexing}
\end{align*}$
Image Sampling

- A continuous image is sampled and ordered into a image grid
- Each grid element is known as a pixel
  - Voxel for volume element

- Consider the pixel as the smallest unit in an image
  - This is not quite a delta because it has a finite size on the CMOS sensor
  - It is possible to do sub-pixel processing (e.g. corner detection)
Sampling and Quantization

- Sampling gives fixed grid of image
- Quantization gives the number of output levels $L$
Quantization Levels

- \( L = \) number of output levels
- \( k = \) number of bits per pixel
- Output range of image
  - \([0, L - 1] = [0, 2^k]\]

- Image storage size
  - \( b = M \times N \times k \)
  - Number of bits to store image with dimensions \( M \times N \)

- 8-bits per channel is typical
  - Provide enough resolution to provide quality visual reproduction

Figure 2.3: Brightness levels. (a) 64. (b) 16. (c) 4. (d) 2. © Cengage Learning 2015.
Resolution

- Spatial resolution is the smallest discernible detail in an image
  - This is controlled by the sampling factor (the size $M \times N$ of the CMOS sensor)

- Gray-level resolution is the smallest discernible change in gray level
  - Based on number of bits for representation
Pixel Neighborhood

- The pixel neighborhood corresponds to nearby pixels

<table>
<thead>
<tr>
<th>(x-1, y-1)</th>
<th>(x, y-1)</th>
<th>(x+1, y-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x-1, y)</td>
<td>(x, y)</td>
<td>(x+1, y)</td>
</tr>
<tr>
<td>(x-1, y+1)</td>
<td>(x, y+1)</td>
<td>(x+1, y+1)</td>
</tr>
</tbody>
</table>

- 4-neighbors
  - Horizontal and vertical neighbors

- 8-neighbors
  - Include 4-neighbors and the diagonal pixels
Connectivity

- Path exists between pixels

- 4-connected

- 8-connected
Image Processing

- Usually the first stage of computer vision applications
  - Pre-process an image to ensure it is in a suitable form for further analysis
- Typical operations include:
  - Exposure correction, color balancing, reduction in image noise, increasing sharpness, rotation of an image to straighten

- Digital Image Processing by Gonzalez and Woods is a great book to learn more
2D Signal Processing

- Image processing is an extension of signal processing to two independent variables
  - Input signal, output signal

- General system

\[
x \rightarrow f \rightarrow y
\]

- Image processing

\[
f(x, y) \rightarrow w \rightarrow g(x, y)
\]
Point Operators/Processes

- Output pixel value only depends on the corresponding input pixel value

- Often times we will see operations like dividing one image by another
  - Matrix division is not defined
  - The operation is carried out between corresponding pixels in the two image
  - Element-by-element dot operation in Matlab
    - \texttt{>> I3 = I1./I2}
    - Where I1 and I2 are the same size
Pixel Transforms

- Gain and bias (Multiplication and addition of constant)
  - \( g(x, y) = a(x, y)f(x, y) + b(x, y) \)
  - \( a \) (gain) controls contrast
  - \( b \) (bias) controls brightness
    - Notice parameters can vary spatially (think gradients)

- Linear blend
  - \( g(x) = (1 - \alpha)f_0(x) + \alpha f_1(x) \)
  - We will see this used later for motion detection in video processing
Color Transforms

• Usually we think of a color image as three images concatenated together
  ▫ Have a red, green, blue slice corresponding to the notion of primary colors

• Manipulations of these color channels may not correspond directly with desired perceptual response
  ▫ Adding bias to all channels may actually change the apparent color instead of increasing brightness

• Need other representations of color for mathematical manipulation
Color Images

- Color comes from underlying physical properties

- However, humans do not perceive color in the same physical process
  - There is some subjectivity (e.g. color similarity)
Human Color Perception

- Cones in human retina are sensitive to color
  - In the center of eye
  - 3 different types for different EM frequency sensitivity
- Rods are monochromatic
  - On outside of the eye and good for low lighting and motion sensing

![Figure 2.26: Relative sensitivity of S, M, L cones of the human eye to wavelength. © Cengage Learning 2015.](image1)

![Figure 2.27: Color matching functions obtained in the color matching experiment. Intensities of the selected primary colors which perceptually match spectral color of given wavelength \( \lambda \). Based on [Wundell, 1995].](image2)
Colors prevention

- Uniform method for defining colors
- Can transform from one to another
  - Want to take advantage of properties and color gamut
- XYZ
  - International absolute color standard
  - No negative mixing
- RGB
  - Additive color mixing for red, green, and blue
  - Widely used in computers
- CMYK
  - Cyan, magenta, yellow, black
  - Used for printers and based on reflectivity
- HSV
  - Hue, saturation, and value = color, amount, brightness
  - Closer to human perception