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# ECG782: Multidimensional Digital Signal Processing

## Filtering in the Frequency Domain

http://www.ee.unlv.edu/~b1morris/ecg782/

# Outline

- Background Concepts
- Sampling and Fourier Transform
- Discrete Fourier Transform
- Extension to Two Variables
- Properties of 2D DFT
- Frequency Domain Filtering Basics
- Smoothing
- Sharpening
- Selective Filtering
- Implementation

# Motivation

- Complicated signals (functions) can be constructed as a linear combination of sinusoids
  - Mathematically compact representation with complex exponentials $e^{j\omega t}$
- Introduced as Fourier series by Jean Baptiste Joseph Fourier
  - Initially considered periodic signals
  - Later extended to aperiodic signals
- Powerful mathematical tool
  - Can go between “time” and “frequency” domain processing

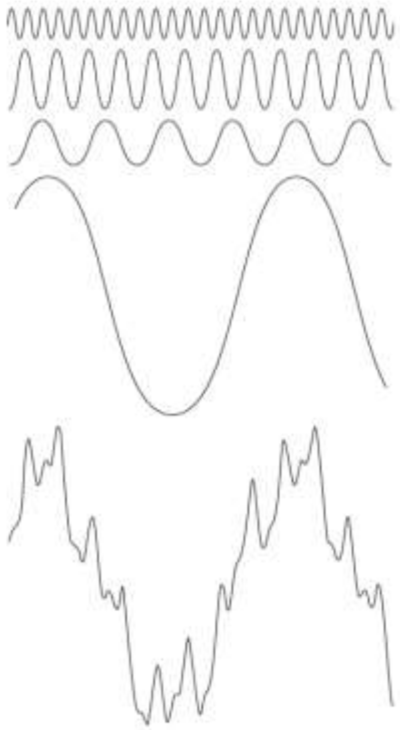


**FIGURE 4.1** The function at the bottom is the sum of the four functions above it. Fourier's idea in 1807 that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism.

# Preliminary Concepts

- Complex numbers
  - $C = R + jI$
  - $C^* = R - jI$
  - $C = |C|e^{j\theta}$
    - Using Euler's formula
    - $e^{j\theta} = \cos\theta + j\sin\theta$
- Fourier Series
  - Express a periodic signal as a sum of sines and cosines
  - $f(t) = \sum_n c_n e^{j\omega_0 nt}$
  - $c_n = \frac{1}{T}\int_T f(t)e^{-j\omega_0 nt}$
    - $\omega_0 = 2\pi/T$
- Fourier Transform
  - $F(\mu) = \mathcal{F}\{f(t)\} = \int f(t)e^{-j2\pi\mu t}dt$
    - $\mu$ : continuous frequency variable
  - $f(t) = \mathcal{F}^{-1}\{F(\mu)\} = \int F(\mu)e^{j2\pi\mu t}d\mu$
  - Notice for real $f(t)$ this generally results in a complex transform

# Rectangle Wave Example

- $F(\mu) = AW \dfrac{\sin \pi\mu W}{\pi\mu W}$
  - Rectangle in time gives sinc in frequency
  - See book for derivation
- Frequency spectrum
  - $|F(\mu)| = AW \left|\dfrac{\sin \pi\mu W}{\pi\mu W}\right|$
    - Consider only a real portion
- Note zeros are inversely proportional to width of box
  - Wider in time, narrow in frequency

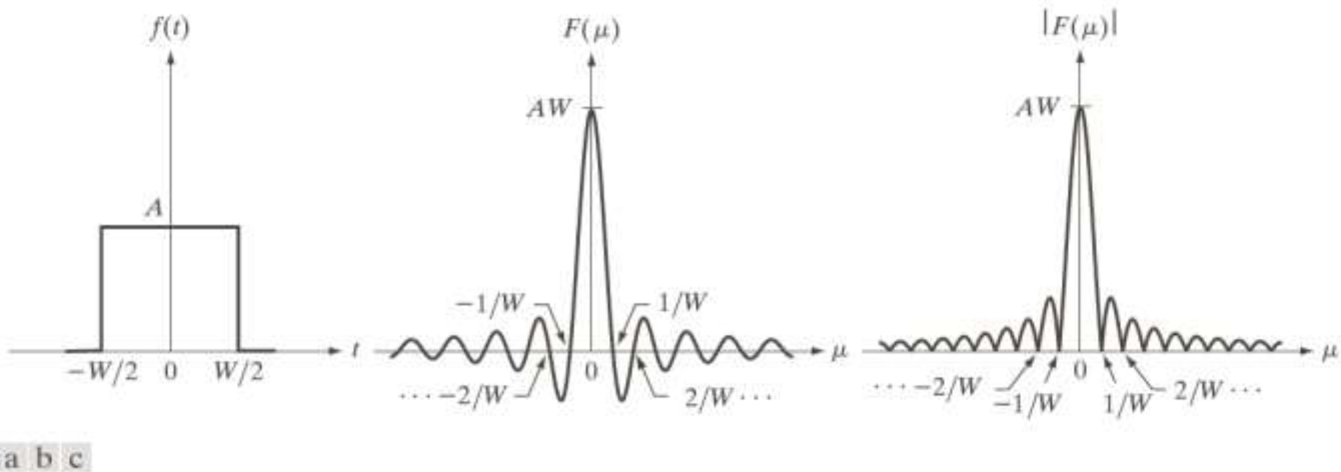



**FIGURE 4.4** (a) A simple function; (b) its Fourier transform; and (c) the spectrum. All functions extend to infinity in both directions.

# Convolution Properties

- Very important input-output relationship between a input signal $f(t)$ and an LTI system $h(t)$
- $f(t) * h(t) = \int f(\tau)h(t-\tau)d\tau$
- Dual time-frequency relationship
  - $f(t) * h(t) \leftrightarrow F(\mu)H(\mu)$
  - $f(t)h(t) \leftrightarrow F(\mu) * H(\mu)$
  - Convolution-multiplication relationship

# Sampling

- Convert continuous signal to a discrete sequence
  - Use impulse train sampling
- $\tilde{f}(t) = f(t)s_{\Delta T}(t) = \sum_n f(t)\delta(t - n\Delta T)$
  - $\delta(t - n\Delta T)$ - impulse response at time $t = n\Delta T$
- Sample value
  - $f_k = f(k\Delta T)$

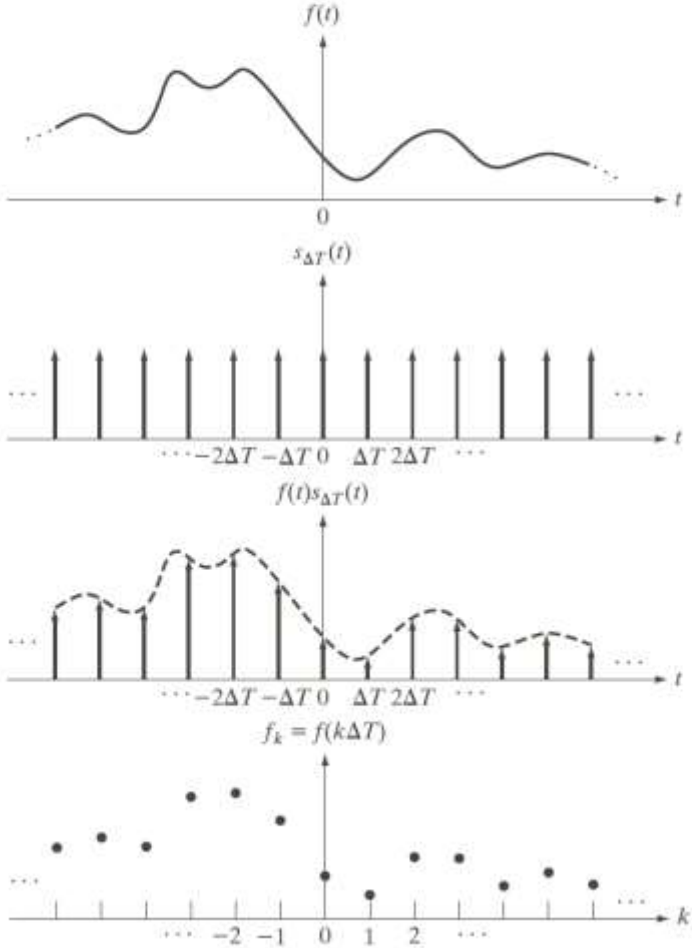



**FIGURE 4.5** (a) A continuous function. (b) Train of impulses used to model the sampling process. (c) Sampled function formed as the product of (a) and (b). (d) Sample values obtained by integration and using the sifting property of the impulse. (The dashed line in (c) is shown for reference. It is not part of the data.)

# Fourier Transform of Sampled Signal

- $\tilde{F}(\mu) = \mathcal{F}\{\tilde{f}(t)\} = F(\mu) * S(\mu)$
  - $S(\mu) = \frac{1}{\Delta T}\sum_n \delta\left(\mu - \frac{n}{\Delta T}\right)$
  - FT of impulse train is an impulse train
    - See section 4.2.3 in the book for details
    - Note spacing between impulses are inversely related
- $\tilde{F}(\mu) = \frac{1}{\Delta T}\sum_n F\left(\mu - \frac{n}{\Delta T}\right)$
  - Sampling creates copies of the original spectrum
  - Must be careful with sampling period to avoid aliasing (overlap of spectrum)

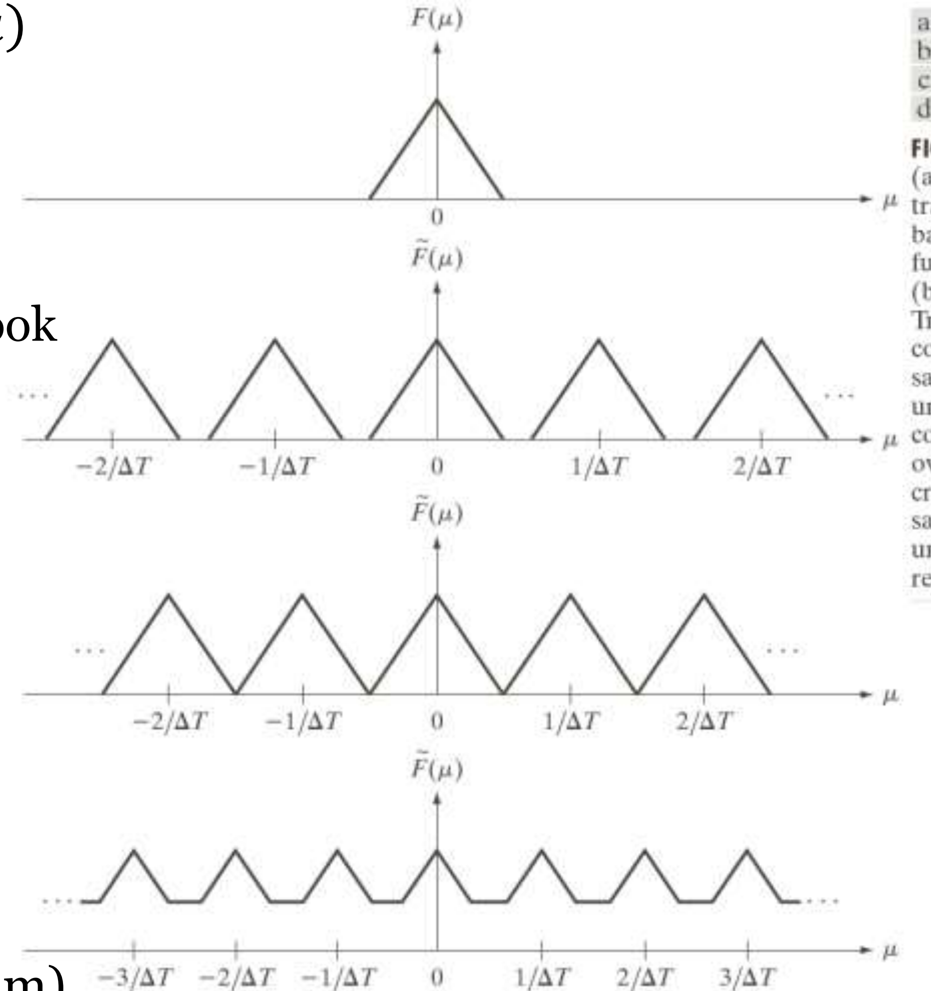



**FIGURE 4.6** (a) Fourier transform of a band-limited function. (b)–(d) Transforms of the corresponding sampled function under the conditions of over-sampling, critically-sampling, and under-sampling, respectively.

# Sampling Theorem

- Conditions to be able to recover $f(t)$ completely after sampling
- Requires bandlimited $f(t)$
  - $F(\mu) = 0$ for $|\mu| > \mu_{\max}$
  - Can isolate center spectrum copy from its neighbors

- Sampling theorem
  - $\frac{1}{\Delta T} > 2\mu_{\max}$
    - Nyquist rate $2\mu_{\max}$
- Recovery with lowpass filter
  - $H(\mu) = \Delta T$ for $|\mu| \leq \mu_{\max}$

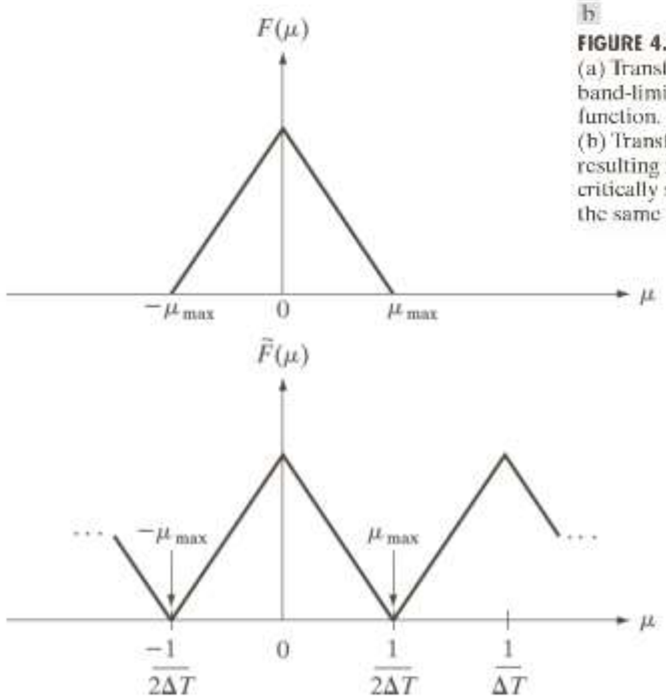



**FIGURE 4.7** (a) Transform of a band-limited function. (b) Transform resulting from critically sampling the same function.

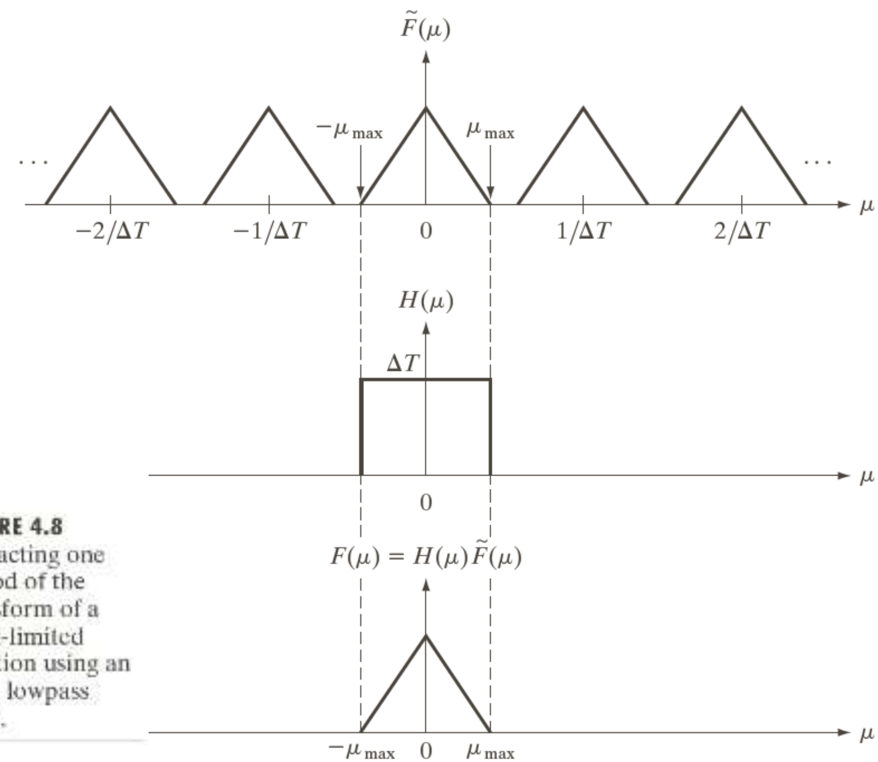



**FIGURE 4.8** Extracting one period of the transform of a band-limited function using an ideal lowpass filter.

# Aliasing

- Corruption of recovered signal if not sampled at rate less than Nyquist rate
  - Spectrum copies overlap
  - High frequency components corrupt lower frequencies
- In reality this is always present
  - Most signals are not bandlimited
  - Bandlimited signals require infinite time duration
    - Windowing to limit size naturally causes distortion
  - Use anti-aliasing filter before sampling
    - Filter reduces high frequency components

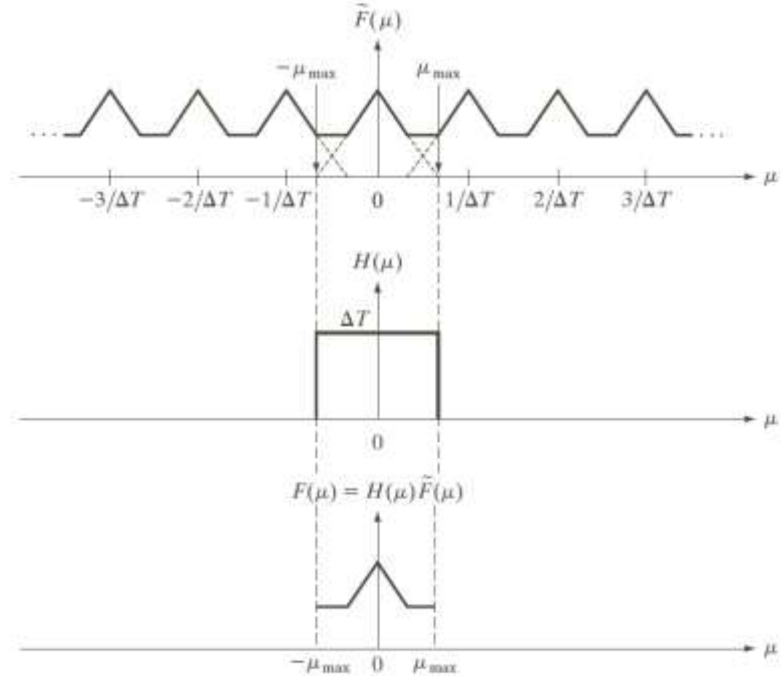



**FIGURE 4.9** (a) Fourier transform of an under-sampled, band-limited function. (Interference from adjacent periods is shown dashed in this figure). (b) The same ideal lowpass filter used in Fig. 4.8(b). (c) The product of (a) and (b). The interference from adjacent periods results in aliasing that prevents perfect recovery of $F(\mu)$ and, therefore, of the original, band-limited continuous function. Compare with Fig. 4.8.

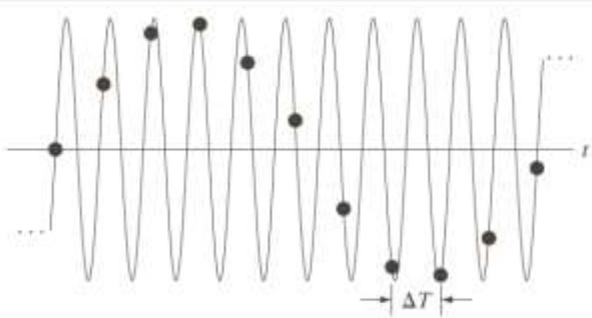



**FIGURE 4.10** Illustration of aliasing. The under-sampled function (black dots) looks like a sine wave having a frequency much lower than the frequency of the continuous signal. The period of the sine wave is 2 s, so the zero crossings of the horizontal axis occur every second. $\Delta T$ is the separation between samples.

# Discrete Fourier Transform

- Discussion has considered continuous signals (functions)
  - Need to operate on discrete signals
- DFT is a sampled version of the sampled signal FT in one period
  - $\tilde{F}(\mu) = \sum_n f_n e^{-j2\pi\mu n\Delta T}$
  - Sample in frequency evenly ($M$) over a period
    - $\mu = \frac{m}{M\Delta T}$
  - $F_m = \sum_n f_n e^{-j2\pi mn/M}$
    - $m = 0,1,2,\dots,M-1$
  - $M$ samples of $f(t)$, $\{f_n\}$, results in $M$ DFT values
  - Note: implicitly assumes samples come from one period of periodic signal
- Inverse DFT
  - $F_n = \frac{1}{M}\sum_m F_m e^{j2\pi mn/M}$

# Sampling/Frequency Relationship

- $M$ samples of signal with sample period $\Delta T$
  - Total time → $T = M\Delta T$
- Spacing in discrete frequency
  - $\Delta u = \frac{1}{M\Delta T} = \frac{1}{T}$
    - Note the switch to $u$ for discrete frequency
  - Total frequency range → $\Omega = M\Delta u = \frac{1}{\Delta T}$
- Resolution of DFT is dependent on the duration $T$ of the sampled function
  - Generally the number of samples

- See `fft.m` in Matlab to test this

# Extensions to 2D

- All discussions can be extended to two variables easily
  - Add second integral or summation for extra variable
- 2D rectangle
  - $F(\mu, \nu) = \text{ATZ} \left[ \frac{\sin(\pi\mu T)}{\pi\mu T} \right] \left[ \frac{\sin(\pi\nu Z)}{\pi\nu Z} \right]$

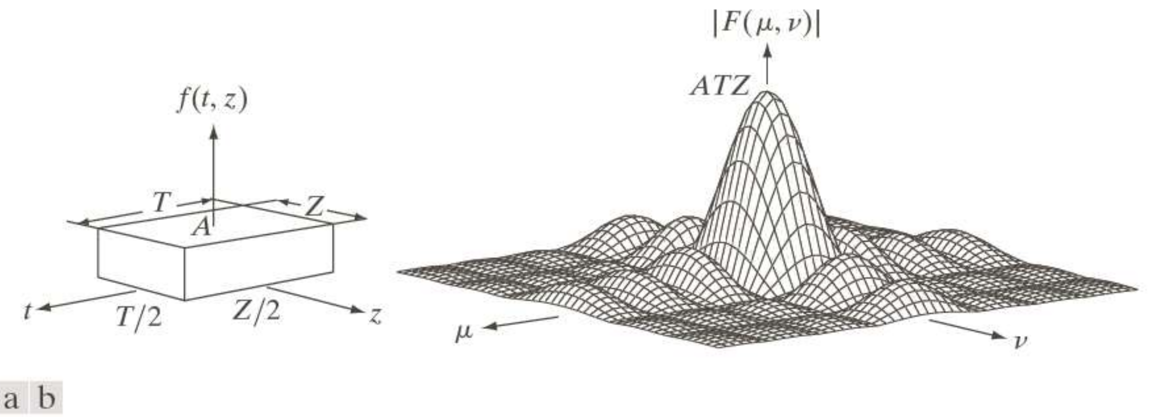


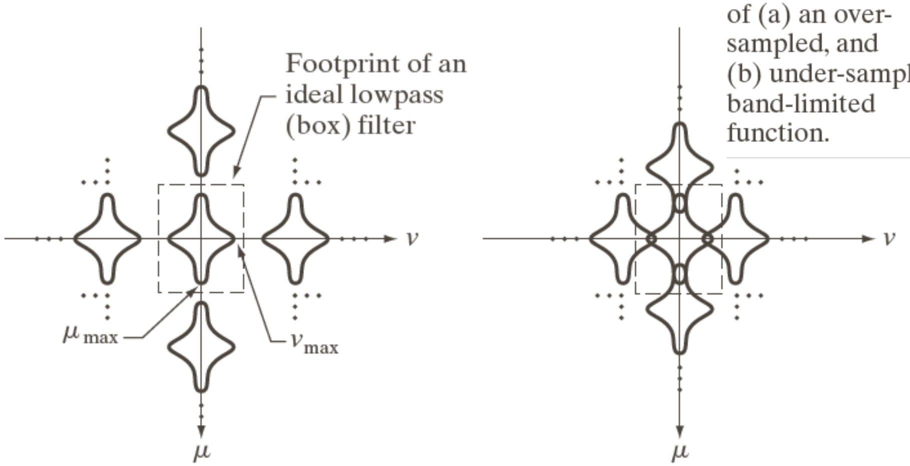

**FIGURE 4.13** (a) A 2-D function, and (b) a section of its spectrum (not to scale). The block is longer along the $t$-axis, so the spectrum is more "contracted" along the $\mu$-axis. Compare with Fig. 4.4.

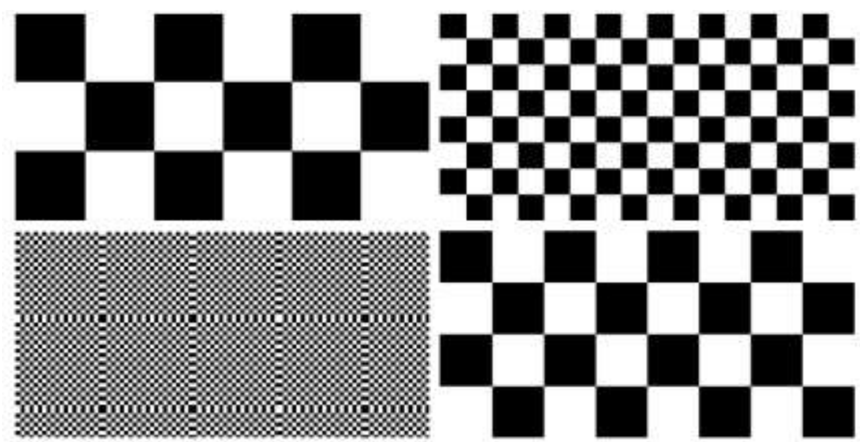
# Image Aliasing

- Temporal aliasing appears in video
  - Wheel effect – looks like it is spinning opposite direction
- Spatial aliasing is the same as the previous discussion→ now in two dimensions

a b

**FIGURE 4.15** Two-dimensional Fourier transforms of (a) an over-sampled, and (b) under-sampled band-limited function.



a b
c d

**FIGURE 4.16** Aliasing in images. In (a) and (b), the lengths of the sides of the squares are 16 and 6 pixels, respectively, and aliasing is visually negligible. In (c) and (d), the sides of the squares are 0.9174 and 0.4798 pixels, respectively, and the results show significant aliasing. Note that (d) masquerades as a "normal" image.

# Image Interpolation and Resampling

- Used for image resizing
  - Zooming – oversample and image
  - Shrinking – undersample an image
    - Must be careful of aliasing
    - Generally smooth before downsample

a b c

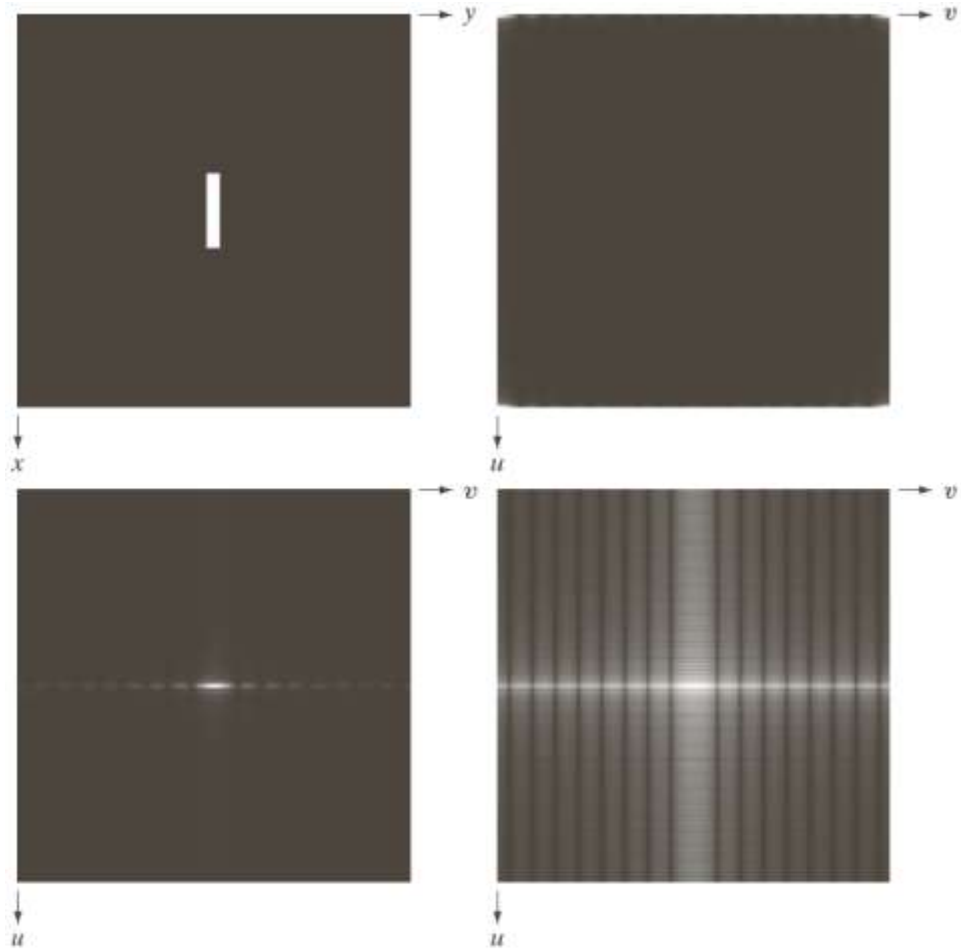
**FIGURE 4.17** Illustration of aliasing on resampled images. (a) A digital image with negligible visual aliasing. (b) Result of resizing the image to 50% of its original size by pixel deletion. Aliasing is clearly visible. (c) Result of blurring the image in (a) with a 3 × 3 averaging filter prior to resizing. The image is slightly more blurred than (b), but aliasing is not longer objectionable. (Original image courtesy of the Signal Compression Laboratory, University of California, Santa Barbara.)

# Fourier Spectrum and Phase Angle

- $F(u, v) = |F(u, v)|e^{j\phi(u,v)}$
  - Magnitude, spectrum
    - $|F(u, v)| = [R^2(u, v) + I^2(u, v)]^{1/2}$
  - Phase angle
    - $e^{j\phi(u,v)} = \arctan\left[\frac{I(u,v)}{R(u,v)}\right]$

- Spectrum is component we naturally specify while phase is a bit harder to visualize

- Spectrum

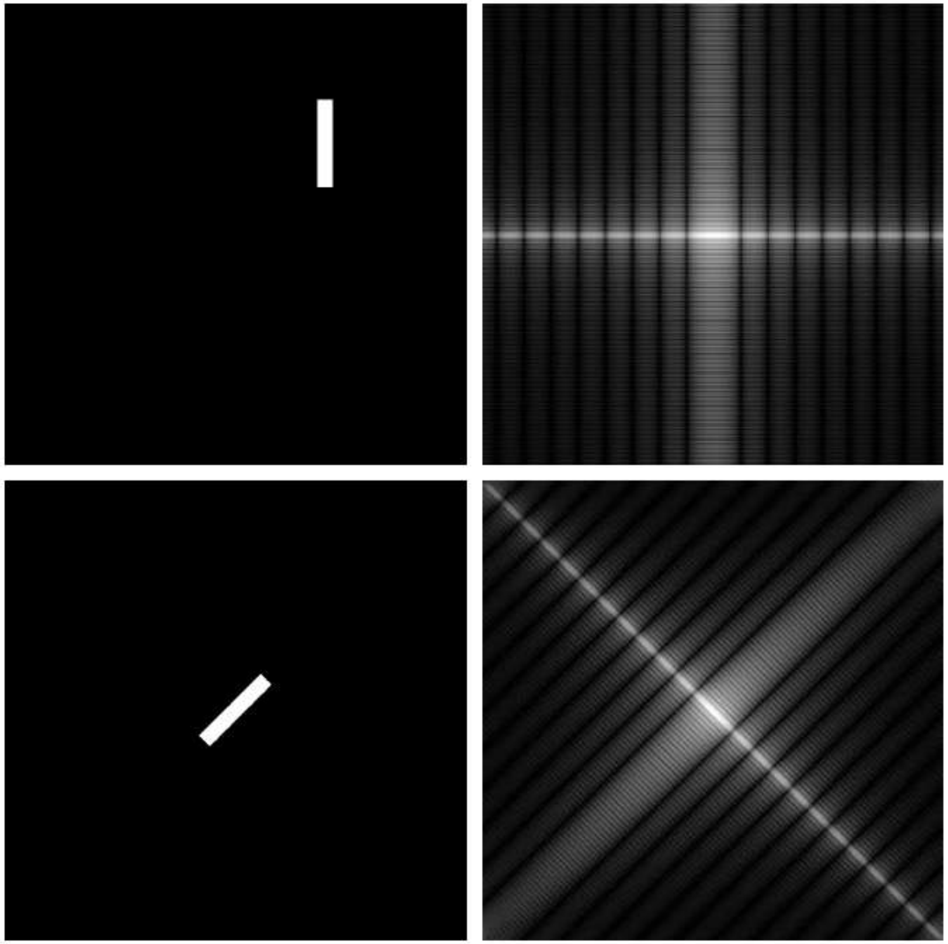



a b
c d

**FIGURE 4.24** (a) Image. (b) Spectrum showing bright spots in the four corners. (c) Centered spectrum. (d) Result showing increased detail after a log transformation. The zero crossings of the spectrum are closer in the vertical direction because the rectangle in (a) is longer in that direction. The coordinate convention used throughout the book places the origin of the spatial and frequency domains at the top left.

# Spectrum

- Translation does not affect spectrum
  - Wide in space → narrow in frequency
- Orientation clearly visible in spectrum



a b
c d

**FIGURE 4.25** (a) The rectangle in Fig. 4.24(a) translated, and (b) the corresponding spectrum. (c) Rotated rectangle, and (d) the corresponding spectrum. The spectrum corresponding to the translated rectangle is identical to the spectrum corresponding to the original image in Fig. 4.24(a).

# Phase

- Difficult to describe phase given image content

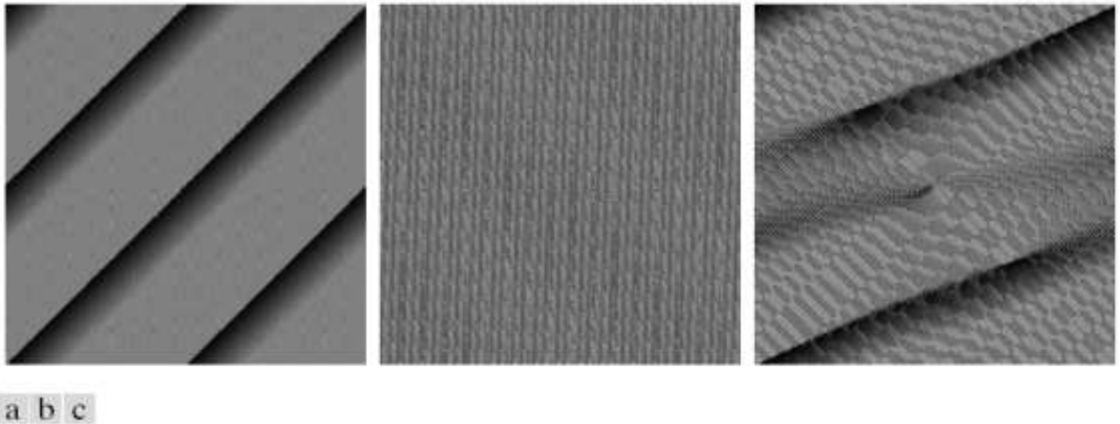



**FIGURE 4.26** Phase angle array corresponding (a) to the image of the centered rectangle in Fig. 4.24(a), (b) to the translated image in Fig. 4.25(a), and (c) to the rotated image in Fig. 4.25(c).

  - a) centered rectangle
  - b) translated rectangle
  - c) rotated rectangle

# Spectrum Phase Manipulation

- Both spectrum and phase are important for image content

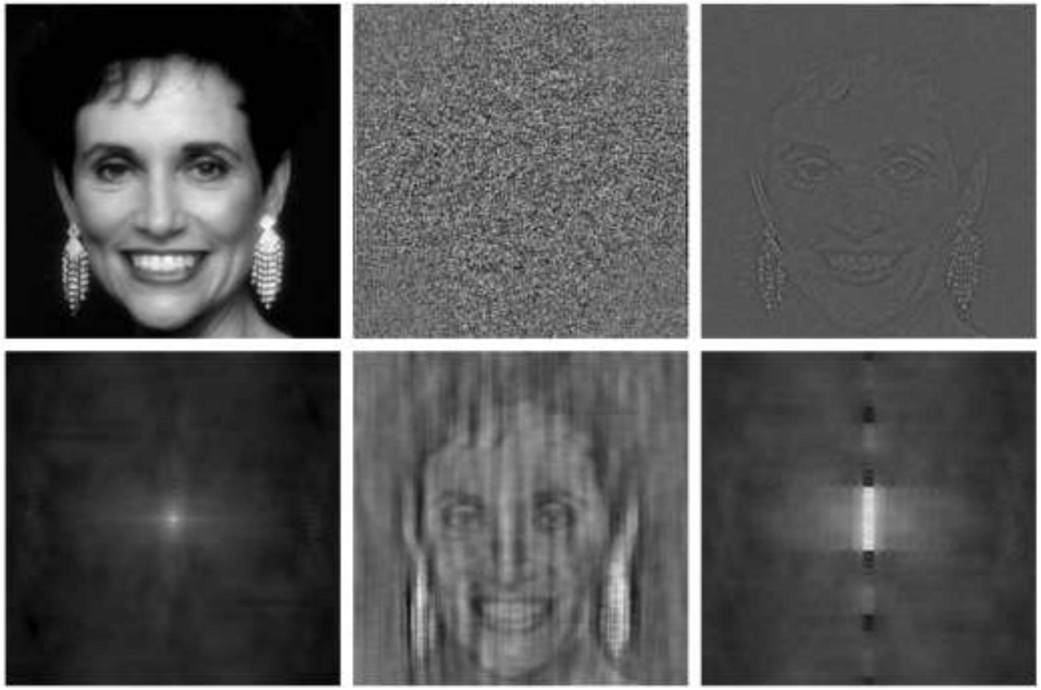



**FIGURE 4.27** (a) Woman. (b) Phase angle. (c) Woman reconstructed using only the phase angle. (d) Woman reconstructed using only the spectrum. (e) Reconstruction using the phase angle corresponding to the woman and the spectrum corresponding to the rectangle in Fig. 4.24(a). (f) Reconstruction using the phase of the rectangle and the spectrum of the woman.

# Frequency Domain Filtering Basics

- Generally complicated relationship between image and transform
  - Frequency is associated with patterns of intensity variations in image
- Filtering modifies the image spectrum based on a specific objective
  - Magnitude (spectrum) – most useful for visualization (e.g. match visual characteristics)
  - Phase – generally not useful for visualization

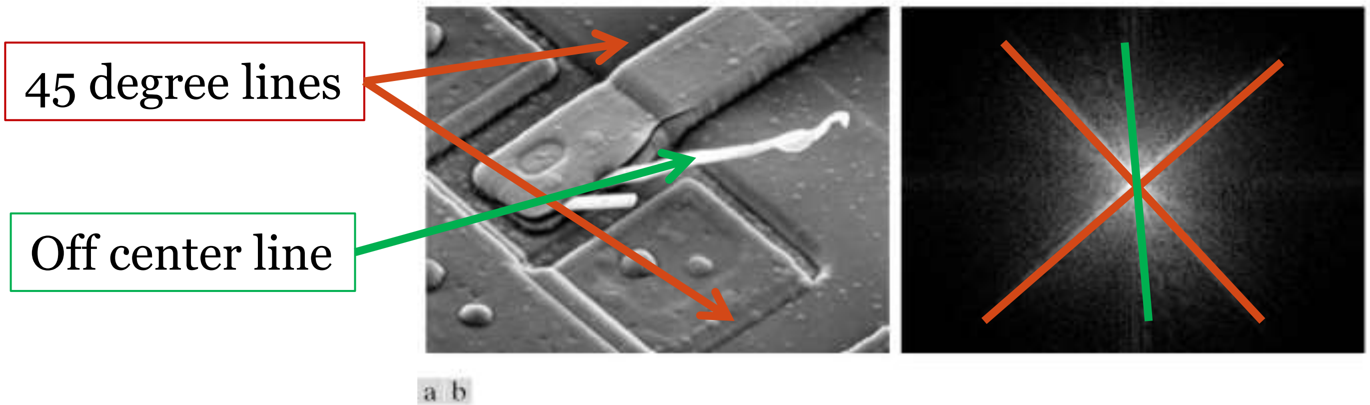



**FIGURE 4.29** (a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

# Fundamentals

- Modify FT of image and inverse for result
  - $g(x,y) = \mathcal{F}^{-1}[H(u,v)F(u,v)]$
    - $g(x,y)$ : output image $[M \times N]$
    - $F(u,v)$ : FT of input image $f(x,y)$ $[M \times N]$
    - $H(u,v)$ : filter transfer function $[M \times N]$
    - $\mathcal{F}^{-1}$ : inverse FT (iFT)
  - Product from element-wise array multiplication

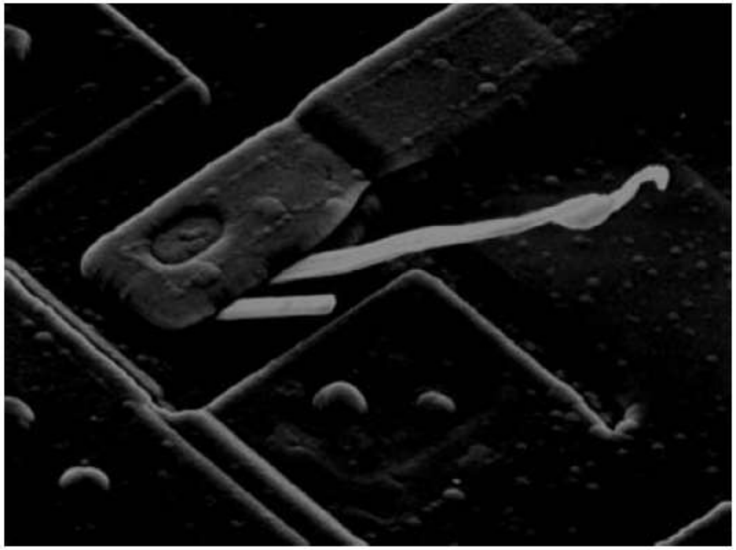
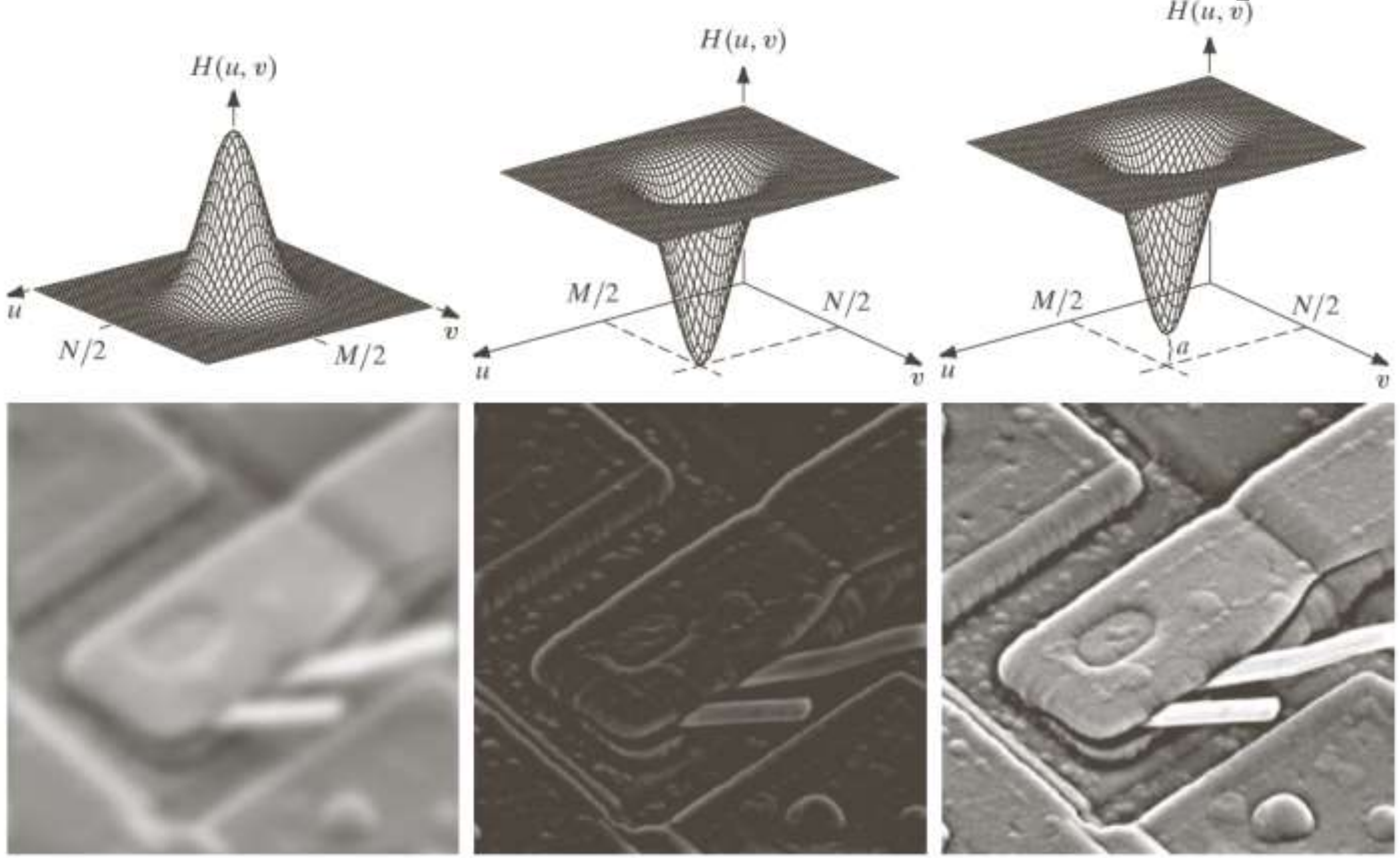


**FIGURE 4.30** Result of filtering the image in Fig. 4.29(a) by setting to 0 the term $F(M/2, N/2)$ in the Fourier transform.

Remove DC (0,0) term from $F(u,v)$

# Example Filters

Addition of small offset to retain DC component after HP




a b c
d e f

**FIGURE 4.31** Top row: frequency domain filters. Bottom row: corresponding filtered images obtained using Eq. (4.7-1). We used $a = 0.85$ in (c) to obtain (f) (the height of the filter itself is 1). Compare (f) with Fig. 4.29(a).

# DFT Subtleties

- Multiplication in frequency is convolution in time
  - Must pad image since output is larger
    - Will pad $f(x,y)$ image but not $h(x,y)$
    - $H(u,v)$ designed and sized for padded $F(u,v)$
  - DFT implicitly assumes a periodic function

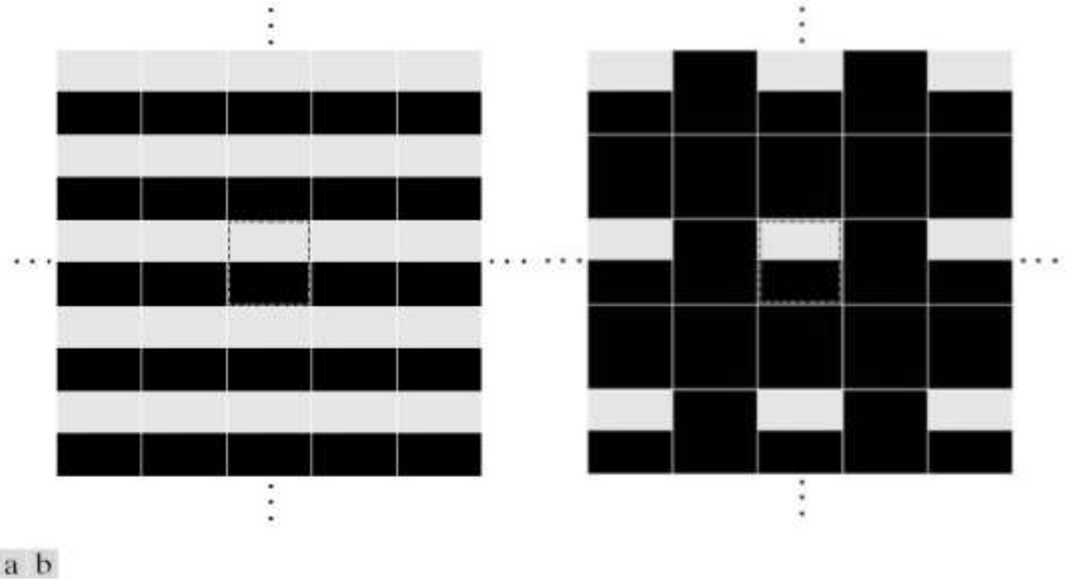
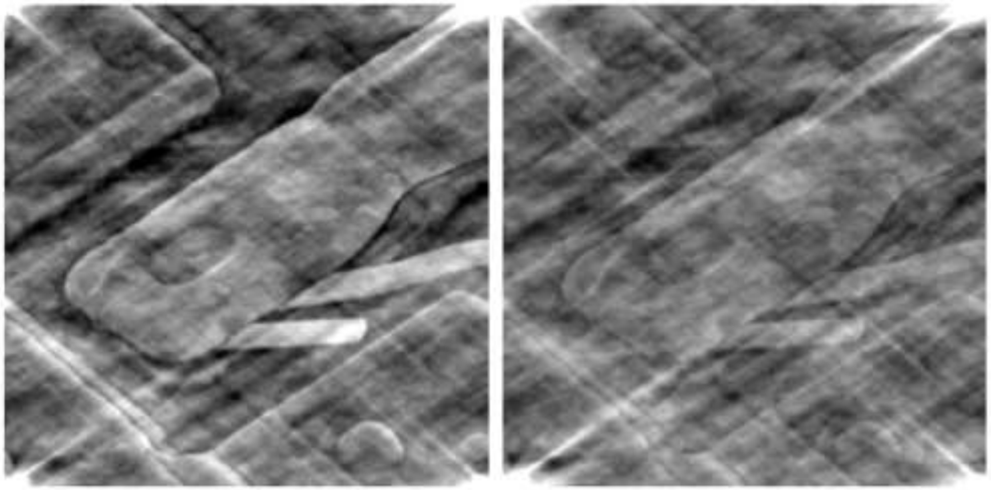



**FIGURE 4.33** 2-D image periodicity inherent in using the DFT. (a) Periodicity without image padding. (b) Periodicity after padding with 0s (black). The dashed areas in the center correspond to the image in Fig. 4.32(a). (The thin white lines in both images are superimposed for clarity; they are not part of the data.)

# Phase Angle

- Generally, a filter can affect the phase of a signal
- Zero-phase-shift filters have no effect on phase
  - Focus of this chapter
- Phase is very important to image
  - Small changes can lead to unexpected results



a b

**FIGURE 4.35**
(a) Image resulting from multiplying by 0.5 the phase angle in Eq. (4.6-15) and then computing the IDFT. (b) The result of multiplying the phase by 0.25. The spectrum was not changed in either of the two cases.

# Frequency Domain Filtering Steps

1. Given image $f(x, y)$ of size $M \times N$, get padding $(P, Q)$
   - Typically use $P = 2M$ and $Q = 2N$
2. Form zero-padded image $f_p(x, y)$ of size $P \times Q$
3. Multiply $f_p(x, y)$ by $(-1)^{x+y}$ to center the transform
4. Compute DFT $F(u, v)$
5. Compute $G(u, v) = H(u, v)F(u, v)$
   - Get real, symmetric filter function $H(u, v)$ of size $P \times Q$ with center at coordinates $\left(\frac{P}{2}, \frac{Q}{2}\right)$
6. Obtain (padded) output image from iFT
   - $g_p(x, y) = \{\text{real}\, [\mathcal{F}^{-1}[G(u, v)]\}(-1)^{x+y}$
7. Obtain $g(x, y)$ by extracting $M \times N$ region from top left quadrant of $g_p(x, y)$

# Steps Example

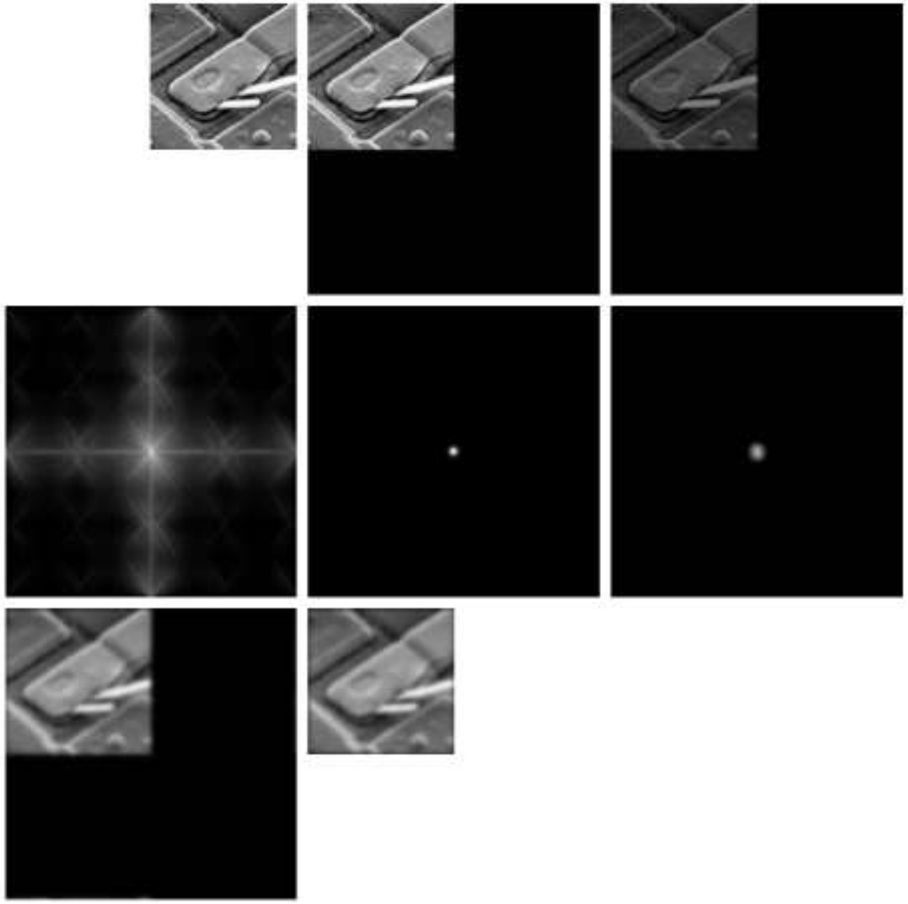



**FIGURE 4.36** (a) An $M \times N$ image, $f$. (b) Padded image, $f_p$ of size $P \times Q$. (c) Result of multiplying $f_p$ by $(-1)^{x+y}$. (d) Spectrum of $F_p$. (e) Centered Gaussian lowpass filter, $H$, of size $P \times Q$. (f) Spectrum of the product $HF_p$. (g) $g_p$, the product of $(-1)^{x+y}$ and the real part of the IDFT of $HF_p$. (h) Final result, $g$, obtained by cropping the first $M$ rows and $N$ columns of $g_p$.

# Relationship to Spatial Filtering

- Frequency multiplication → convolution in spatial domain
  - $h(x, y) \leftrightarrow H(u, v)$
  - Use of a finite impulse response
- Generally use small filter kernels which are more efficient to implement in spatial domain
- Frequency domain can be better for the design of filters
  - More natural space for definition
  - Use iFT to determine the "shape" of the spatial filter

# Smoothing

- High frequency image content comes from edges and noise
- Smoothing/blurring is a lowpass operation that attenuates (removes) high frequency content
- Consider three smoothing filters
  - Ideal lowpass – sharp filter
  - Butterworth – filter order controls shape
  - Gaussian – very smooth filter

# Ideal Lowpass Filter

- $H(u,v) = \begin{cases} 1 & D(u,v) \le D_0 \\ 0 & D(u,v) > D_0 \end{cases}$
  - $D(u,v) = \left[\left(u - \frac{P}{2}\right)^2 + \left(v - \frac{Q}{2}\right)^2\right]$
  - Pass all frequencies $D_0$ distance from DC
    - $D_0$ is the cuttoff frequency

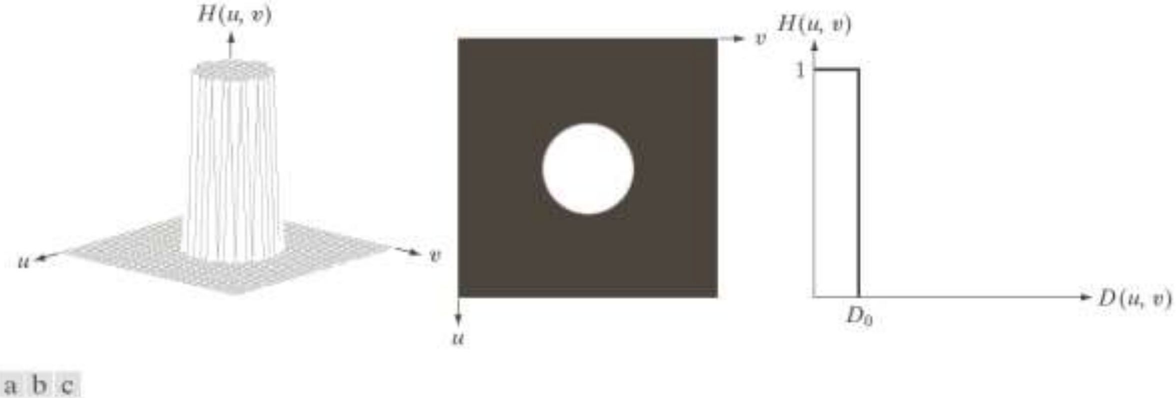
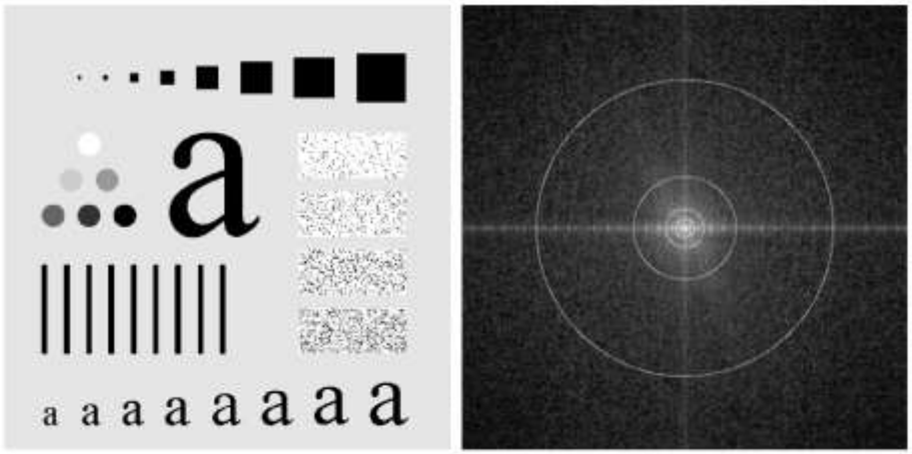


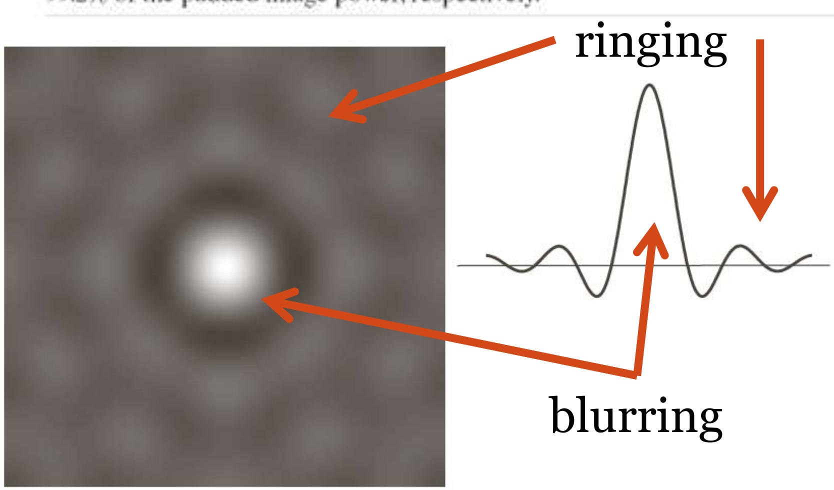

**FIGURE 4.40** (a) Perspective plot of an ideal lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

# Ideal Lowpass Example



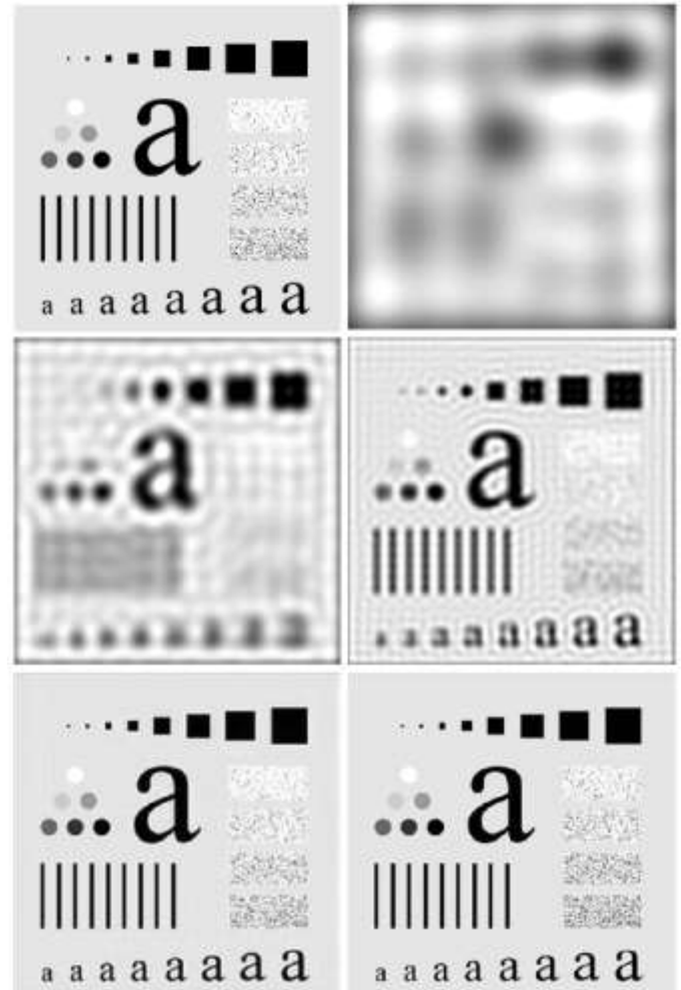
a b

**FIGURE 4.41** (a) Test pattern of size 688 × 688 pixels, and (b) its Fourier spectrum. The spectrum is double the image size due to padding but is shown in half size so that it fits in the page. The superimposed circles have radii equal to 10, 30, 60, 160, and 460 with respect to the full-size spectrum image. These radii enclose 87.0, 93.1, 95.7, 97.8, and 99.2% of the padded image power, respectively.




a b

**FIGURE 4.43** (a) Representation in the spatial domain of an ILPF of radius 5 and size 1000 × 1000. (b) Intensity profile of a horizontal line passing through the center of the image.



a b
c d
e f

**FIGURE 4.42** (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.

# LP Spectrum View

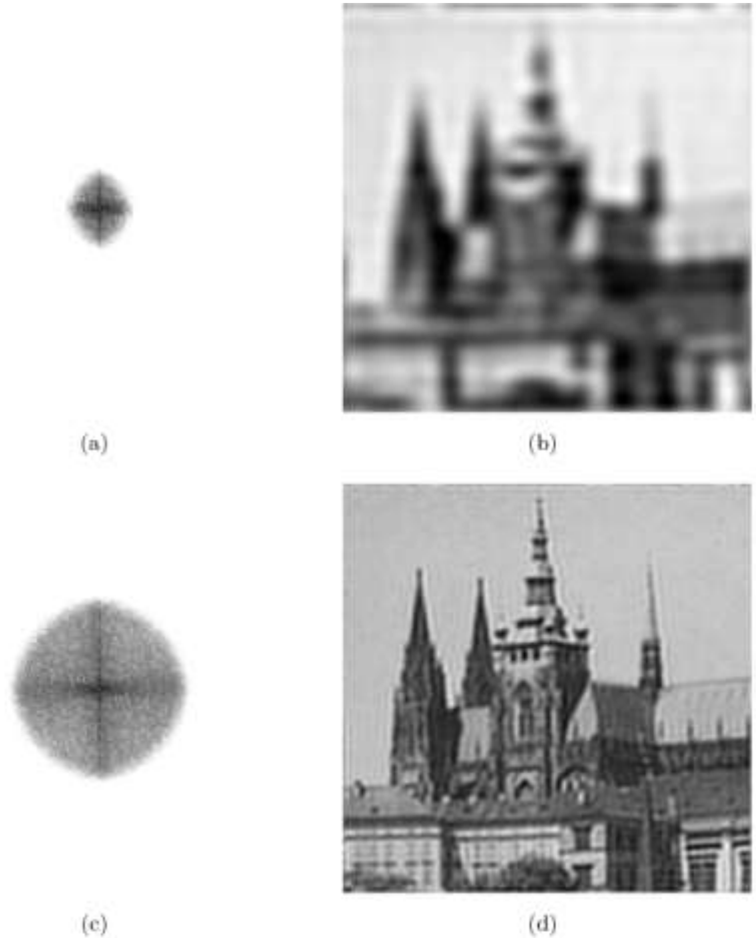



**Figure 5.25**: Low-pass frequency-domain filtering—for the original image and its spectrum see Figure 3.7. (a) Spectrum of a low-pass filtered image, all higher frequencies filtered out. (b) Image resulting from the inverse Fourier transform applied to spectrum (a). (c) Spectrum of a low-pass filtered image, only very high frequencies filtered out. (d) Inverse Fourier transform applied to spectrum (c). *© Cengage Learning 2015.*

# Butterworth LP Filter

- $H(u,v) = \frac{1}{1+[D(u,v)/D_0]^{2n}}$
  - $n$ – order of the filter (controls sharpness of transition)
  - Cutoff generally specified as the 50% of max ($D_0 = 0.5$)

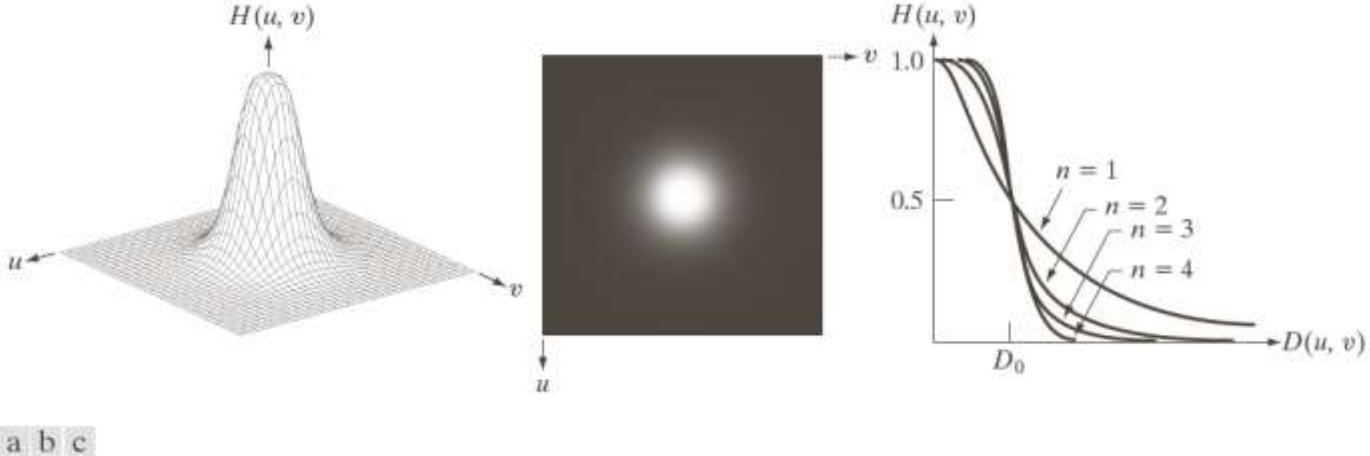
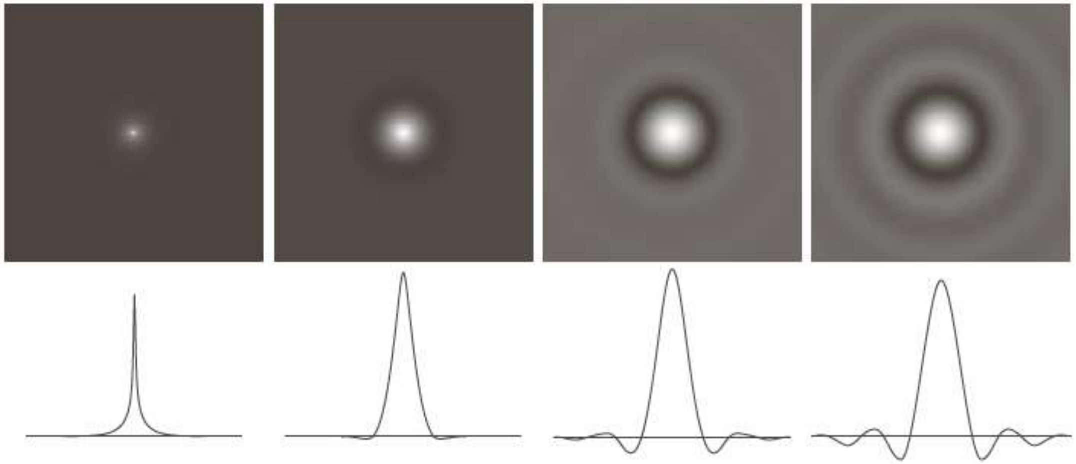



**FIGURE 4.44** (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

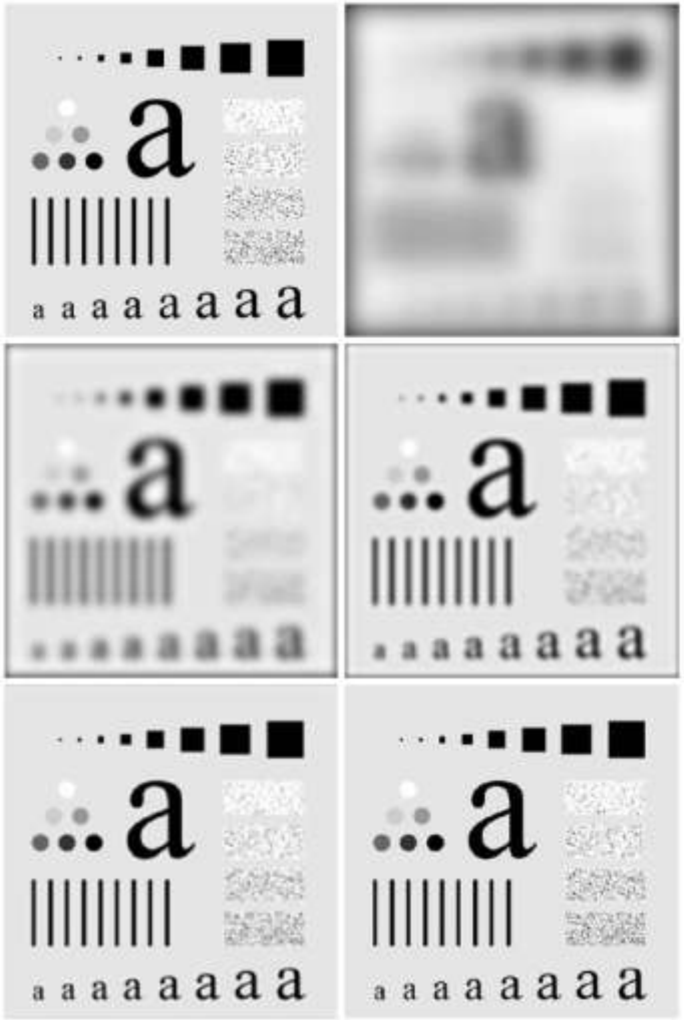
# Butterworth LP Example

- No ringing is visible because of the gradual transition from high to low frequency in filter
  - May be visible in higher-order filters ($n > 2$)
  - Trade-off frequency narrow main lobe with sidelobe height



a b c d

**FIGURE 4.46** (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding intensity profiles through the center of the filters (the size in all cases is $1000 \times 1000$ and the cutoff frequency is 5). Observe how ringing increases as a function of filter order.



a b
c d
e f

**FIGURE 4.45** (a) Original image. (b)–(f) Results of filtering using BLPFs of order 2, with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Fig. 4.42.

# Gaussian Lowpass Filter

- $H(u,v) = e^{-D^2(u,v)/2\sigma^2}$
  - $\sigma$ – measure of spread
    - $\sigma = D_0$ is the cutoff frequency
  - iFT is also a Gaussian
    - No ringing because of smooth function
  - A favorite filter for smoothing

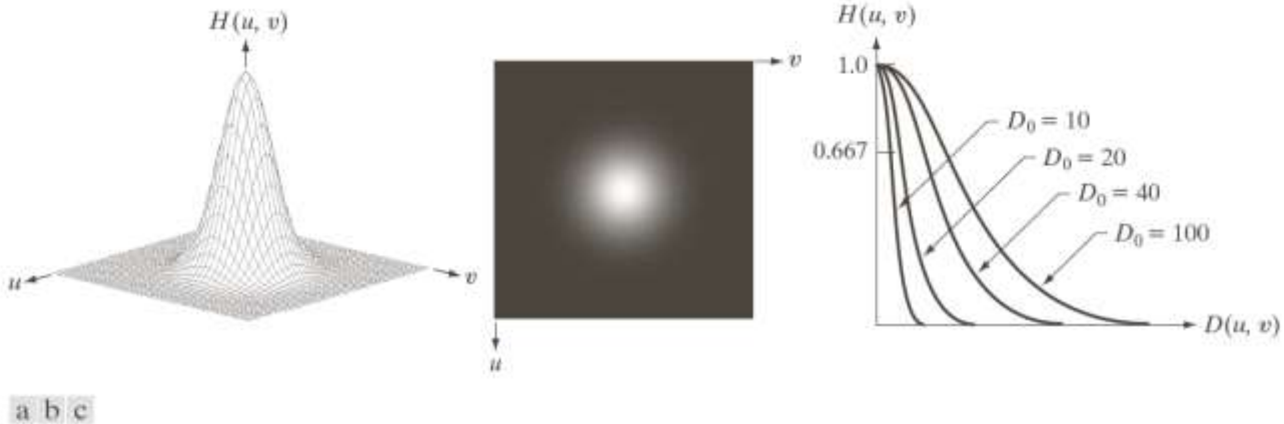
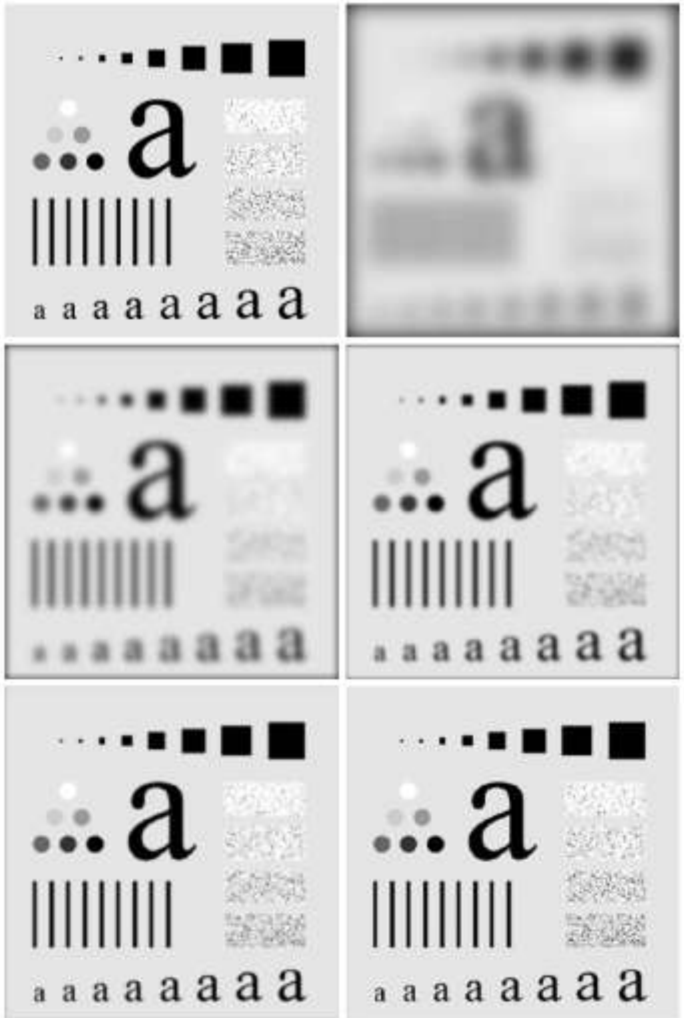



**FIGURE 4.47** (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of $D_0$.

# Gaussian LP Example

- No ringing
- Not as much smoothing as Butterworth 2

- Best for use when ringing is unacceptable
- Butterworth better when tight control of transition between high and low frequency is required



a b
c d
e f

**FIGURE 4.48** (a) Original image. (b)–(f) Results of filtering using GLPFs with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Figs. 4.42 and 4.45.

# Sharpening

- Use a highpass filter
  - $H_{HP}(u,v) = 1 - H_{LP}(u,v)$
- Ideal
  - $H(u,v) = \begin{cases} 0 & D(u,v) \le D_0 \\ 1 & D(u,v) > D_0 \end{cases}$
- Butterworth
  - $H(u,v) = \frac{1}{1+[D_0/D(u,v)]^{2n}}$
- Gaussian
  - $H(u,v) = e^{-D^2(u,v)/2\sigma^2}$

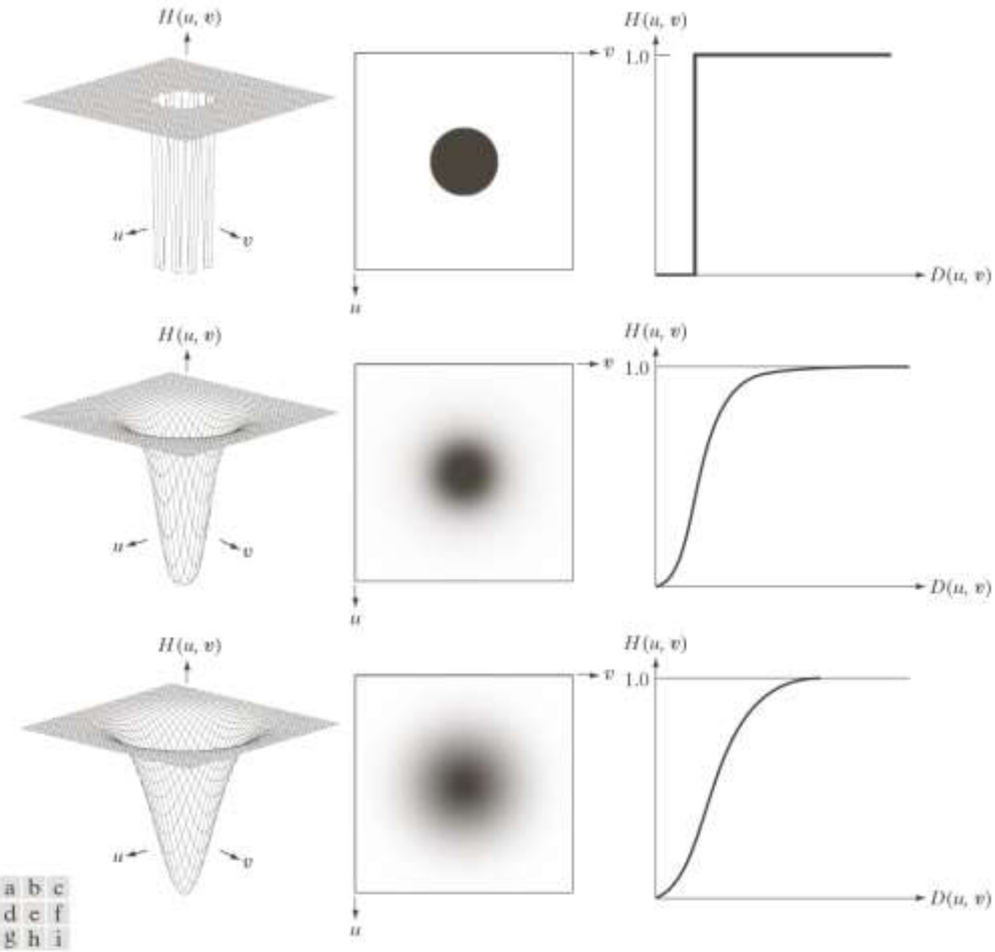
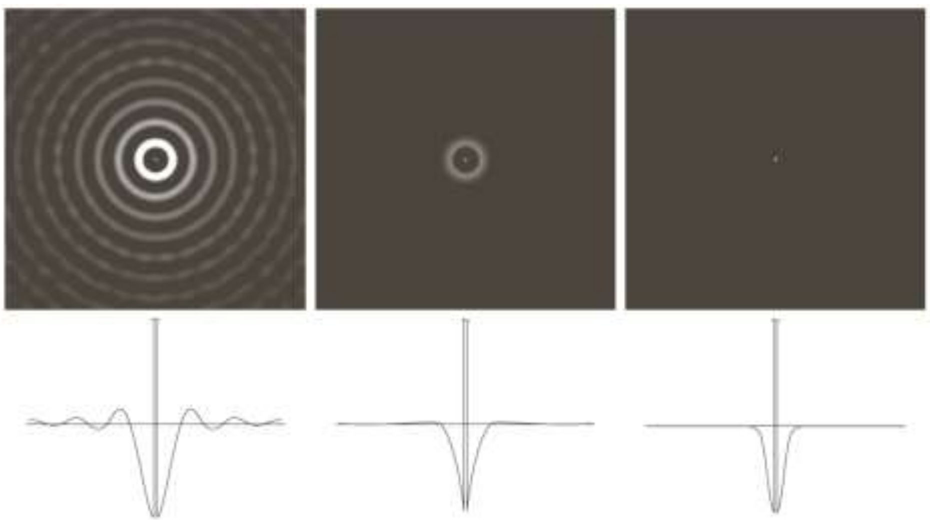



**FIGURE 4.52** Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

# Highpass Examples



a b c

**FIGURE 4.53** Spatial representation of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding intensity profiles through their centers.

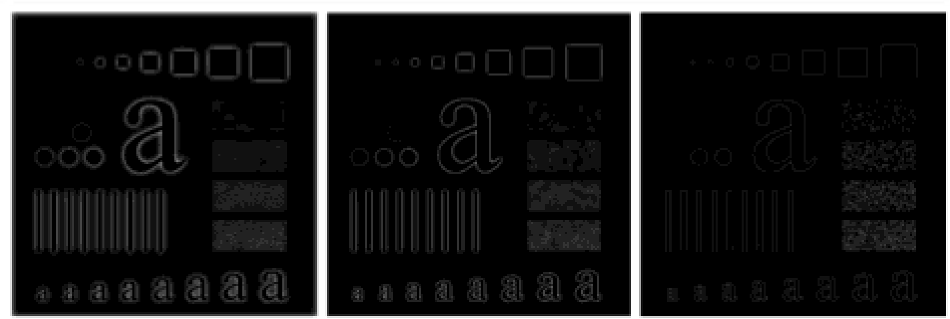
Same ringing artifacts as ideal lowpass

a b c

**FIGURE 4.54** Results of highpass filtering the image in Fig. 4.41(a) using an IHPF with $D_0 = 30$, 60, and 160.

a b c

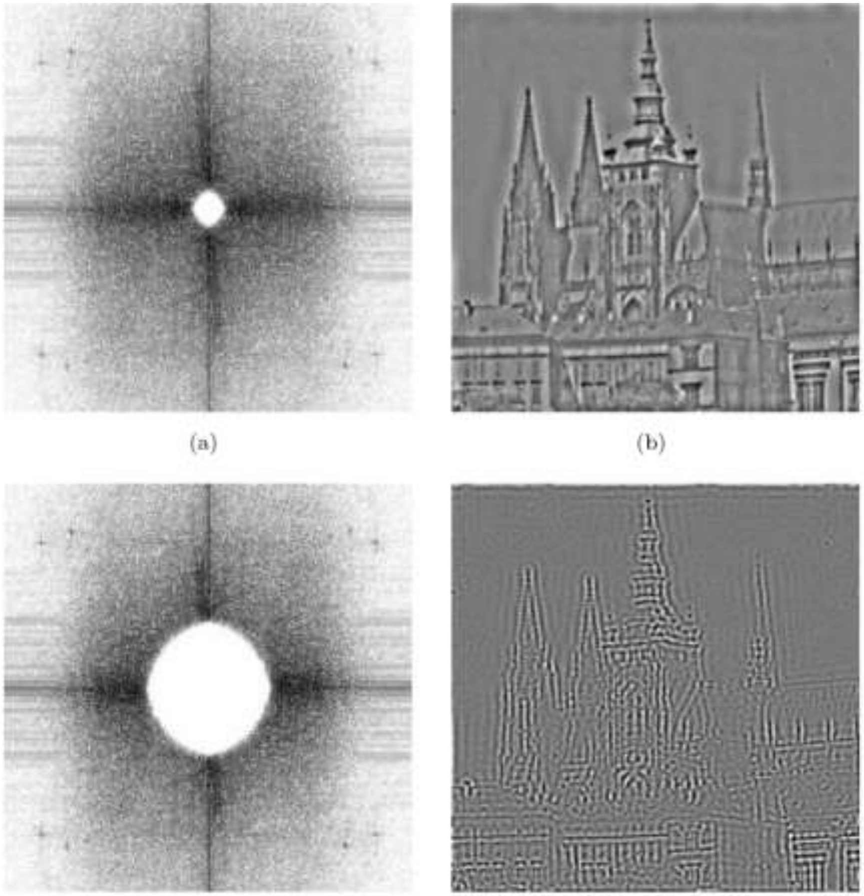
**FIGURE 4.55** Results of highpass filtering the image in Fig. 4.41(a) using a BHPF of order 2 with $D_0 = 30$, 60, and 160, corresponding to the circles in Fig. 4.41(b). These results are much smoother than those obtained with an IHPF.



a b c

**FIGURE 4.56** Results of highpass filtering the image in Fig. 4.41(a) using a GHPF with $D_0 = 30$, 60, and 160, corresponding to the circles in Fig. 4.41(b). Compare with Figs. 4.54 and 4.55.

# HP Spectrum View

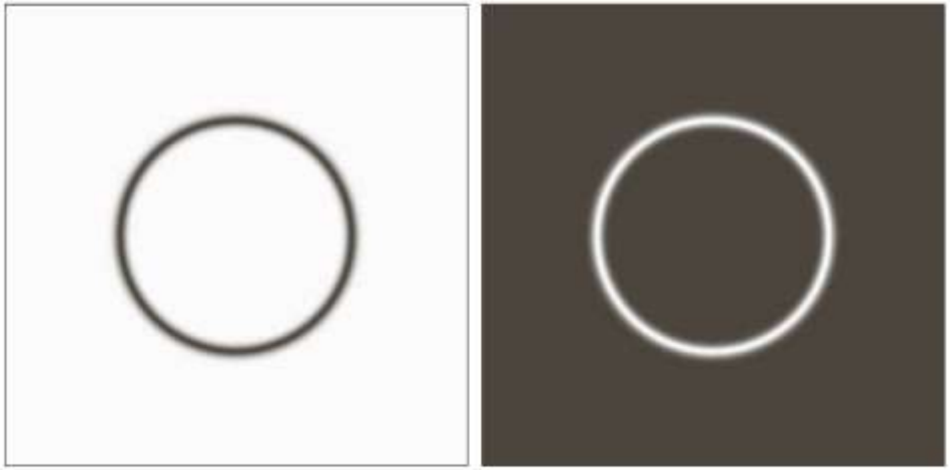


(a) (b) (c) (d)

**Figure 5.26**: High-pass frequency domain filtering. (a) Spectrum of a high-pass filtered image, only very low frequencies filtered out. (b) Image resulting from the inverse Fourier transform applied to spectrum (a). (c) Spectrum of a high-pass filtered image, all lower frequencies filtered out. (d) Inverse Fourier transform applied to spectrum (c). 

# Selective Filtering

- Bandpass/reject – operate on a ring in the frequency spectrum
  - See Table 4.6 for definitions

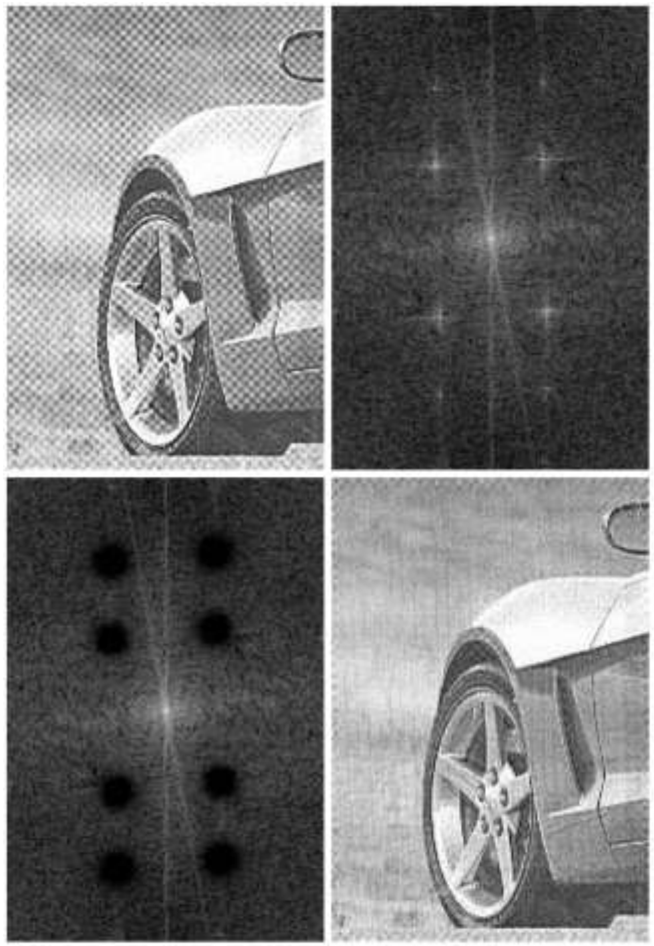


a b

**FIGURE 4.63** (a) Bandreject Gaussian filter. (b) Corresponding bandpass filter. The thin black border in (a) was added for clarity; it is not part of the data.

- Notch filters – operate on specific regions in the frequency spectrum
  - Move center of HP filter appropriately

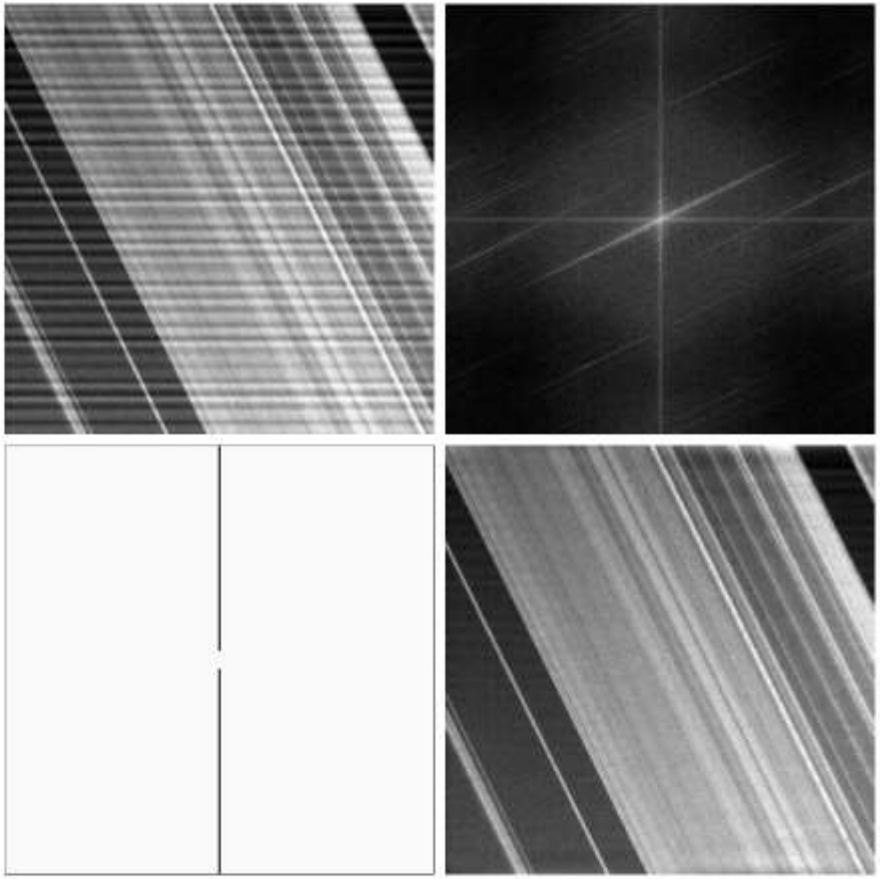
# Notch Examples



a b
c d

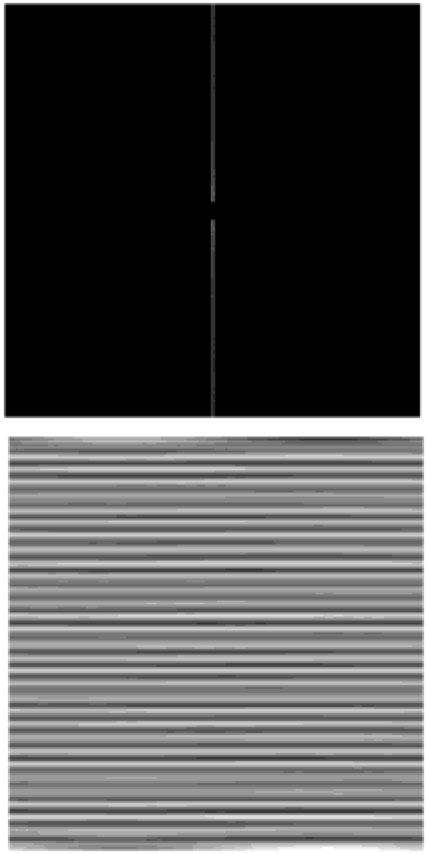
**FIGURE 4.64** (a) Sampled newspaper image showing a moiré pattern. (b) Spectrum. (c) Butterworth notch reject filter multiplied by the Fourier transform. (d) Filtered image.

# Notch Examples II



a b
c d

**FIGURE 4.65** (a) 674 × 674 image of the Saturn rings showing nearly periodic interference. (b) Spectrum: The bursts of energy in the vertical axis near the origin correspond to the interference pattern. (c) A vertical notch reject filter. (d) Result of filtering. The thin black border in (c) was added for clarity; it is not part of the data. (Original image courtesy of Dr. Robert A. West, NASA/JPL.)

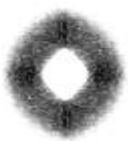


a b

**FIGURE 4.66** (a) Result (spectrum) of applying a notch pass filter to the DFT of Fig. 4.65(a). (b) Spatial pattern obtained by computing the IDFT of (a).

# BP Spectrum View

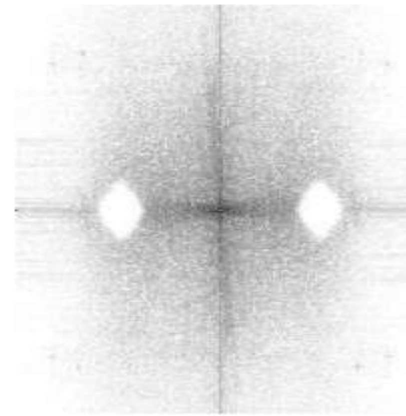
**Figure 5.27**: Band-pass frequency domain filtering. (a) Spectrum of a band-pass-filtered image, low and high frequencies filtered out. (b) Image resulting from the inverse Fourier transform applied to spectrum (a). © *Cengage Learning 2015.*



(a)

(b)

(a)



(b)

(c)

**Figure 5.28**: Periodic noise removal. (a) Noisy image. (b) Image spectrum used for image reconstruction—note that the areas of frequencies corresponding with periodic vertical lines are filtered out. (c) Filtered image. © *Cengage Learning 2015.*

# Implementation Issues

- DFT is separable
  - Can compute first a 1D DFT over rows followed by the 1D DFT over columns
  - Simplifies computations in 1D
- Practically use Fast Fourier Transform (FFT) to computer all DFT
  - Computationally efficient algorithm that simplifies problem by halving sequence repeatedly
  - Efficiency requires $M$ and $N$ (size of image) to be multiples of 2