Homework #1
Due Th. 9/06

1. (OS 2.22)
   Consider a discrete-time LTI system with impulse response $h[n]$. If the input $x[n]$ is a periodic sequence with period N (i.e. if $x[n] = x[n+N]$), show that the output $y[n]$ is also a periodic sequence with period N.

2. (OS 2.23 a,b,c) + additional systems
   For each of the following systems, determine whether the system is (1) stable, (2) causal, (3) linear, and (4) time-invariant.

   (a) $T(x[n]) = (\cos \pi n)x[n]$
   (b) $T(x[n]) = x[n^2]$
   (c) $T(x[n]) = x[n] \sum_{k=0}^{\infty} \delta[n-k]$
   (d) $T(x[n]) = e^{x[n]}$
   (e) $T(x[n]) = ax[n] + b$

3. (OS 2.31)
   If the input and output of a causal LTI system satisfy the difference equation

   $$y[n] = ay[n-1] + x[n],$$

   then the impulse response of the system must be $h[n] = a^n u[n]$.

   (a) For what values of a is this system stable?
   (b) Consider a causal LTI system for which the input and output are related by the difference equation

   $$y[n] = ay[n-1] + x[n] - a^N x[n-N],$$

   where N is a positive integer. Determine and sketch the impulse response of this system. *Hint*: Use linearity and time-invariance to simplify the solution.
   (c) Is the system in part (b) and FIR or an IIR system? Explain.
   (d) For what values of a is the system in part (b) stable? Explain.

4. (OS 2.34 a,b,d)
   An LTI system has the frequency response

   $$H(e^{j\omega}) = \frac{1 - 1.25e^{-j\omega}}{1 - 0.8e^{-j\omega}} = 1 - \frac{0.45e^{-j\omega}}{1 - 0.8e^{-j\omega}}.$$

   (a) Specify the difference equation that is satisfied by the input $x[n]$ and output $y[n]$.
   (b) Use one of the above forms of the frequency response to determine the impulse response $h[n]$.
   (d) If the input to the above system is $x[n] = \cos(0.2\pi n)$, the output should be of the form $y[n] = A\cos(0.2\pi n + \theta)$. What are A and θ.

5. (OS 2.74)
   The overall system in the dotted box in Figure P2.74 can be shown to be linear and time-invariant.

(a) Determine an expression for $H(e^{j\omega})$, the frequency response of the overall system from the input $x[n]$ to the output $y[n]$, in terms of $H_1(e^{j\omega})$, the frequency response of the internal LTI system. Remember that $(-1)^n = e^{j\pi n}$.

(b) Plot $H(e^{j\omega})$ for the case when the frequency response of the internal LTI system is

$$H_1(e^{j\omega}) = \begin{cases} 1, & |\omega| < \omega_c \\ 0, & \omega_c < |\omega| \leq \pi \end{cases}$$

6. (a) Prove that

$$\sum_{n=0}^{N} a^n = \frac{1 - a^{N+1}}{1 - a}$$

(b) Prove that, for $|a| < 1$,

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1 - a}$$

(c) Find a closed form expressed for

$$\sum_{n=N_1}^{N_2} a^n$$

for any $0 < N_1, N_2 < \infty$.

7. Use the convolution sum formula to find $y[n] = h[n] * x[n]$ for

$$h[n] = \begin{cases} 1 & n \geq -3 \\ 3^n & n < -3 \end{cases} \qquad x[n] = \begin{cases} (1/3)^n & n \geq 3 \\ 3^n & n < 3 \end{cases}$$