Midterm 02 Review

Administrative

- Exam Thursday 3/29 in class
- 1 page of double-sided notes (Tables 4.2 and 5.2 will be provided)
- No calculators
- Emphasis on Fourier Transform (Chapter 4: CTFT, Chapter 5: DTFT)

Fourier Transform

The Fourier Transform is the aperiodic extension of the Fourier Series.

- would like to represent an aperiodic signal as a combination of exponentials
- in the FT, the exponentials are not generally harmonically related but encompass all frequencies

Fourier Series Review

The following highlights the equations for Fourier Series analysis. Notice there is duality in the discrete-time FS.

	Continuous Time	Discrete Time
Synthesis	$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$	$x[n] = \sum_{k=<N>} a_k e^{jk\omega_0 n}$
Analysis	$a_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 t}$	$a_k = \frac{1}{N}\sum_{n=<N>} x[n] e^{-jk\omega_0 n}$

Chapter 4: Continuous Time Fourier Transform

The synthesis equation shows that a signal $x(t)$ can be represented by a linear combination of complex exponentials $e^{j\omega t}$ for all infinitesimally small ω values.

Synthesis	$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$	Inverse FT
Analysis	$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$	FT

FT of periodic signals

First fin the FS of the periodic signal, then the FT is found as impulses at the harmonic frequencies with scaling of $2\pi a_k$ at the k^{th} harmonic.

$$x(t) = \sum_{-\infty}^{\infty} a_k e^{jk\omega_0 t} \longleftrightarrow X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

FT Properties

Be sure to know the properties especially the the convolution and multiplication properties.

$$x(t) * y(t) \longleftrightarrow X(j\omega)Y(j\omega)$$
$$x(t)y(t) \longleftrightarrow \frac{1}{2\pi}X(j\omega) * Y(j\omega)$$

Linear Constant-Coefficient Differential Equations

Systems defined by differential equations can be efficiently solved using Fourier techniques. The general procedure for a differential problem are highlight in the steps below.

1. Take FT of both sides of differential equation.
2. Solve for
$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}.$$
3. Use partial fraction expansion on $H(j\omega)$ to get easy terms for inverse FT.
4. Inverse FT partial terms of $H(j\omega)$ to obtain $h(t)$.
5. Given an input signal $x(t)$, find the FT $X(j\omega) = \mathcal{F}\{x(t)\}$.
6. Find the output response
$$Y(j\omega) = H(j\omega)X(j\omega)$$
and do partial fractions.
7. Take iFT to obtain $y(t)$.

Chapter 5: Discrete Time Fourier Transfer

This is basically the same idea as the CTFT, however there are some small differences.

Synthesis	$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$	Inverse FT
Analysis	$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$	FT

The synthesis equation explains that only 2π frequencies exist in the DTFT. This results in a loss of the duality relationship we was between FT and iFT in the continuous case. The DTFT is periodic with period 2π,

$$X(e^{j(\omega+2\pi)}) = X(e^{j\omega})$$

therefore the spectrum is repeated every 2π interval (this is the same as the DTFS case).
The periodic nature of the DTFT results in slight modifications of the FT properties we saw in the CTFT. In particular

$$\begin{aligned} x[n] * y[n] &\longleftrightarrow X(e^{j\omega})Y(e^{j\omega}) \\ x[n]y[n] &\longleftrightarrow \underbrace{\frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})Y(e^{j(\theta-\omega)})d\theta}_{\text{periodic convolution}} \end{aligned}$$

The linear constant-coefficient difference equation is the DT analogy to the CT differential equation. The difference equation is handled in the same fashion, however, partial fraction expansion in the CTFT case had polynomials in $j\omega$ while the DTFT has polynomials in $e^{j\omega}$.