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ECG782: Multidimensional Digital Signal Processing

Morphology

Outline

- Mathematical Morphology
- Erosion/Dilation
- Opening/Closing
- Grayscale Morphology
- Morphological Operations
- Connected Components

Morphological Image Processing

- Filtering done on binary images
  - Images with two values [0,1], [0, 255], [black,white]
  - Typically, this image will be obtained by thresholding
    - $g(x,y) = \begin{cases} 1 & f(x,y) > T \\ 0 & f(x,y) \le T \end{cases}$
- Morphology is concerned with the structure and shape
- In morphology, a binary image is filtered with a structuring element *s* and results in a binary image

- Matlab Notes
  - http://www.mathworks.com/help/images/pixel-values-and-image-statistics.html

Mathematical Morphology

- Tool for image simplification while maintaining shape characteristics of objects
  - Image pre-processing
    - Noise filtering, shape simplification
  - Enhancing object structure
    - Skeletonizing, thinning, thickening, convex hull
  - Segmenting objects from background
  - Quantitative description of objects
    - Area, perimeter, moments

Set Representation for Binary Images

- The language of mathematical morphology is set theory
  - A set represents an object in an image
- Example
  - $X = \begin{Bmatrix} (1,0), (1,1), (1,2), \\ (2,2), (0,3), (0,4) \end{Bmatrix}$

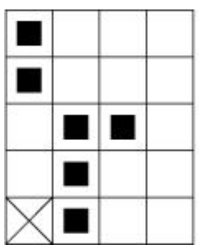


Figure 13.1: A point set example. *Learning 2015.*

- Morphological transformation Ψ
  - Relationship between image X and structuring element B
  - Structuring element B is expressed with respect to a local origin O

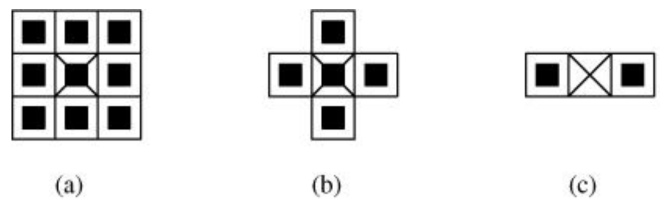
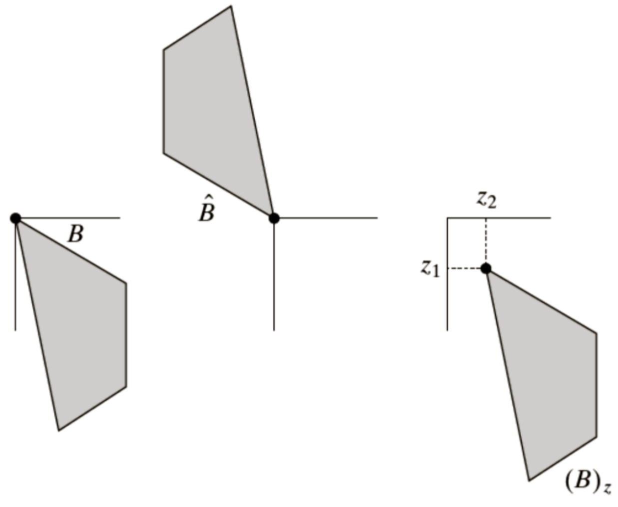


Figure 13.2: © *Cengage Le*

- Relationship computed as B is moved across the image in a raster scan
  - Similar to filtering but with zero/one output
  - Current pixel corresponds to O

Set Reflection/Translation

- Reflection
  - $\hat{B} = \{w | w = -b, \text{for } b \in B\}$
  - Negative of coordinates (flip across origin)
- Translation
  - $(B)_z = \{c | c = b + z, \text{for } b \in B\}$
  - Shift of set



a b c

FIGURE 9.1 (a) A set, (b) its reflection, and (c) its translation by z.

Structuring Elements

- The structuring element (SE) can be any shape
  - This is a mask of “on” pixels within a rectangular container
  - Typically, the SE is symmetric

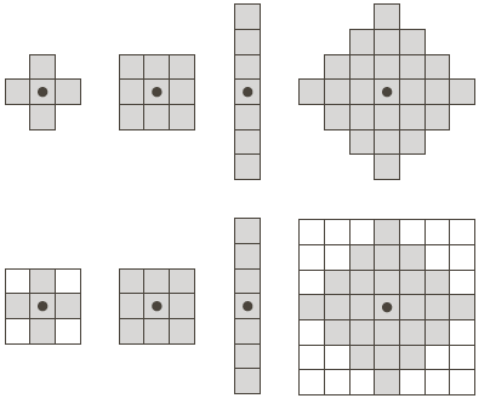


FIGURE 9.2 First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.

Binary Image Logic Operations

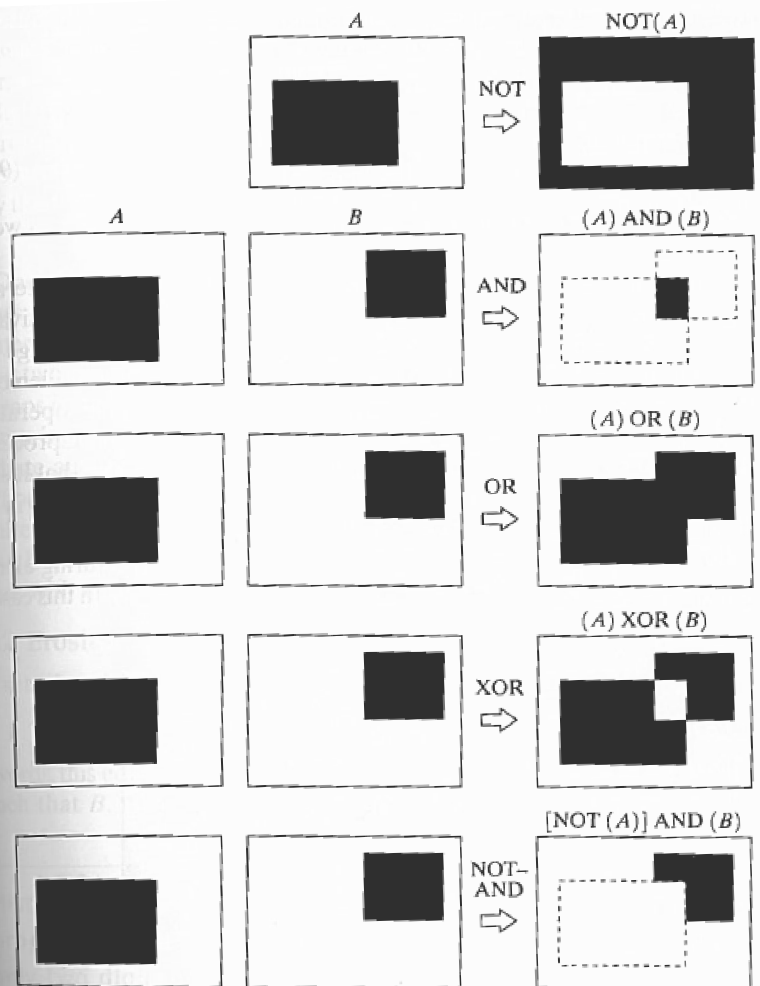


FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

- Simple operations
  - Does not require structuring element or raster scan
- Extension of basic logic operators
  - NOT, AND, OR, XOR
- Often use for “masking”

Basic Morphology Operations

- Erosion
- Dilation
- Opening
- Closing

Dilation

- Morphological combination of two sets using vector addition
  - $X \oplus B = \{p \in E^2 : p = x + b, x \in X, b \in B\}$

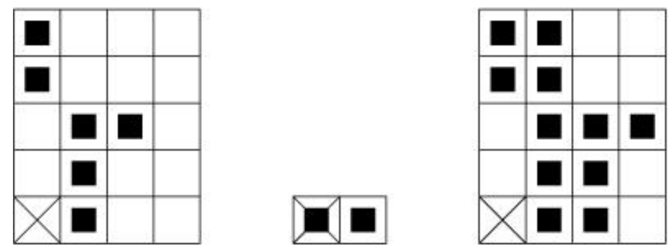
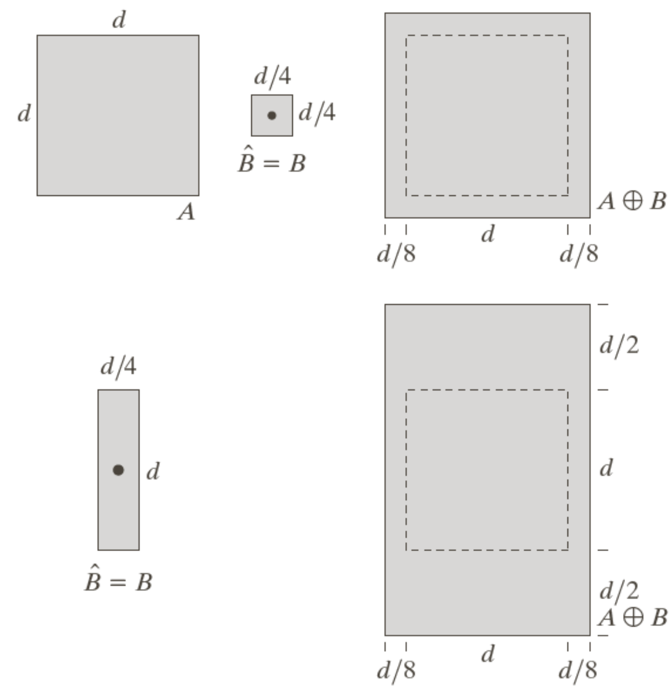


Figure 13.4: *Learning 2015*

- Output image is "on" anywhere the SE touches an "on" pixel
  - $A \oplus B = \{z | (\hat{B})_z \cap A \neq \emptyset\}$
    - A – image
    - B – SE
    - z – displacements (x,y locations)



a b c
d e

FIGURE 9.6
(a) Set A.
(b) Square structuring element (the dot denotes the origin).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element. (e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference

Dilation Properties

- Use of 3×3 SE is an isotropic expansion
  - Called fill or grow operation

Figure 13.5: Dilation as isotropic expansion. © *Cengage Learning 2015.*

- Commutative and associative
  - $X \oplus B = B \oplus X$ $\quad$ $X \oplus (B \oplus D) = (X \oplus B) \oplus D$
- Can be used to fill small holes and gulfs in objects
  - Increases size of an object
- Not an invertible operation

Erosion

- Combine two sets using vector subtraction
  - $X \ominus B = \{p \in E^2 : p = x + b,$ $b \in B\}$

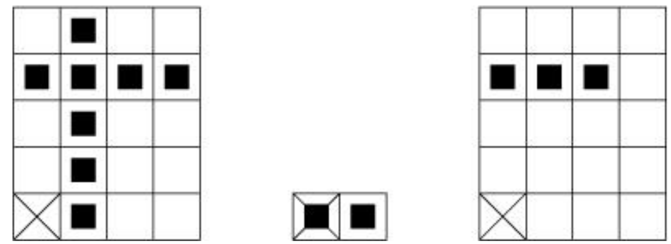


Figure 13.7: © Cengage Le

- Retain only pixels where the entire SE is overlapped
  - $A \ominus B = \{z | (B)_z \subseteq A\}$
    - A – image
    - B – SE
    - z – displacements (x,y locations)

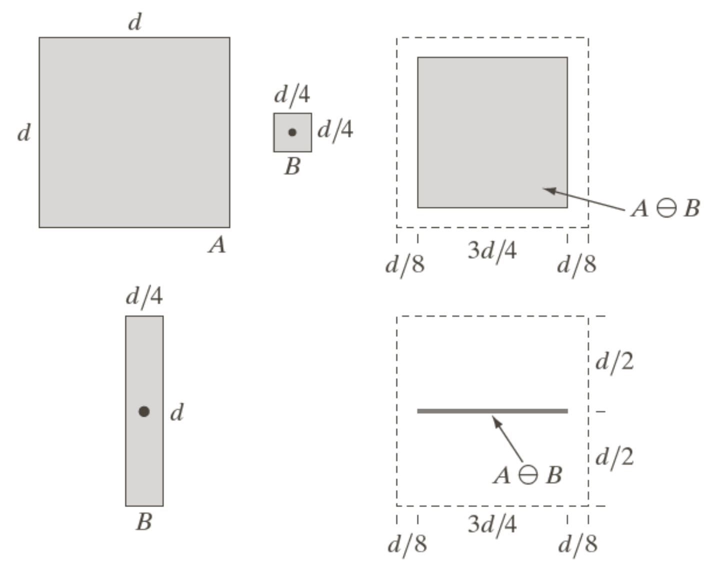


FIGURE 9.4 (a) Set A. (b) Square structuring element, B. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A by B using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference.

- Not an invertible operation

Erosion Properties

- Use of 3 × 3 SE is an isotropic reduction
  - Called shrink or reduce operation
- Dual operation for dilation

Figure 13.8: shrink. © *Cer*

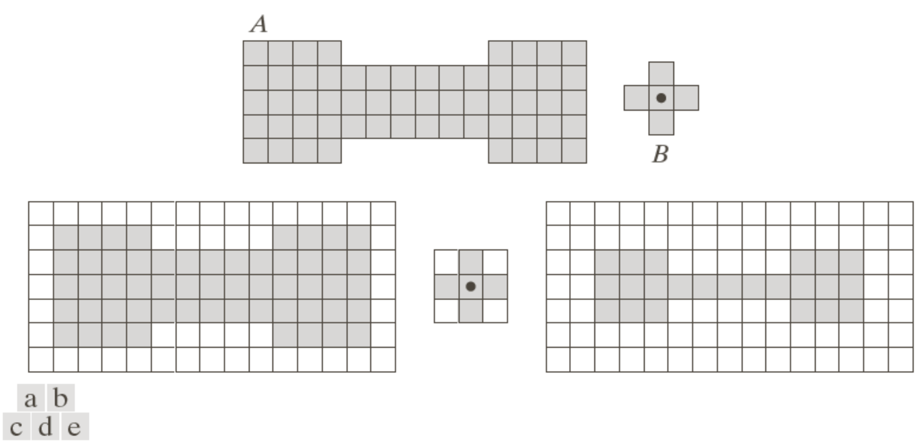


FIGURE 9.3 (a) A set (each shaded square is a member of the set). (b) A structuring element. (c) The set padded with background elements to form a rectangular array and provide a background border. (d) Structuring element as a rectangular array. (e) Set processed by the structuring element.

- Can be used to get contours
  - Subtract erosion from original

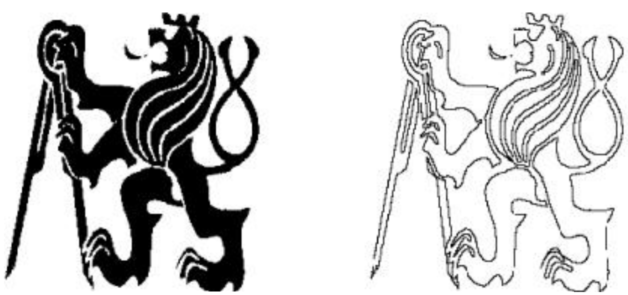


Figure 13.9: Contours obtained by subtraction of an eroded image from an original (left). © *Cengage Learning 2015.*

Opening and Closing

- Opening
- Erosion followed by dilation (note they are not inverses)
  - $X \circ B = (X \ominus B) \oplus B$
- Simplified, less detailed version
- Removes small objects
- Retains "size"
- Idempotent
  - Repeated application does not change results

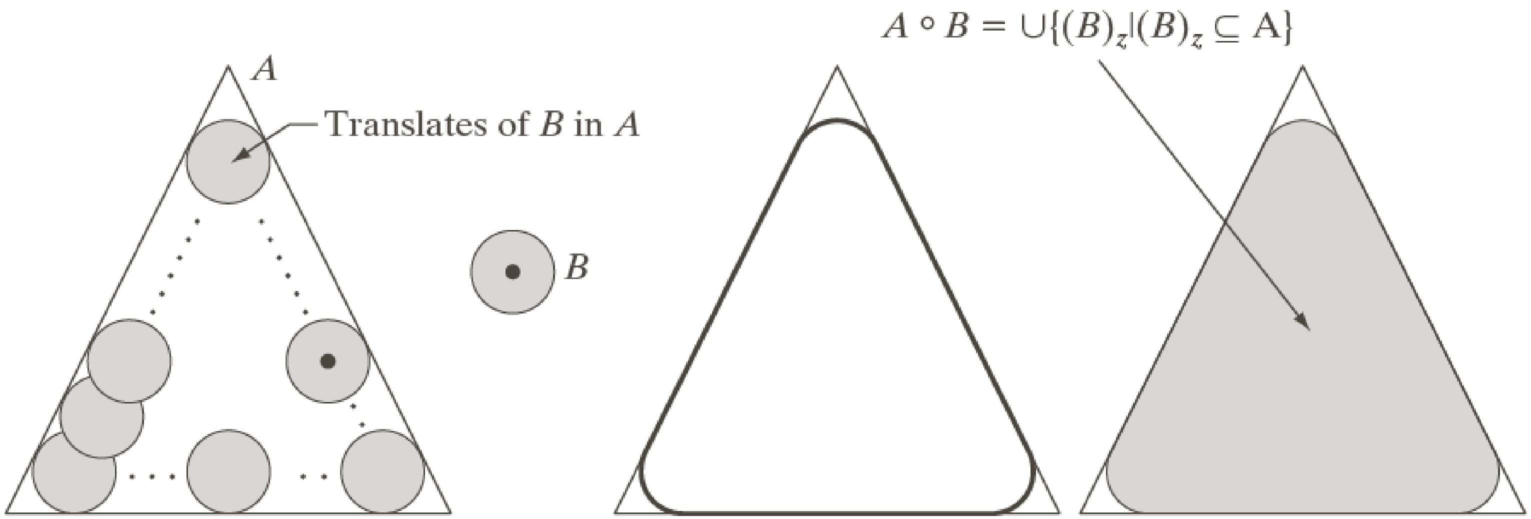
Figure 13.10:
on the left). ©

- Closing
- Dilation followed by erosion
  - $X \cdot B = (X \oplus B) \ominus B$
- Connects objects that are close
- Fills small holes (gulfs)
- Smooths object outline
- Retains "size"
- Idempotent

Figure 13.11:
on the left). ©

Opening

- All pixels that fit inside when the SE is "rolled" on the inside of a boundary
  - $A \circ B = \cup \{(B)_z | (B)_z \subseteq A\}$



a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade A in (a) for clarity.

Closing

- All pixels that fit inside when the SE is "rolled" on the outside of a boundary

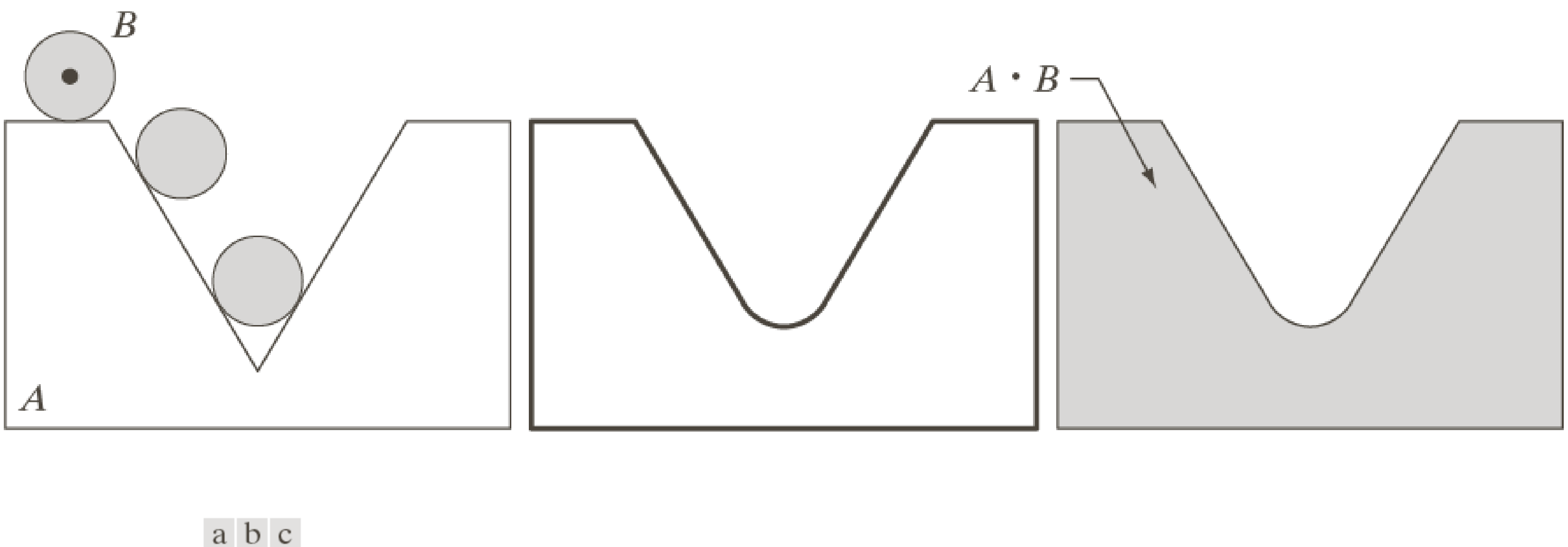


FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade A in (a) for clarity.

Examples

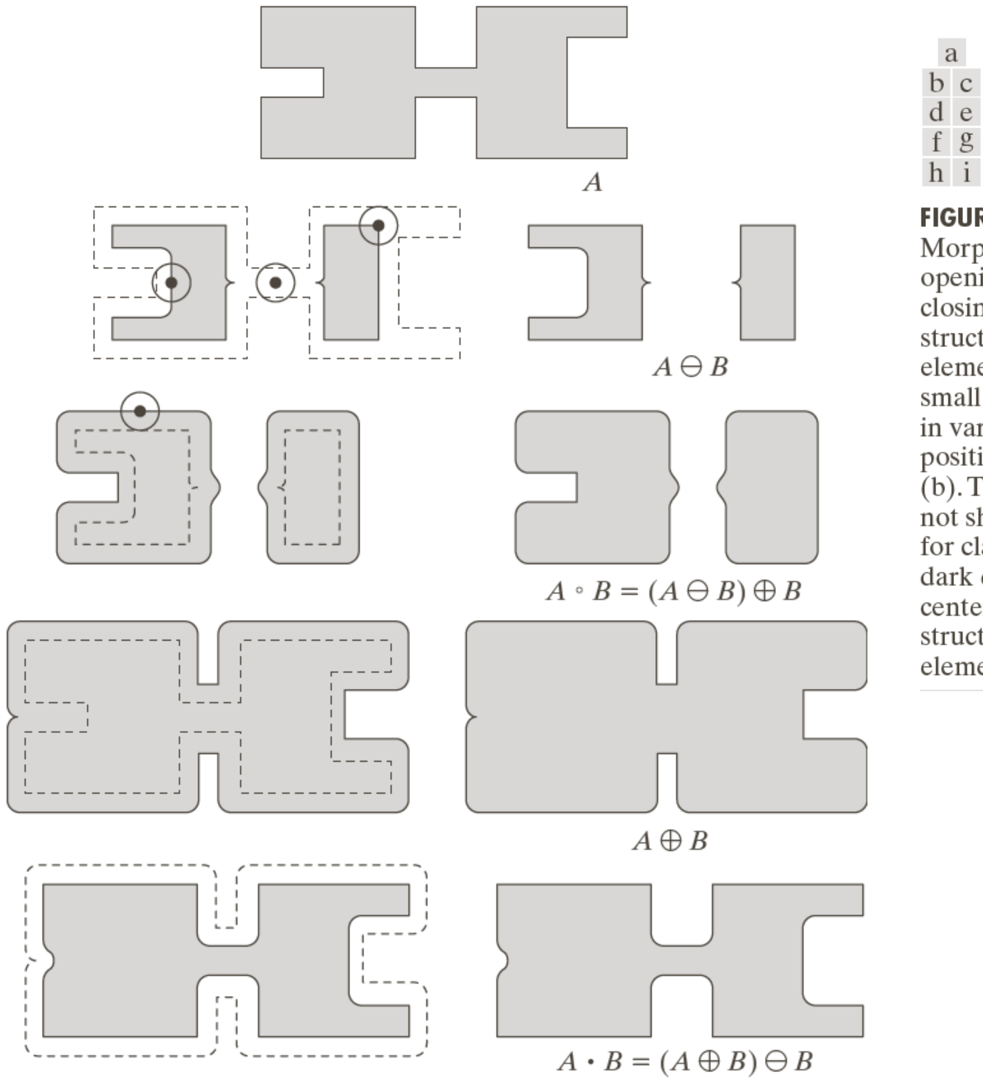


FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.

Examples II

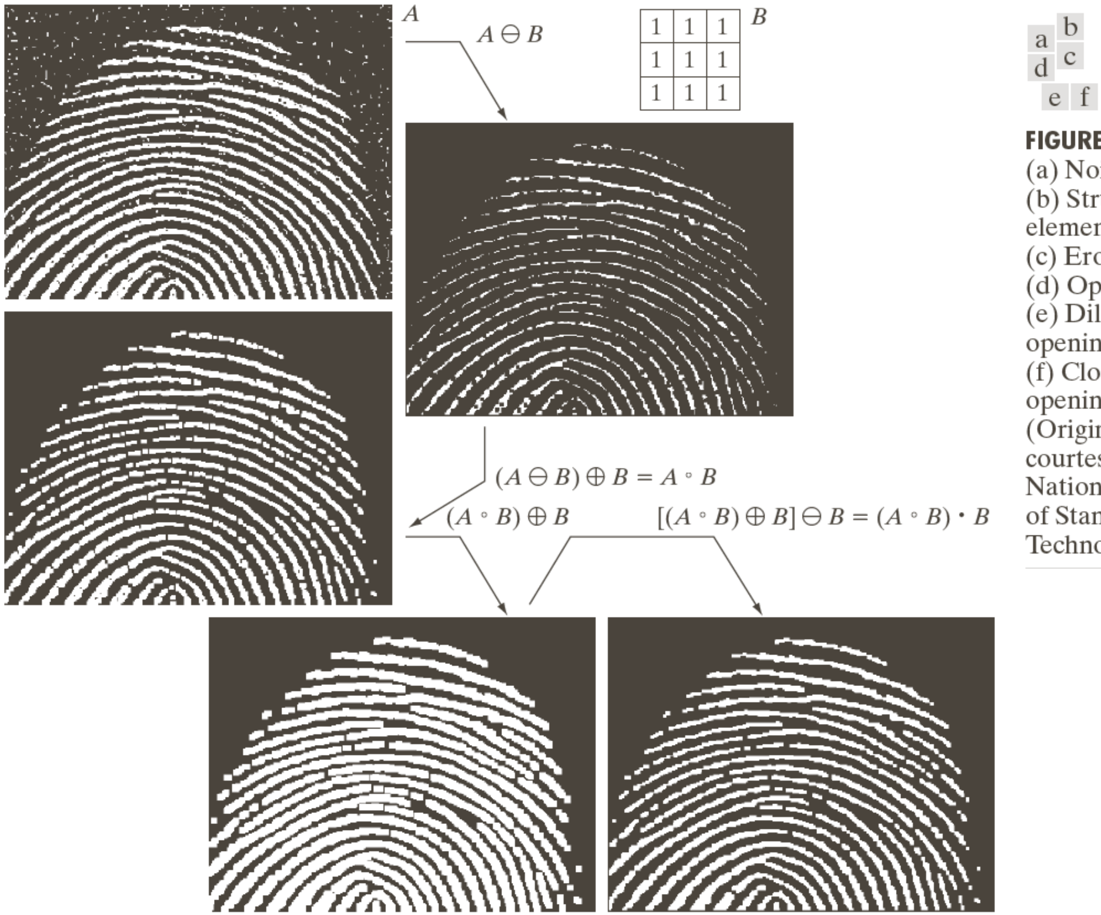


FIGURE 9.11
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of *A*.
(e) Dilation of the opening.
(f) Closing of the opening.
(Original image courtesy of the National Institute of Standards and Technology.)

Grayscale Morphology

- Can extent binary morphology to grayscale images
  - Min operation – erosion
  - Max operation – dilation
- The structuring element not only specifies the neighborhood relationship
- It specifies the local intensity property
- Must consider image as a surface in 2D plane

Top Surface and Umbra

- Top surface is the highest intensity in a set
  - $T[A](x) = \max\{y, (x, y) \in A\}$

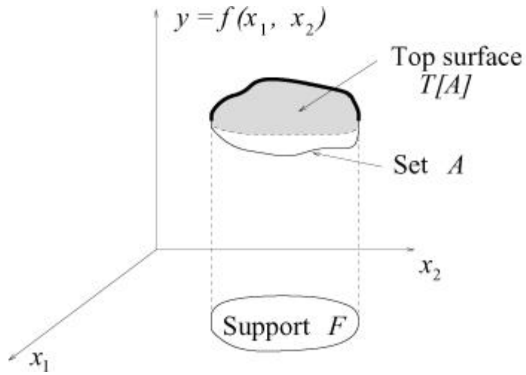


Figure 13.12: Top surface of the set A corresponds to maximal values of the function $f(x_1, x_2)$. © *Cengage Learning 2015.*

- Umbra is the “shadow” points below top surface
  - $U[f] = \{(x, y) \in F \times E, y \leq f(x)\}$

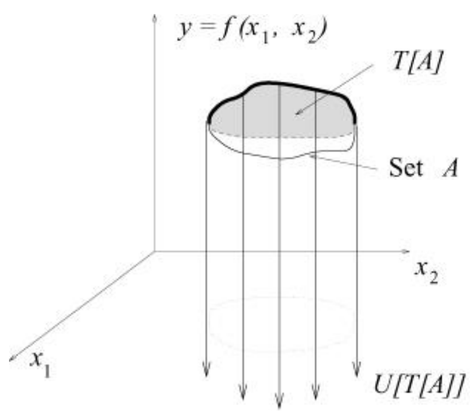


Figure 13.13: Umbra of the top surface of a set is the whole subspace below it. © *Cengage Learning 2015.*

Grayscale Morphology Definitions

- Dilation - Top surface of dilation of umbras
  - $f \oplus k = T\{U[f] \oplus U[k]\}$
    - Left-side is grayscale dilation
    - Right-side is binary dilation
- Erosion
  - $f \ominus k = T\{U[f] \ominus U[k]\}$

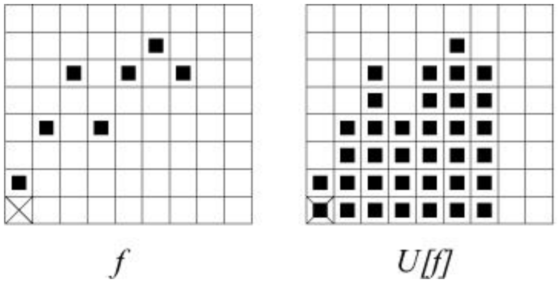


Figure 13.14: (left) and its Learning 2015.

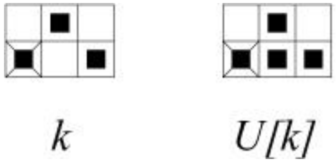


Figure 13.15: A its umbra (right)

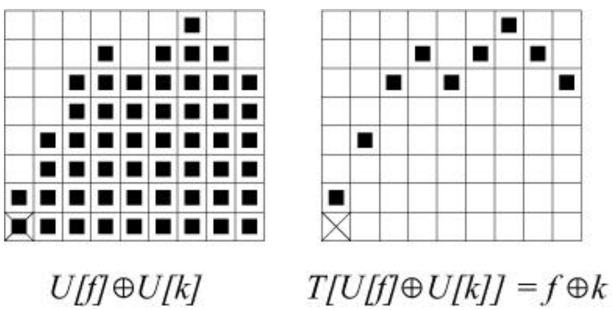


Figure 13.16: 1D example of gray-scale dilation. The umbras of the 1D function f and structuring element k are dilated first, $U[f] \oplus U[k]$. The top surface of this dilated set gives the result, $f \oplus k = T\left[U[f] \oplus U[k]\right]$. © *Cengage Learning 2015.*

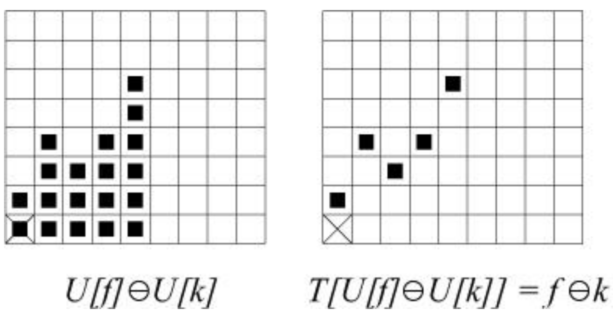


Figure 13.17: 1D example of gray-scale erosion. The umbras of 1D function f and structuring element k are eroded first, $U[f] \ominus U[k]$. The top surface of this eroded set gives the result, $f \ominus k = T\left[U[f] \ominus U[k]\right]$. © *Cengage Learning 2015.*

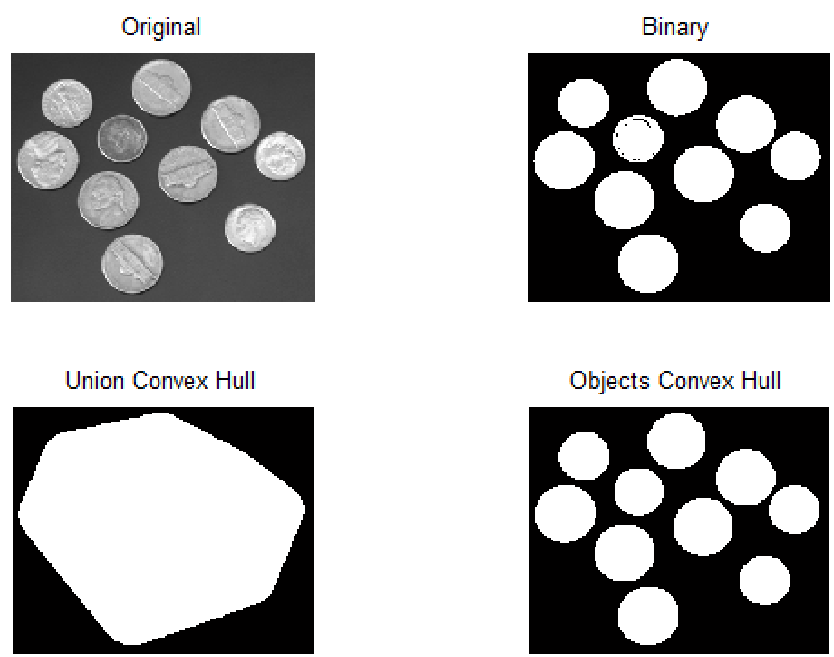
Other Morphological Operations

- Boundary extraction
  - $\beta(A) = A - (A \ominus B)$
  - Subtract erosion from original
  - Notice this is an edge extraction

a b

FIGURE 9.14 (a) A simple binary image, with 1s represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

- Convex hull (H)
  - Smallest convex set that contains another set S
  - This is often done for a collection of 2D or 3D point
  - `bwconvhull.m`



More on Morphological Operations

- Definitions from Szeliski book
- Threshold operation
  - $\theta(f,t) = \begin{cases} 1 & f \geq t \\ 0 & \text{else} \end{cases}$
- Structuring element
  - s – e.g. 3 x 3 box filter (1's indicate included pixels in the mask)
  - S – number of "on" pixels in s
- Count of 1s in a structuring element
  - $c = f \otimes s$
  - Correlation (filter) raster scan procedure
- Basic morphological operations can be extended to grayscale images

- Dilation
  - $\text{dilate}(f,s) = \theta(c,1)$
  - Grows (thickens) 1 locations
- Erosion
  - $\text{erode}(f,s) = \theta(c,S)$
  - Shrink (thins) 1 locations
- Opening
  - $\text{open}(f,s) = \text{dilate}(\text{erode}(f,s),s)$
  - Generally smooth the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions
- Closing
  - $\text{close}(f,s) = \text{erode}(\text{dilate}(f,s),s)$
  - Generally smooth the contour of an object, fuses narrow breaks/separations, eliminates small holes, and fills gaps in a contour

Morphology Example

Note: Black is “1” location

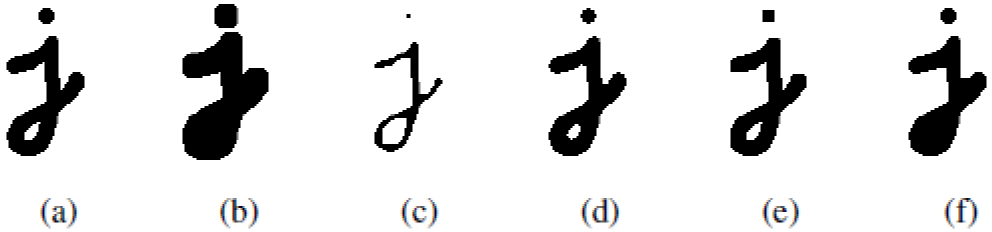
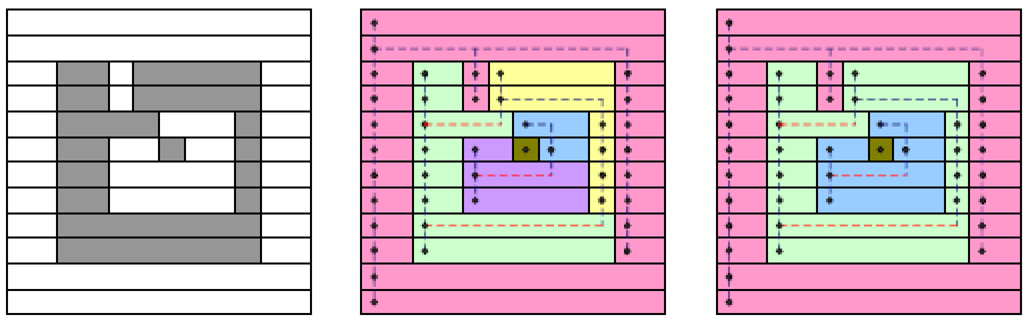


Figure 3.21 Binary image morphology: (a) original image; (b) dilation; (c) erosion; (d) majority; (e) opening; (f) closing. The structuring element for all examples is a 5×5 square. The effects of majority are a subtle rounding of sharp corners. Opening fails to eliminate the dot, since it is not wide enough.

- Dilation - grows (thickens) 1 locations
- Erosion - shrink (thins) 1 locations
- Opening - generally smooth the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions
- Closing - generally smooth the contour of an object, fuses narrow breaks/separations, eliminates small holes, and fills gaps in a contour

Connected Components

- Semi-global image operation to provide consistent labels to similar regions
  - Based on adjacency concept
- Most efficient algorithms compute in two passes



(a) (b) (c)

Figure 3.23 Connected component computation: (a) original grayscale image; (b) horizontal runs (nodes) connected by vertical (graph) edges (dashed blue)—runs are pseudocolored with unique colors inherited from parent nodes; (c) re-coloring after merging adjacent segments.

- More computational formulations (iterative) exist from morphology
  - $X_k = (X_{k-1} \oplus B) \cap A$

Set

Connected component

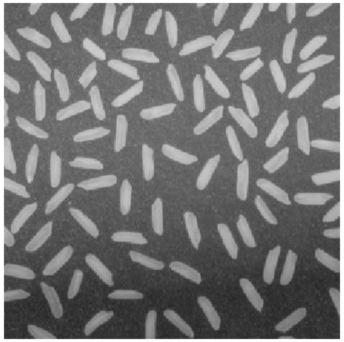
Structuring element

More Connected Components

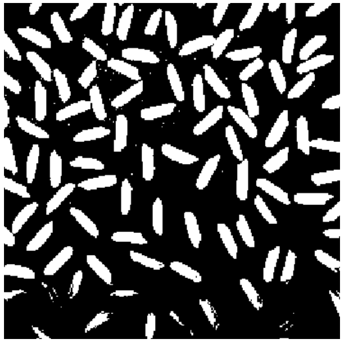
- Typically, only the "white" pixels will be considered objects
  - Dark pixels are background and do not get counted
- After labeling connected components, statistics from each region can be computed
  - Statistics describe the region – e.g. area, centroid, perimeter, etc.

- Matlab functions
  - `bwconncomp.m`, `labelmatrix.m (bwlabel.m)`- label image
  - `label2rgb.m` – color components for viewing
  - `regionprops.m` – calculate region statistics

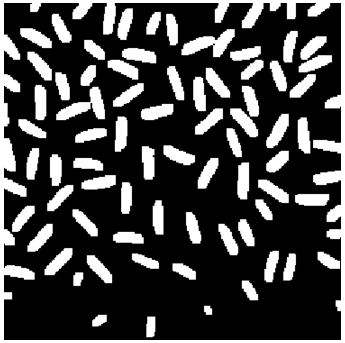
Connected Component Example



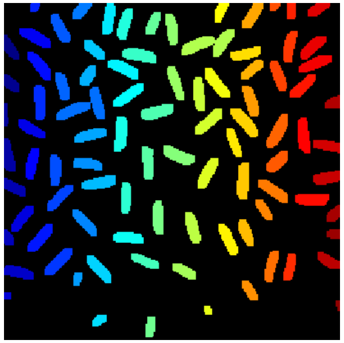
Grayscale image



Threshold image



Opened Image



Labeled image – 91 grains of rice