Logistics

- **Thursday Oct. 4\textsuperscript{th}**
  - In normal lecture (13:00-14:15)
  - 1 hour and 15 minutes
- **Chapters 1-2.6**
- Closed book, closed notes
- No calculators
- Must show work and be legible for credit

- Boolean Axioms and Theorems will be provided
Preparation

• Read the book (2\textsuperscript{nd} Edition)
  ▫ Then, read it again
• Do example problems
  ▫ Use both Harris and Roth books
• Be sure you understand homework solutions

• Come visit during office hours for questions
Chapter 1.2 Managing Complexity

• Abstraction – hiding details that aren’t important

• Digital discipline – restricting design choices to digital logic for more simple design

• Hierarchy – dividing a system into modules and further submodules for easier understanding

• Modularity – modules have well-defined functions and interfaces for easy interconnection

• Regularity – uniformity among modules for reuse
Chapter 1.3 Digital Abstraction

- Analog $\rightarrow$ digital computing

- Information in a discrete variable
  - $D = \log_2 N$ bits

- Introduction to binary variables

- Example 1: Information in 9-state variable
  - $D = \log_2 9 = 3.1699$ bits
    - Note 3 bits can represent 8 values so requires just more than 3 bits
Chapter 1.4 - Number Systems

• Number representation
  ▫ N-digit number \( \{a_{N-1} a_{N-2} \ldots a_1 a_0\} \) of base \( R \) in decimal
    • \( a_{N-1} R^{N-1} + a_{N-2} R^{N-2} + \ldots + a_1 R^1 + a_0 R^0 \)
    • \( = \sum_{i=0}^{N-1} a_i R^i \)
  ▫ Range of values

• Base 2, 10, 16, etc. conversion
  ▫ Often from base \( R_0 \) to decimal to \( R_1 \)
  ▫ Two methods:
    • Repeatedly remove largest power of 2
    • Repeatedly divide by two
Number Examples

• Convert $10110_2$ to decimal

• Convert $10110_2$ to base 5

• Convert $10110_2$ to hex and octal
Chapter 1.4.5 - Binary Addition

- Signed number representation
  - Unsigned, two’s complement, sign-magnitude

- Addition
  - Binary carries
  - Potential for overflow

- Subtraction
  - Find negative of number and add

- Zero/Sign extension
Example Binary Addition

- Assume 6-bit 2’s complement and indicate if overflow occurs
- Add $13_{10} + 11_{10}$

- Add $21_{10} + 11_{10}$

- Add $-25_{10} + 18_{10}$

- Add $-12 + 13$
Chapter 1.5 - Logic Gates

- **NOT, BUF**

  **NOT**
  
  ![Not Gate Diagram]
  
  $$Y = \overline{A}$$
  
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  **BUF**
  
  ![Buffer Gate Diagram]
  
  $$Y = A$$
  
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- **AND, OR**

  **AND**
  
  ![And Gate Diagram]
  
  $$Y = AB$$
  
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  **OR**
  
  ![Or Gate Diagram]
  
  $$Y = A + B$$
  
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- **XOR, NAND**

  **XOR**
  
  ![Xor Gate Diagram]
  
  $$Y = A \oplus B$$
  
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  **NAND**
  
  ![Nand Gate Diagram]
  
  $$Y = \overline{A \cdot B}$$
  
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- **NOR, XNOR**

  **NOR**
  
  ![Nor Gate Diagram]
  
  $$Y = \overline{A + B}$$
  
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  **XNOR**
  
  ![Xnor Gate Diagram]
  
  $$Y = \overline{A \oplus B}$$
  
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Example

- Give truth table for logic gate
Chapter 1.6 Beneath Digital Abstraction

- Noise margins

\[
NM_H = V_{OH} - V_{IH}
\]
\[
NM_L = V_{IL} - V_{OL}
\]
Example 1.18

• What is the inverter low and high noise margins

\[ V_{DD} = 5, V_{IL} = 1.35, V_{IH} = 3.15, V_{OL} = 0.33, V_{OH} = 3.84 \]
Chapter 1.7 - Transistors

- Voltage controlled switch
  - NMOS – pass 0’s
    - Connect to GND
  - PMOS – pass 1’s
    - Connect to VDD

- CMOS logic gates

![Diagram of CMOS logic gates]

\[ g = 0 \quad \text{OFF} \]
\[ g = 1 \quad \text{ON} \]
Example

- Give the truth table and function
Chapter 1.7 - Power Consumption

- Two types of power consumption
- Dynamic – power required to charge gate capacitances (turn on/off transistor switches)

\[ P_{\text{dynamic}} = \frac{1}{2}CV_{DD}^2f \]

- Static – power consumed when no gates switching

\[ P_{\text{static}} = I_{DD}V_{DD} \]
Chapter 2.2 - Boolean Equations

• Terms: variable/complement, literal, product/implicant

• Order of operations: NOT $\rightarrow$ AND $\rightarrow$ OR

• Sum-of-product (SOP) form
  ▫ Determined by minterms of truth table

• Product-of-sums (POS) form
  ▫ Determined by maxterms of truth table
Chapter 2.3 - Boolean Algebra

• Boolean algebra is very much like our normal algebra

• Need to know Boolean Axioms and Theorems
  ▫ Distributivity, covering, De Morgan’s

• Proving equations
  ▫ Perfect induction/proof by exhaustion – show truth tables match
  ▫ Simplification – use theorems/axioms to show both sides of equation are equal
Chapter 2.3.5 - Simplifying Equations

- Practice, practice, practice
Chapter 2.4 - Logic to Gates

- Schematic diagram of digital circuit

Figure 2.23 Schematic of $y = \overline{A} \overline{B} \overline{C} + \overline{A} \overline{B} \overline{C} + \overline{A} \overline{B} \overline{C}$
Chapter 2.5 - Multilevel Combinational Logic

- Convert gate level schematic into Boolean equation

- Bubble pushing – application of De Morgan’s in schematic

\[ Y = \overline{ABC} + \overline{D} \]
Chapter 2.6 - Real Circuit Issues

- Don’t cares: X
  - Truth table flexibility

- Contention: X
  - Illegal output value
  - Output could be 1 or 0 in error

- Floating: Z
  - High impedance, high Z
  - Output between 0, 1 by design

\[ \begin{array}{cc|c}
E & A & Y \\
0   & 0   & Z \\
0   & 1   & Z \\
1   & 0   & 0 \\
1   & 1   & 1 \\
\end{array} \]